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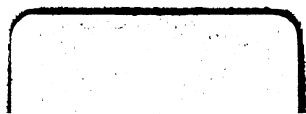
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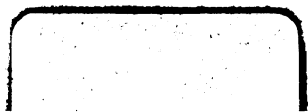
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Halsbury

1844

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~~Second~~
A

Course
of
Lectures
in

Natural Philosophy

By the late
Richard Helsham, M.D.

Professor of Physick and Natural
Philosophy in the University of Dublin.

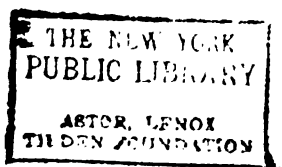
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P R E F A C E.

THAT the reader may be duly prepared for the perusal of the following Treatise, it will be necessary that he first acquaint himself with the genuine Method and Rules of Philosophizing, as they have been delivered by Sir ISAAC NEWTON.

His Method of Philosophizing is thus laid down in his *Opticks**.

“ As in mathematicks, so in natural philosophy, the investigation of difficult things by way of *Analysis*, ought ever to precede the method of composition. This *Analysis* consists in making experiments and observations, and in drawing general conclusions from them by induction, and admitting of no objections against the conclusions, but such as are taken from experiments or other certain truths. And although the arguing from experiments and observations by induction be no demonstration of general conclusions; yet it is the best way of arguing which the nature of things admits of, and may be looked upon as so much the stronger, by how much the induction

* Opt. p. 380.

“ is more general. And if no exception oc-
 “ cur from *Phænomena*, the conclusion may
 “ be pronounced generally. But if at any
 “ time afterwards, any exceptions shall occur
 “ from experiments, it may then be pro-
 “ nounced with such exceptions as shall occur.
 “ By this way of *Analysis*, we may proceed
 “ from compounds to ingredients, and from
 “ motions to the forces producing them;
 “ and in general from effects to their causes,
 “ and from particular causes to more general
 “ ones, till the argument ends in the most
 “ general. This is the method of *Analysis*;
 “ And the *Syntbesis* consists in assuming the
 “ causes discovered, and established as prin-
 “ ciples, and by them explaining the *Phæ-*
 “ *nomena* proceeding from them, and proving
 “ the explanations.”

His Rules of Philosophizing, delivered in his *Principles**, are these four.

R U L E I.

“ *More causes of natural things are not to be*
 “ *admitted, than are both true and sufficient for*
 “ *explaining their phænomena.*

“ Thus Philosophers say; nature does no-
 “ thing in vain, and in vain that is done by
 “ more causes, which can be done by fewer.
 “ For nature is simple, and delights not in
 “ superfluous causes of things.”

* *Philos. Natur. Princip. Mathemat.* p. 387.

P R E F A C E.

R U L E II.

“ *Of natural effects therefore of the same
kind the same causes are to be assigned, as far
as it can be done.*

“ As of respiration in a man and in a
beast; of the descent of stones in *Europe*
and in *America*; of light in a culinary fire
and in the sun; of the reflexion of light in
the earth and in the planets.”

R U L E III.

“ *The qualities of bodies which cannot be
increased and diminished, and which agree
to all bodies in which experiments can be
made, are to be reckoned as qualities of all
bodies whatsoever.*

“ For the qualities of bodies are not known
but by experiments; and therefore, as ma-
ny are to be reckoned general as generally
agree with experiments, and those which
cannot be diminished cannot be taken
away. Certainly dreams are not to be de-
vised at pleasure contrary to the tenor of
experiments; nor must we depart from the
analogy of nature, since she is wont to be
simple, and always consonant to herself.
The extension of bodies is not known but
by the senses, nor is it perceived in all bo-
dies: but because it agrees to all bodies
which are perceivable, it is affirmed of all

“ whatsoever. We experience many bodies
 “ to be hard. But the hardness of the whole
 “ arises from the hardness of the parts, and
 “ thence with good reason we conclude the
 “ undivided parts not only of those bodies
 “ which are perceived, but also of all others,
 “ to be hard. We gather all bodies to be
 “ impenetrable, not by reason, but by sense.
 “ We find the bodies we handle to be im-
 “ penetrable, and thence conclude impene-
 “ trability to be a property of all bodies what-
 “ soever. That all bodies are moveable, and
 “ by certain forces (which I call *vires iner-*
 “ *tiaë*) persevere in motion or rest, we gather
 “ from these same properties in bodies which
 “ are seen. Extension, hardness, impenetra-
 “ bility, mobility, and *vis inertiaë* of the
 “ whole, arise from the extension, hardness,
 “ impenetrability, mobility, and *vis inertiaë*
 “ of the parts; and thence we conclude that
 “ all the least parts of all bodies are extend-
 “ ed, and hard, and impenetrable, and move-
 “ able, and endued with *vires inertiaë*. And
 “ this is the foundation of all Philosophy.
 “ Farther we know from the *Phænomena*, that
 “ the parts of bodies which are divided
 “ and mutually contiguous to one another,
 “ may be separated from one another; and it
 “ is certain from mathematicks, that the un-
 “ divided parts may by reason be distin-
 “ guished

“ guished into less parts. But whether those
 “ parts distinct, and not yet divided, can
 “ by the powers of nature be divided and
 “ separated from one another, is uncertain.
 “ But if it should appear, even by one single
 “ experiment, that by breaking a hard and
 “ solid body, any undivided particle suffered
 “ a division; we might conclude by the
 “ force of this Rule, that not only the di-
 “ vided parts were separable, but that the un-
 “ divided parts might be divided *in infinitum*.

“ *Lastly*, If it be universally evident by
 “ experiments and astronomical observations,
 “ that all bodies round the earth gravitate
 “ towards the earth, and that in proportion
 “ to the quantity of matter in each, and that
 “ the moon gravitates towards the earth in
 “ proportion to its quantity of matter, and in
 “ like manner our sea gravitates towards the
 “ moon, and that all the planets mutually
 “ gravitate towards one another, and that
 “ there is a similar gravity of comets to-
 “ wards the sun; we must pronounce by
 “ this Rule, that all bodies gravitate mutu-
 “ ally towards one another. For the argu-
 “ ment from the *Phænomena* will be stronger
 “ for an universal gravity, than for the impe-
 “ netrability of bodies, concerning which in
 “ the heavenly bodies we have no experiment,
 “ no observation at all.”

R U L E IV.

“ *In experimental Philosophy propositions*
 “ *collected from the Phænomena by induction*
 “ *are to be deemed, notwithstanding contrary*
 “ *Hypotheses, either accurately or very nearly*
 “ *true, till other Phænomena occur, by which*
 “ *they may be rendered either more accurate*
 “ *or liable to exceptions.*

“ This ought to be done, lest arguments of
 “ induction should be destroyed by *Hypotheses.*”

This Method and these Rules, have been carefully observed by our Author in these LECTURES; which, from the clearness and diffusiveness of the stile, and the easy and just manner of reasoning, are, in my opinion, better fitted for the instruction of youth, than any thing which I have seen on this subject.

I have added a few *Problems* by way of APPENDIX.

THE

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LEC.

LECTURE I.

OF ATTRACTION.

LECT.
I.

AS Natural Philosophy is a science in its own nature entertaining and delightful, and withal conducive in many instances to the ease and convenience of life; it is not to be wondered that there have been men in all ages who have laid themselves out in the improvement and cultivation of it. But it is a matter of no small surprize to think how inconsiderable a progress the knowledge of nature had made in former ages, when compared with the vast improvements it has received from the numberless discoveries of later times; insomuch, that some of the branches of natural philosophy, which at this day is almost compleat in all its parts, were utterly unknown before the last century. If we look into the reason of this, we shall find it to be chiefly owing to the wrong measures that were taken by philosophers of former ages in their pursuits after natural knowledge; for they disregarding experiments, the only sure foundation whereon to build a rational philosophy, busied themselves in framing hypotheses, for the solution of natural appearances, which as they were creatures of the brain, without any foundation in nature, were generally speaking so lame and defective, as in many cases not to answer those very phænomena for whose sakes they had been contrived. Whereas the philosophers of later times, laying aside those false lights, as being of no other use than to misguide the understanding

in

LECT.

I.



in its searches into nature, betook themselves to experiments and observations; and from thence collected the general powers and laws of nature; which with a proper application, and the assistance of mathematical learning, enabled them to account for most of the properties and operations of bodies; and to solve many difficulties in the natural appearances, which were utterly inexplicable on the foot of hypotheses. By this means has natural philosophy within the compass of one century been brought out of the greatest darkness and obscurity into the clearest light; and this has been chiefly owing to the unparalleled abilities, and indefatigable industry of that great and accurate philosopher Sir ISAAC NEWTON; who to his great honour has in his principles of natural philosophy, and his incomparable treatise of light and colours, cleared more difficulties, and disclosed more and more important truths relating to nature, than are to be met with in the voluminous writings of all that went before him. To illustrate some of these truths by experiments is the design of this course, which consists of four parts. In the first are considered solid bodies and their properties. In the second water and watery fluids. In the third the elastic fluid of air. And in the last the subtile fluid of light. But before I proceed to these particulars, it will be necessary to say something concerning certain principles, forces, or powers, wherewith all parts of matter of what kind soever, so far as experience reaches, seem to be endued; and whereby they act upon one another for producing a great part of the phænomena of nature.

Such is first that power whereby the minute particles of matter do in some circumstances tend towards one another, which is commonly called attraction; the cause whereof is in a great measure unknown, tho' the thing itself is manifest from experiments. For if two polished plates of brass be

Exp. 1.

laid

laid one upon another, having their contiguous sides smeared with oil, they will cohere in *vacuo*, and with such firmness, that when they are suspended, the force of gravity in the lower plate will not suffice to separate and pull them asunder.

LECT.
I.

That the cohesion of these plates is to be attributed to the mutual attractions of their contiguous parts, cannot I think admit of a doubt, since the pressure of the outward air on their external surfaces, (to whose force this effect might otherwise have been attributed) is in this case taken off.

The use of the oil is to fill up the minute cavities in the surfaces, and by so doing to prevent the lodgment of air between the plates; which upon the removal of the outward air would expand itself by reason of its elasticity, and thereby force the plates asunder.

The forementioned attraction is in like manner collected from the following experiments.

If two plane polished plates of glass be laid together, so that their sides be parallel, and at a very small distance from one another; and their lower edges be dipped in water; the water will rise up between them, and the less the distance of the glasses is, the greater will the height be to which the water rises. If the distance be about the hundredth part of an inch, it will rise to the height of about an inch; and if the distance be greater or less in any proportion, the height will be reciprocally proportional to the distance very nearly.

Exp. 2.

The reason why the water ascends between the plates is, that those parts of the surfaces of the glasses which lye next above the surface of the water, and are contiguous thereto, attract the water, and by that means cause it to ascend; and this ascent continues till the weight of the elevated water becomes equal to the force of the attracting surfaces, and then the motion ceases, the water tending as much downward by the force of its own gravity,

LECT. I. gravity, as it doth upward by the attraction of the glasses.

The reason why the water rises to heights which are inverſly as the diſtances of the glaſſes, is this: the abſolute attractive force of the glaſſes, whereby the water is raiſed, continues unvaried whatever be the diſtance of the glaſſes; for the height and length of the glaſs ſurfaces, whoſe attractions influence the aſcent of the water, are always the ſame, and conſequently the attractive force muſt be ſo too; and for that reaſon will conſtantly ſupport the ſame weight of water; but the quantity and conſequently the weight of the elevated water will always be the ſame, if its height be reciprocally as its baſe, that is, in this caſe, as the diſtance of the plates; for the length of the baſe being equal to the length of the plates, it continues unvaried; and therefore the baſe will ever be as its breadth, that is, as the interval between the plates.

Eſp. 3. If the glaſs plates inſtead of being ſet parallel to one another, be made to meet at one of their ends, and kept at a little diſtance at the other; and their lower edges be then dipped in water, ſpirit of wine, or any other convenient liquor; the inward ſides of the plates being firſt moiſtened with a clean cloth dipped in the liquor; the liquor will riſe between the plates; and the upper ſurface of the elevated liquor will form a curve, the heights of whoſe ſeveral points above the ſurface of the ſtagnating liquor will be to one another reciprocally as their perpendicular diſtances from the concurrence of the plates. For the illuſtration of which, let AE be the ſurface of the ſtagnating liquor wherein the lower edges of the plates are immerſed, AH the concurrence of the plates, and F, G, I, K, L the curve formed by the ſurface of the elevated liquor; from any points in the curve as G, I, K, L taken at pleaſure, let fall the right lines GB, IG, KD, LE perpendicular to AE, and thoſe lines will expreſs the heights

Fig. 1.

heights of the respective points of the curve above the surface of the stagnant liquor; whilst AB, AC, AD, AE denote the perpendicular distances of the same points from the concourse of the glasses; now these heights and distances are to one another in a reciprocal proportion: for if we suppose the lines GB, IC, KD, LE to be so many pillars of liquor consisting of four sides, two of which are terminated by the plates, and the other two by the contiguous liquor; and if those sides which lye next the plates be of an equal but exceedingly small breadth in all the pillars, then will the attracting surfaces of the plates which support those pillars be likewise equal, and consequently the quantities supported, that is, the pillars must be so too. But in order to have them equal, their heights must be reciprocally proportional to their bases; which bases inasmuch as they are supposed to be equally broad must be as their lengths, that is, as the intervals between the glasses in those parts where the pillars are taken; and therefore the heights of the pillars must be reciprocally as the intervals between the plates; but from the nature of similar triangles the intervals between the glasses at different distances from the concourse are to one another directly as those distances; whence it follows, that the heights of the pillars are to one another reciprocally as their respective distances from the concourse of the plates; that is, if GB be double of IC, then is AC double of AB.

From what has been said it is plain that the curve formed by the upper surface of the elevated liquor must be an hyperbola; for from the nature of the hyperbola the external ordinates are reciprocally as the abscissæ; wherefore if AB, AC, AD, AE be taken for the abscissæ; then will BG, CI, DK, EL, be the respective ordinates; and consequently the curve which passes through the points G, I, K, L is an hyperbola.

As

LECT.

I.

Exp. 4.

As water or any other proper fluid ascends between polished plates of glass by the force of their attractions; so does it likewise in slender pipes of glass open at both ends; for if such tubes be dipped at one end into water, spirit of wine, or any other convenient fluid, the liquor will rise within the pipes to a considerable height, and this experiment (as also those before made) succeeds in the very same manner in *vacuo*, as in the open air, for the liquor constantly ascends to the same height in both.

That the ascent of liquor in these small tubes, as also between polished plates of glass, is to be attributed to some power in the glass strongly acting on the liquor, and not to the pressure either of the stagnating liquor or incumbent atmosphere, is evident from this consideration; that as much of the liquor remains suspended in the pipes, and between the plates, when they are lifted out of the stagnating fluid, either in *vacuo* or the open air, as was elevated above the surface of the fluid, while they were immersed therein: and therefore whatever cause concurred to the elevating of the liquor while the plates and pipes were therein immersed, and exposed to the air; the same contributes as powerfully to keep it up, when the stagnating liquor is removed, and the pressure of the atmosphere taken off, and consequently must be some power inherent in the glass.

The heights to which the liquor rises in slender pipes, are to one another reciprocally as the diameters. For the power which raises the liquor in a slender pipe, being the attractive force of that part only of the internal concave surface which lies next above the liquor, and constitutes a ring of an indefinitely small height, which height is ever the same whatever be the diameter of the ring, because the distance to which the attractive force of glass reaches is unvaried; and the attractive force of such an annular surface being as the number of attracting parts whereof it is composed, that is, as the surface, which

which because its height is given is as the periphery, that is, as the diameter, the attractive force of the pipe must be as the diameter. Wherefore if in comparing the forces of two such pipes we make F to denote the attractive force of the larger, and f the attractive force of the smaller, and also D and d to denote their diameters; we shall have this analogy, *viz.* $F : f :: D : d$, that is, the force of the larger pipe is to that of the smaller as the diameter of the larger to the diameter of the smaller: but these forces are likewise to one another in the same ratio with the quantities of liquor which they keep suspended; for they continue to elevate the liquor till such time as the weights, and consequently the quantities of liquor drawn up, become a balance to the attracting forces. Wherefore if H be put for the height of the liquor in the pipe, whose diameter is D , and h for its height in the pipe whose diameter is d ; then will H multiplied into the square of D be as the quantity of liquor in the larger pipe; and h multiplied into the square of d as the quantity of liquor in the smaller pipe; whence we have this second analogy $F : f :: H \times D^2 : h \times d^2$; and by substituting D and d in the room of F and f , to which they are proportional, as appears from the first analogy, we shall have $D : d :: HD^2 : hd^2$; and then multiplying extremes and means, and throwing off similar quantities, we shall have $HD = hd$, and by resolving this equation into an analogy, we shall have $H : h :: d : D$, that is, the height to which the liquor rises in the larger pipe is to the height to which it rises in the smaller, as the diameter of the smaller pipe to that of the larger; so that the heights of the liquor are reciprocally proportional to the diameters of the pipes.

By virtue of this attractive force, wherewith small pipes are endued, plants receive nourishment from the earth; the slender tubes whereof their roots are composed, sucking in various juices according to
B
their

LECT.
I.

their different natures and constitutions. From the same attractive force it is that sponges take in water; and that water ascends in loaf sugar, when any part of it is dipped therein; those parts of the sugar which lie next above the water attracting, and thereby raising the same. And here it must be observed that the water rises by the action of those particles alone which are contiguous to, and lie next above the surface of the elevated water; those particles which are at any the least sensible distance above the water being too far removed to influence the water by their attractions: and what has been thus observed of sugar, is likewise true of polished plates, slender pipes, and every other attracting body, by virtue of whose attractions fluids are raised. For if those parts of attracting surfaces which are at any sensible distance above the surface of the fluid, do in any measure contribute to the ascent; it is evident that the fluid *ceteris paribus* must rise to a greater height when the attracting surfaces are continued to a considerable height above the elevated fluid, than when they terminate at a very little distance above the same. But the

Exp. 5. contrary appears from experiment. For if two polished plates of glass set parallel to one another at the distance of about the hundredth part of an inch, be immersed in water so far that only an inch and one tenth be suffered to remain above the water, the water will rise up between them to the height of about an inch; and if the surface of the stagnating water be then depressed by drawing off some of the water, the elevated water will likewise descend between the plates so as still to preserve the height of about an inch and no more.

Exp. 6. If a polished plate of glass be laid parallel to the horizon, and another plate of the same kind be laid thereon, so as that they may touch at one of their ends, and be kept at a very small distance at the other: being first moistened on their inward sides with

with a clean cloth or feather dipped in oil of oranges ; and if a drop of the oil be placed between the plates at that end where they are at some distance from each other, so as that it may be touched by both the plates, it will begin to move towards the concourse of the glasses, and will continue to go on with an accelerated motion till it arrives at the concourse. And if during the motion of the drop, that end of the glasses where they meet, and towards which the drop moves, be lifted up, the drop will nevertheless continue its motion, and of consequence must be attracted ; but as the end of the glasses is raised higher and higher, the drop will ascend more and more slowly, till at last upon a certain elevation of the plates the motion ceases, the gravity of the drop, wherewith it tends downward, becoming equal to the attractive force which draws it upward ; as appears from this, that upon giving the plates the least degree of elevation beyond what is necessary to stop the drop, it straightway begins to descend, its gravity in that case overcoming the attraction.

By the help of this phænomenon the force may be determined, wherewith the drop is attracted at all distances from the concourse of the glasses. For that part of a body's gravity whereby it is carried down an inclined plane, is to its absolute weight, as the sine of the angle of the plane's elevation, to the Radius, or as the perpendicular height of the plane to the length thereof ; and therefore may be denoted by the perpendicular height applied to the length ; and where the length of the plane is given, that force will be every where as the sines of the angles of elevation, or the perpendicular altitudes of the plane ; as shall be made appear when I come to treat of the descent of bodies on inclined planes. If therefore the sines of such elevations of the plates as are necessary to stop the motion of the drop, be taken at two different distances of the drop from

B 2

the

LECT.

I.



the concourse of the plates; those lines will denote the respective gravities of the drop, and consequently the attractive forces, wherewith the plates act upon the drop at each of those distances. Thus for instance, if the distances of the drop from the concourse of the glasses be as one and two; and the line of the elevation necessary to stop the motion of the drop when at the smaller distance be as four, and when at the greater distance as one; the gravity of the drop, wherewith it endeavours to descend at the forementioned distances of one and two, will be as four and one. For the illustration of

Fig. 2.

which, let AB and AC represent the plates at different elevations; F and G the places where the drop stands upon those elevations; then will BD and CE denote the forces of gravity wherewith the drop endeavours to descend along the plates in the points F and G, which forces are equal to the attractions of the glasses in those points; and if BF and CG the distances of the drop from the concourse of the plates be as one and two, and BD and CE as four and one; then is the attractive power wherewith the glasses act upon the drop at F, to the force wherewith they act upon it at G, as four to one, that is, reciprocally as the squares of the distances of the drop from the concourse of the glasses; and this is nearly the case, as will appear from the experiment.

Exp. 6.

Tho' the drop be attracted by forces that are in the reciprocal duplicate ratio of the distances of the drop from the concourse of the glasses; yet are the attractions within the same quantities of attracting surface in the reciprocal simple ratio only of those distances: for as the drop moves towards the concourse of the glasses, it must spread and touch each glass in a larger surface; and this spreading is always proportional to the lessening of the interval between the glasses; and of consequence from the nature of similar triangles, it is likewise proportional to

to the diminution of the distance from the concourse. So that the force which acts upon the drop is increased as the drop approaches the concourse in the simple reciprocal ratio of the distance, on account of the enlargement of the attracting surface in that proportion; and therefore in a given quantity of attracting surface the force must be in the reciprocal simple ratio of the distance from the concourse; that is to say, any given portion of the glass surfaces taken at the distance of one inch from their concourse must act with twice the force that it does at the distance of two inches, and with thrice the force that it does at the distance of three inches, and so on. Hence it will be found that the attractive force of one and the same slender pipe of a conical figure is given; or in other words, that the attractive force wherewith a conical pipe is indued at any one distance from the vertex of the cone, is equal to the attractive force of the same at any other distance from the vertex; so that the attractive force of a conical pipe is in every part equal throughout the whole length of the pipe, and may be expressed by the diameter of a circular section of the pipe taken at any distance from the vertex, applied to that distance. For the attraction in any part of such a pipe, is as the quantity of attracting surface in that part multiplied into the absolute force; but the quantity of attracting surface in any part is as the diameter of that part, and the absolute force is reciprocally as the distance from the vertex; wherefore if A be put to denote the distance of any part from the vertex and D the diameter, $\frac{D}{A}$ will express the attraction of that part; but from the nature of similar triangles the diameters of the circular sections of a cone taken at different distances from the vertex are to one another as the distances, consequently $\frac{D}{A}$ is a standing quantity.

B 3

Wherefore

LECT.

I.

Wherefore since the attractive force in every part of a conical tube is denoted by a quantity which is invariable, it follows that the force is so too; so that in this respect conical pipes do not differ from those of a cylindrical form; but herein lies the difference, that in very slender pipes where the diameters are equal, the attractions of such as are conical do far surpass the attractions of those which are cylindrical. And indeed so exceeding great does this attractive force become with respect to the quantity of attracting surface in that part of a conical pipe, where the diameter is but one part of an inch divided into ten millions (if such minuteness may be supposed) that if the attraction of a cylindrical tube, whose diameter is an inch, were as great with respect to its quantity of attracting surface, it would be able to support a column of water an inch in diameter and upwards of three miles in height. For let us suppose a conical tube whose base is an inch in diameter to be continued till the diameter is so far diminished as to equal only one part of an inch divided into ten millions; it is evident from what was just now said, that the whole attractive force of such a pipe where its diameter is an inch, is equal to the whole attractive force of the same, where the diameter is but the ten millionth part of an inch; consequently if a portion of the larger attracting surface be taken equal to the smaller attracting surface, the force of that will be to the force of this, as the force of the smaller surface divided by the number of parts in the larger surface, to the force of the smaller surface, that is, as one divided by ten millions to one. If therefore a conical, or indeed a cylindrical tube an inch in diameter (for where the diameter is so large there is scarcely any difference) was indued with an attractive force as great in proportion to its quantity of attracting surface, as is a conical tube of the ten millionth part of an inch in diameter, its force would be ten millions of times greater

greater than it is, and of consequence would raise the water ten millions of times higher than it doth at present: but it has been found by experience that in a cylindrical tube of an inch in diameter, the water will rise to the height of about the fiftieth part of an inch, and therefore if the force by which it rises was augmented in the forementioned proportion, it must rise to the height of two hundred thousand inches, which being divided by sixty-three thousand three hundred and sixty, the number of inches in a mile, gives three and a little more in the quotient.

The quantities of liquor supported by the attractions of slender conical pipes are to one another, as the diameters of the little circular surfaces of the elevated liquor, applied to the respective distances of the same circular surfaces from the vertices of the several cones whereof the pipes are portions. For it has been proved that the attractive forces of conical pipes are as those quantities; and therefore the weights which they support must be so too. Hence it follows that the less the proportion is, which the distance of the elevated liquor's surface from the vertex of the cone bears to the diameter of the same surface, or, which amounts to the same thing, the faster the sides of the pipe converge, the stronger is its attractive force, and the greater the quantity of liquor which is supported.

The firm union and strong cohesion of the particles of solid bodies seems to arise from this force, wherewith they mutually attract each other; which as it appears to be exceeding strong in the immediate contact of the particles, so is it found by experience to reach but a very little way beyond the same with any sensible effect. At very small distances indeed it is sufficient to raise up liquors, as also to produce the many odd and surprizing appearances which are to be met with in chymical operations, and which without the assistance of this and

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I.

some other principles, which I shall hereafter have occasion to mention, are utterly inexplicable. For want of a due knowledge of these powers chymists have fallen into gross mistakes and absurdities in their reasonings. Thus, for instance, some who were unacquainted with the principle of attraction, have attempted to give a reason for the floating of the minute particles of solid bodies in menstrooms specifically lighter than themselves; by saying that there is an intestine motion in the parts of the menstrooms, by virtue whereof the particles of the solid bodies are driven perpetually from place to place, and by that means are kept from falling: not considering that Sir ISAAC NEWTON has demonstrated in the nineteenth proposition of the second book of his Principles, that fluids have not naturally any intestine motion; but that, setting aside all external causes of motion, the particles of fluids are as perfectly at rest as those of solid bodies. There is indeed during the time of the solution a considerable motion; but as this is occasioned by the mutual attraction between the menstruum and the body, by means of which attraction the parts of the fluid are driven with great force between the parts of the solid, so as to loosen and divide them one from another; as soon as the solution is over, the motion ceases, and all the parts are at rest again, and the particles of the dissolved body are kept suspended by their close adhesion to the parts of the menstruum, and not by any imaginary motion, wherewith they are tossed to and fro in the manner of a shuttle-cock; and in truth, could such an intestine motion be allowed, as it must be made in all manner of directions, it would be as apt, nay more apt, considering the conspiring gravity of the particles, to precipitate and cast them down, than to raise and keep them up.

Were it not beside my present purpose, I could produce many more instances of false reasonings in the writings of chymists, occasioned by their ignorance

nerance of the true principles of nature ; but as chymistry is at present out of my province, I shall rest contented with the single instance which I have given.

LECTURE II.

OF ATTRACTION.

HAVING in my former Lecture proved from experiments, that there is a power in nature whereby the parts of matter, which are brought so near as to touch, do in some circumstances mutually attract each other ; I shall now treat of such kinds of attraction as extend themselves to considerable distances beyond the point of contact, and on that account affect the mind more strongly, so as to convince it more fully of the reality of such a principle. Of this kind is, First, that attraction which obtains between glass and glass. Secondly, that of electricity. Thirdly, the attraction of magnetism. And lastly, that of gravity ; of all which in their order.

LECT.
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And first, if a glass bubble be set to float on water contained in a glass vessel, at a small distance from the side of the vessel, it will from a state of rest begin to move towards the side of the vessel ; and its motion will be continually accelerated, so as to make it upon its arrival at the side of the vessel to strike the same with some force.

Exp. I.

Perhaps it may be thought that the motion of the bubble arises from some declivity in the water towards the sides of the vessel : but whoever observes the surface of the water will find, that it rises all about the sides of the glass, so as to become of a concave figure, and for that reason may retard, but can by no means promote the motion of the bubble ; and this rising of the liquor about the sides of the vessel is to be attributed to the same cause with the motion of the bubble, namely, the attraction of the glass.

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II.



The acceleration observable in the bubble's motion arises from two causes; the first is, the continued and uninterrupted action of the attractive force of the glass; for if we suppose the time of the bubble's motion to be divided into a number of equal parts, as for instance ten; and if the attraction of the glass be supposed to make equal impressions on the bubble in each of those parts of time, it is plain that whatever be the motion which is excited in the bubble by the impression of attraction in the first portion of time, the same will be doubled in the second, tripled in the third, and so on continually thro' the several portions of time; for the motion produced in the first portion of time is not lost, and therefore by the addition of as much more in the second portion of time it becomes double, and in the third triple, and so on. Now if instead of ten parts we suppose the time of the motion to be divided into numberless parts indefinitely small, in each of which the attraction of the glass makes equal impressions on the bubble, as before; the motion will be continually accelerated, tho' the attractive force of the glass should continue the same at all distances of the bubble; but the attractive force acts more strongly the nearer the bubble approaches, on which account the motion is more and more accelerated the nearer the bubble comes to the glass.

By electrical attraction, I mean that kind of attraction which is excited in bodies when their parts are heated by friction, and which doth not discover itself by any sensible effect when the bodies are cold. Of this sort are the attractive forces, which amber, rosin, sealing-wax, and indeed most sulphureous substances when heated by rubbing, have been found to exert towards chaff, feathers, gold-leaf, lamp-black, and many other light substances. But as the attraction of these bodies have fallen within the notice of vulgar eyes, I think it needless to make any experiment for the proof thereof; but choose

rather

rather to lay before you some experiments which plainly shew this power to obtain in glass, and that to a very notable degree, tho' it has not till of late been commonly observed. And first,

If a cylindrical tube of flint glass be rubbed briskly with brown paper, or woollen cloth till it acquires some degree of heat, and be then held near to small pieces of gold or brass leaf; they will begin to move, and some of them will fly towards the tube with great swiftness, and fix themselves upon it so as to adhere thereto, being acted upon by the attractive force of the glass: whilst others during their ascent towards the tube, will before they can reach the same, be driven backward with great violence, as will likewise some of those which touch the glass, being actuated by another force very different from that of attraction, which I shall endeavour to explain to you hereafter. The hotter the tube is made by rubbing, the farther doth its power reach, so as in some cases to act upon the leaf at the distance of a foot or more.

This electrical attraction of glass doth in like manner appear from the following experiments.

If over a globe of glass fixed on an axis, whose position is horizontal, a parcel of woollen threads be suspended from a semicircular wire, so as that their lower ends may be distant an inch or a little more from the globe, they will suitably to the nature of all heavy bodies, hang down perpendicular to the horizon, and parallel to each other; if then the globe be moved pretty briskly round its axis, the threads will immediately change their position, so as to have their ends bent a little upward, pointing that way towards which the motion tends; the rotatory motion of the globe being communicated to the circumambient air wherein the threads hang, and by means thereof in some measure to the threads themselves. Let then an hand be applied to the lower part of the globe, so as to rub the same, and

Exp. 2.

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as soon as it grows warm from the friction, the threads which were before crooked will dart themselves out into so many strait lines, all pointing towards the center of the globe; but as soon as the attrition ceases, and the globe cools, they quit this direction, and return to their former position; whence it evidently appears that they are attracted by the glass, since they are made to point towards its center, notwithstanding the contrary directions that were given them by the motion of the air and the force of gravity. In this and the two following experiments there is one remarkable circumstance, which tho' it does not concern the matter in hand, yet because I shall have occasion to have recourse to it hereafter, I shall, to prevent the repetition of experiments, take notice of it here. And it is this; if while the threads are extended and acted upon by the attraction of the globe, a finger be moved towards the extremity of any of them, they will immediately recede and fly from the touch, and this they will do upon every approach of the finger.

Exp. 3.

If the axis of the globe instead of being parallel to the horizon be placed perpendicular thereto, and the semicircular wire which supports the threads be in the plane of a circle parallel to the horizon, the threads must by reason of their gravity hang down in lines parallel to the axis of the globe, yet as soon as the motion and attrition are given to the globe as before, the threads will begin to raise and extend themselves towards the center of the globe, and appear like so many rays converging towards that center in a plane parallel to the horizon: so that in this case the attractive force of the glass does not only draw the threads out of the parallel position they have to each other, but likewise raises them up in a position parallel to the horizon, notwithstanding the force of gravity which is constantly acting upon them to carry them down.

If

If the threads, instead of being placed without the globe, be fixed to the axis at the center, and be of such a length as to reach within about an inch of the surface; when the globe is turned round, they will bend backward contrary to the direction of the motion; because the included air, tho' it does in some measure partake of the rotation of the globe, yet doth it not move with equal swiftness, and for that reason must resist the rotation of the threads and bend them backward. When the threads are in this state, if the attraction of the glass be excited by attrition as in the two last experiments, they will straightway extend themselves towards the concave surface of the globe, constituting as it were so many rays issuing from the center, and diverging from one another in a regular manner.

LECT.
II.
Exp. 4.

The reason why the threads in all these experiments are stretched into lines tending either to or from the center of the globe, seems to be this. Whatever be the force wherewith the globe acts on the threads, the direction of it must be perpendicular to the surface of the globe; consequently in the same direction must the threads move; but from the nature of the globe those and those lines only are perpendicular to its surface, which either issue from or tend towards the central point.

Exp. 5.

Having said thus much concerning electrical attraction, I now proceed to that of magnetism. Many and surprising are the properties both of the loadstone and magnetical needle, which however I shall not here consider; my intent at present being only to shew from experiment the law of magnetical attraction; or, in other words, to shew in what proportion the attractive power of the loadstone varies according to the different distances of the iron which it attracts. And in order to this, let a loadstone be suspended at one end of a balance, and counterpoised by weights at the other; let a flat piece of iron be placed beneath it at the distance of four tenths

Exp. 6.

LECT.
II.

tenth parts of an inch, the stone will immediately descend, and adhere to the iron: let the stone again be removed to the same distance, and a weight of four grains and four tenth parts of a grain be thrown into the scale at the other end of the balance; this weight will be an exact counterbalance to the attractive force, and prevent the descent of the stone; but if any part of the weight be taken out, the attraction will prevail, and carry the stone down. If the stone be placed at half the former distance, that is to say, at the distance of two tenth parts of an inch above the iron, the weight necessary to hinder its descent will be about seventeen grains and an half, that is four times as much as before. Consequently, the attractive force of the stone at the single distance from the iron, is to the same at the double distance as four to one, that is reciprocally as the squares of the distances.

Perhaps it may be objected that Sir ISAAC NEWTON (to whose judgment in natural affairs the utmost regard is due) has said that the power of the loadstone decreases nearly in the triplicate ratio of the increase of the distance. But whoever considers his words in the fifth corollary of the sixth proposition of the third book of his Principles, where he mentions this law, will find that he speaks of it with diffidence, as a thing which he rather guessed at from some rude observations, than collected from accurate experiments; for his words are, *Et in recessu a magnete decrescit in ratione distantie non duplicatâ, sed fere triplicatâ, quantum ex crassis quibusdam observationibus animadvertere potui.* So that notwithstanding this objection I shall still venture to affirm the law of magnetical attraction to be such as makes it act with forces which are in the reciprocal duplicate ratio of the distance. Because this law is deduced from an experiment made with sufficient exactness, and which does not seem liable to any exception.

Tho'

Tho' the principle of gravity, which comes next to be treated of, be diffused throughout the solar system, and may probably be extended so far as to reach the other systems of the universe; yet shall I consider it at present with respect only to the globe of earth, which we inhabit; the parts whereof would by reason of the diurnal rotation be apt to fly asunder, were they not kept together by the influence of this principle; whereby likewise all bodies on or near the surface of the earth are made to tend towards its center. This power at equal distances from the center of the earth is always proportional to the quantity of matter in the body whereon it acts; for all bodies, the light as well as heavy, being let fall from the same height, descend with equal swiftness, provided they meet with no resistance from the air, as will appear from the following experiment. Let a piece of gold and a feather be let fall from the top of an exhausted receiver at the same instant of time, and they will both arrive at the bottom at the same time very nearly.

Exp. 7.

The reason why the feather doth not reach the bottom quite so soon as the gold, is, that the receiver cannot be perfectly exhausted, and therefore the small portion of air which remains within, though very much rarified, gives some small resistance to the descending bodies, which suitably to the nature of all resistance must retard the lighter body more than the heavier, and consequently cause some little difference in the times of the descent, which otherwise would be exactly equal. This then being the case, it evidently follows, that the forces of gravity, whereby bodies descend, must at equal distances from the center be as the quantities of matter in the descending bodies; for if a certain force of gravity be requisite to carry down a certain quantity of matter with a certain swiftness, then is double the force necessary to carry down a double quantity of matter with the same

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same swiftneſs; and triple the force to carry down a triple quantity, and ſo in proportion whatever be the quantity of matter: ſo that the weights of bodies at equal diſtances from the center of the earth are always proportional to the quantities of matter which they contain; and therefore the quantity of matter in any body may be meaſured by its weight.

The gravity of a body at any place beneath the ſurface of the earth has been proved by Sir ISAAC NEWTON, to be directly as the diſtance from the center: that is, ſuppoſing the earth's radius to be four thouſand miles, a body which on the ſurface of the earth weighs a pound, will within the earth at the diſtance of two thouſand miles from the center weigh only half a pound, at the diſtance of one thouſand miles only a quarter, and ſo on till at the center it loſes all its gravity.

It has been likewiſe proved that the force of gravity on the ſurface of the earth, and at all diſtances beyond it, is in the reciprocal duplicate ratio of the diſtance from the center; that is, if a body weighs a pound at the ſurface of the earth, whoſe diſtance from the center is four thouſand miles, it will at double that diſtance weigh only a quarter of a pound, and at triple the diſtance, only the ninth part of a pound, and ſo on; whatever be the diſtance, the force of gravity will be reciprocally as the ſquare of the diſtance. For is it not highly rational that the power of gravity, whatever it be, ſhould exert itſelf more vigoroſly in a ſmall ſphere, and weaker in a greater, in proportion as it is contracted or expanded? and if ſo, ſeeing that the ſurfaces of ſpheres are as the ſquares of their *radii*, this power at ſeveral diſtances muſt be as the ſquares of thoſe diſtances reciprocally. Tho' ſtrictly ſpeaking, this be the law of gravity, yet where the diſtances from the ſurface are inconfiderable with reſpect to the earth's radius

the force of gravity may be looked upon as equal at all those distances ; thus for instance, the gravity of a body at the distance of half a mile from the earth may be looked upon as equal to the gravity thereof at the distance of a quarter of a mile ; or at the very surface ; because the difference is so small, that if it be rejected it will not occasion any error in calculations. And indeed on this supposition are founded most of the reasonings of GALILÆO, TORRICELLIUS, HUYGENS, and other naturalists concerning the descent of heavy bodies ; and by the help of the same supposition have the several theorems been formed relating to the acceleration of falling bodies, the spaces described, the times of the fall, and the velocities thereby acquired ; as I shall now shew you.

If the force of gravity whereby a body descends remains unvaried, the motion of a body falling by such a force will be accelerated, and that uniformly ; that is, the velocity will increase, and the increments thereof in equal times will be equal. For let us suppose the time of the descent to be divided into a number of equal parts indefinitely small, in each of which by supposition, the force of gravity makes equal impressions on the body to carry it down ; whatever velocity therefore the body receives from the impression of gravity in the first portion of time, it must receive as much in every other portion ; since therefore, setting aside all outward lets and obstacles, the effect of every impression remains, the velocity given in the first portion of time, will be doubled in the second, tripled in the third, quadrupled in the fourth, and so on continually thro' the several portions of time. So that the velocity of a body falling, by the force of gravity will constantly increase in the same proportion with the time of the descent. Or, in other words, the motion of a body carried down by the force of gravity will be uniformly accelerated : and the velocities

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locities acquired will be as the times of the descent from the beginning of the fall.

Fig. 3.

From what has been said it follows, that a right line as AB be supposed to denote the time of a body's fall, and another right line as BC set at right angles to the former, to express the velocity acquired by the falling body in the time denoted by AB : the triangle ABC being completed, and another right line as DE drawn parallel to BC , then will DE denote the velocity acquired by the falling body in a portion of time which is to the time denoted by AB , as AD to AB : for, from the nature of similar triangles AB is to AD as BC to DE ; but BC expresses the velocity acquired, where the time is as AB : consequently, since the velocities are as the times of descent, DE will express the velocity acquired in the time denoted by AD .

And what has been thus proved of the line DE , is in like manner true of any other right line, as FG , or HI , drawn within the triangle parallel to the base; for FG and HI will express the velocities acquired in the times denoted by AF and AH .

Fig. 4.

The spaces described by bodies falling from a state of rest by the force of gravity are to one another as the squares of the times from the beginning of the fall. In the triangle ABC , let AB express the time of a body's fall, and BC the velocity acquired at the end of the fall, let AB be divided into a number of equal parts indefinitely small; and from each of those divisions suppose lines, as DE drawn parallel to BC ; it is evident from what has been said, that those lines will express the velocities of the falling body in the several respective points of time; which velocities, inasmuch as the body is given, and the portions of time are indefinitely small, will be as the respective spaces described in those times: but the sum of the spaces described in all the small portions of time is equal to



the space described from the beginning of the fall; and the sum of all the lines, as DE taken infinitely near each other constitute the area of the angle. And therefore the space described by a falling body in the time expressed by AB , and where the velocity acquired at the end of the fall is denoted by BC , will be as the area of the triangle ABC . And for the same reason the space described by a falling body in the time expressed by AD will be as the area of the triangle ADE . But from the nature of similar triangles these areas are to one another as the squares of their homologous sides; that is, as AB^2 to AD^2 , or as BC^2 to DE^2 . But AB and AD express the times of the fall, and BC and DE the velocities acquired by the fall; wherefore the spaces described by a falling body are as the squares of the times from the beginning of the fall, or as the squares of the velocities at the end of the fall. And what has been thus demonstrated from the nature of gravity is likewise confirmed by experiments. For if a weight of eleven hundred grains be let fall from the height of three inches, so as to strike one end of a balance, its force will be just sufficient to raise a pound weight at the other end of the balance to the height of about the eighth or tenth part of an inch; whereas if the same body be required to raise a weight of two pounds to the same height, it must be let fall from the height of twelve inches; and if the weight to be raised, be three pounds, then must the moving body fall from the height of twenty-seven inches; for lesser heights will not suffice, as will appear from the experiment.

Exp. 3;

The forces wherewith the descending body strikes the end of the balance are measured by the weights that are raised; which in this case are as one, two, and three; but the forces wherewith one and the same body strikes, are as the velocities of the body: wherefore in the case before us the velocities acquired

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by the falling body are as one, two, and three ; but the heights from which it descends in order to acquire those velocities are as one, four, and nine ; that is, as the squares of the velocities.

Exp. 9. If this experiment be repeated with a body double in weight to the former, to wit, with one of twenty-two hundred grains ; the weights raised by the strokes will be two, four, and six pounds, to wit, double the former.

From this experiment appears the truth of that rule, which collects the quantity of motion in any body by multiplying the velocity of the body into its quantity of matter. For the force of a stroke is, *cæteris paribus*, always proportional to the quantity of motion in the striking body ; consequently in like circumstances the motions of bodies may be measured by the force of their strokes : but it has appeared from the experiment that where the striking body is as unity, and the velocities wherewith it moves at the times of the strokes, as one, two, and three ; the forces of the respective strokes are likewise as one, two, and three. But where the body is as two, the strokes are as two, four, and six ; that is, in both cases the strokes are as the products arising from the multiplication of the quantities of matter in each body into the respective velocities : wherefore the quantities of motion are as those products. Whence as a corollary it follows, that if the weight of one body multiplied into its velocity gives an equal product to what arises from the multiplication of the weight of another body by its velocity, the motions of those two bodies are equal ; and this will ever be where the weights of the bodies are reciprocally proportional to their velocities. Thus when the body whose weight was as unity, was let fall from the height of twelve inches, and thereby acquired a velocity which was as two ; it raised a two pound weight, which was likewise raised by the body whose weight was as two, when by fall-

ing from the height of three inches, it had acquired a velocity which was as unity.

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From what has been proved concerning the spaces described by falling bodies it follows, that if the time of a body's fall be divided into a number of equal parts, the spaces thro' which it falls in each of those parts of time taken separately and in their order, beginning from the first, are as the odd numbers taken likewise in their order, beginning from unity. For instance, if the time of the fall be four seconds, the space described in the first of those seconds will be as one, in the second as three, in the third as five, and in the fourth as seven; for where the times of the fall are as one, two, three, and four; the spaces described are as one, four, nine, and sixteen; and therefore if from the space described in two seconds, to wit, four, be subducted the space described in the first second, to wit, one, the remainder, to wit, three, will be the space described in the next second. And if from nine, which is the space described in three seconds, be taken four, which is the space described in two seconds, the remainder, which is five, will be the space described in the third second. In like manner subducting nine, the space described in three seconds, from sixteen, which is the space described in four seconds, the remainder, to wit, seven, will be the space described in the fourth second; and so on according to the number of parts into which the time of the fall is divided.

From what has been said it likewise follows, that the velocity acquired by a falling body at the end of the fall is such as with an equable motion would in the same time in which the body fell, carry it thro' a space double that of the fall. That the truth of this may be made appear, it is necessary that some things be premised concerning the spaces described by bodies carried with an equable motion. And first, if the velocity of a body moving uniformly be given,

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the space described will be as the time of the motion; for if a body with a given velocity moves thro' a certain space, a foot, for instance, in a second of time, it will in two seconds, with the same velocity, move thro' two feet, and thro' three feet in three seconds, and so on; whatever be the time, the space described will be proportional thereto. On the other hand, if the time be given, the space described will be as the velocity: for if a body in a given time moves thro' the space of a foot with a certain velocity, with double the velocity it will pass thro' the space of two feet, and with triple the velocity thro' the space of three feet, and so on, whatever be the velocity, the space described will be in the same proportion. But if neither the time of a body's motion, nor the velocity wherewith it moves be given, the space described will be as the time and velocity conjointly: for if a body moving with a certain velocity runs thro' a certain space in a certain time, it follows from what has been said, that if the time be increased or diminished in any proportion, in the same also will the space be increased or diminished, supposing the velocity to remain the same; but if that likewise be changed, it is plain that the space will be changed in the same proportion: and therefore universally the space described by a body moving equably is as the time and velocity conjointly. For which reason, if in the rectangle, one side, as AB , be supposed to denote the time wherein a body moves equably, and BC the velocity wherewith it moves, the rectangle $ABCD$ will be as the space described; but the triangle ABC of the same figure, is as the space described by a falling body in the time denoted by AB , and BC is as the velocity acquired at the end of the fall; and the rectangle $ABCD$ is double the triangle ABC , consequently the velocity acquired by a falling body is such as will carry the body with an equable motion in the time of the fall thro' double the space of the fall.

As

Fig. 5.

As the motion of bodies falling from a state of rest is uniformly accelerated; so likewise the motion of bodies thrown upward is uniformly retarded; for the same force of gravity which conspires with the motion of descending bodies, acts in direct opposition to the motion of such as ascend; and therefore in whatever manner it accelerates the one, in the very same manner must it retard the other. Whence it follows, that if a body be thrown directly upward, the time of its rise will be equal to that wherein a body falling freely from a state of rest, acquires the same velocity wherewith the body is thrown up. For since the action of gravity is constant and uniform, in whatever time it generates any velocity in a falling body, in the same time must it destroy that velocity in a rising body; and therefore the time of the rise must be equal to that of the fall. It likewise follows that the height to which a body thrown upward rises is equal to that from which a body falling freely does at the end of the fall acquire a velocity equal to that wherewith the body is thrown up. For since the times in which the velocity of the falling body is generated, and that of the rising body is destroyed, are equal; and since of the two equal velocities one is generated and the other destroyed by the constant uniform action of one and the same power; it is manifest, that whatever be the space thro' which the falling body moves in order to acquire its velocity, the rising body must ascend thro' an equal space in order to lose its velocity; that is, it must rise to the same height from which the other falls.

The force of gravity at the surface of the earth is such as, setting aside the resistance of the air, makes a body falling from a state of rest to descend thro' a space of sixteen feet and an inch in a second of time. For the time wherein a pendulum performs its smallest vibrations is to the time in which a body falls thro' half the length of the pendulum as the

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circumference of a circle to its diameter (as shall be shewn when I come to treat of the pendulum) wherefore since the spaces described by falling bodies are as the squares of the times, and since the diameter of a circle expresses the time which a body takes to fall thro' half the length of a pendulum vibrating seconds, when the circumference expresses a second; it follows, that as the square of the diameter is to the square of the circumference, so is half the length of the pendulum to the space thro' which a body falls in a second of time. So that putting D to denote the diameter of a circle, which is as unity, P the periphery, which is as 3,1416, L the length of the pendulum vibrating seconds, which is $39\frac{1}{8}$ inches, and S to denote the space sought; we shall have this analogy, $D^2 : P^2 :: \frac{L}{2} : S$.

Consequently $S = \frac{P^2 \frac{1}{2} L}{D^2}$, or rejecting the divisor as being equal to unity, $S = P^2 \frac{1}{2} L = 193$ inches, or sixteen feet and an inch.

Before I quit this subject I must observe to you, that bodies do not every where descend at the rate of sixteen feet and an inch in a second of time, but in such places only as are in or near the latitude of forty-nine degrees; in places more distant from the line the descent is quicker, and more slow in those less distant. For the force of gravity is less towards the æquator than towards the poles, as has been collected from observations made on pendulums; for they have been found to vibrate more slowly near the line than in places farther removed; insomuch that a pendulum which in the latitude of Paris vibrates seconds, must be shortened one sixth of an inch French measure, in order to its vibrating seconds under the line. And the length of a pendulum which in the latitude of Paris performs its vibrations in a second, is to the length of a pendulum whose vibrations are performed in the same time

time under the line as 220 to 219. Since therefore the forces of gravity which actuate pendulums that vibrate in equal times are to one another as the lengths of the pendulums (as shall be shewn when I come to treat of pendulums) it is evident that the force of gravity in the latitude of Paris is to the same force under the line as 220 to 219. And indeed it has appeared from a great number of observations, that the force of gravity is least at the æquator, and that it continually increases as we recede from thence and approach the poles, under which it is greatest of all. And the chief cause of this difference is the rotation of the earth about its axis, whereby all bodies on or near the surface of the earth are indued with a centrifugal force, which acts in opposition to that of gravity, and of course must lessen the same; and the diminution of gravity arising from this cause must be greatest under the æquator, and grow less and less in the approach to the poles: and that for two reasons, first, because the centrifugal force is greatest at the æquator, and from thence towards the poles is continually diminished so as at last to vanish in the polar points. For all parts of the earth's surface with the bodies thereto adjacent revolve in the same time either in the æquator or in circles parallel thereto; but the æquator is the largest of all those circles, and the others grow less and less as they are more and more distant from the æquator. Now the centrifugal forces of bodies revolving in the same time in different circles being to one another as the *radii* of the circles (as shall be shewn when I come to treat of those forces) it follows that the centrifugal force must be greatest at the æquator, and thence be continually diminished towards the poles. To illustrate this, let A B be the axis of the earth, C K

Fig. 6.

the radius of the æquator, D I, E H and F G the radii of so many circles parallel to the æquator, the centrifugal forces in the points K, I, H, G, are
as

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as those radii; so that the centrifugal force is greatest in the point K, that is at the æquator, and at I it is less than at K, and at H less than at I, and less again at G, and so on, till at length it vanishes at the polar point, where there is no rotation. Whence it is evident that the force of gravity must be smallest under the line, and must increase towards the poles, inasmuch as the force which acts in opposition to it is greatest under the line, and lessens in the approach to the poles. The force of gravity must likewise be less under the æquator than in any other place, because under the line the centrifugal force acts in direct opposition to the force of gravity; whereas in other places it acts in an oblique direction to that of gravity, and of consequence must act less powerfully against it. Thus in the point K the force of gravity pulleth from K towards C, whilst the centrifugal force pulleth directly contrary from C towards K; whereas in the point L gravity pulleth from L towards C, whilst the direction of the centrifugal force is from O towards L. Let the centrifugal force in the point L be expressed by the line LM, and to CL continued to N let fall the perpendicular MN. The force LM, according to the known method of resolving forces (of which I shall speak hereafter) may be resolved into two forces denoted by the lines NM, and LN; whereof the latter only acts in opposition to gravity, as pulling directly against it; the other no way affecting the same; consequently, supposing the centrifugal force at L to be the same as at K, yet will the force of gravity be less diminished by it at L than at K, because at L part only of the centrifugal force resists that of gravity, whereas at K the whole centrifugal force acts in opposition thereto.

From what has been said it follows, that the force whereby gravity is lessened in the æquator is to the force whereby it is lessened in any other part of the earth's

earth's surface as the square of radius to the square of the sine of the complement of latitude. For the centrifugal force in the point K, the whole of which acts in opposition to gravity, is to the centrifugal force in the point L, as CK or CL to OL; but the whole centrifugal force in L is to that part of it which opposes gravity, as LM to LN, that is, because the triangles LNM and COL are similar, as CL to OL; wherefore the centrifugal force, or the force which opposes gravity in the point K, is to that part of the centrifugal force which opposes gravity in the point L in the duplicate ratio of CL to OL, that is, as the square of radius to the square of the sine of the complement of latitude.

LECTURE III.

OF REPULSION AND CENTRAL FORCES.

AS experience has convinced us that there are Powers in nature, whereby not only the larger systems and collections, but likewise the smaller parcels and particles of matter are in some cases made to tend to one another; the same experience will inform us of other powers in nature, whereby the parts of matter do in some circumstances recede and fly from each other. For if the disagreeing pole of a loadstone be moved towards a magnetical needle floating on water, the needle will recede; and the nearer the stone is brought to it, with the greater violence and precipitation will it fly off; the repelling power, like the attractive, exerting itself with greater vigor at smaller distances.

This repelling power is likewise evident from the experiments which were made relating to electrical attraction: for it was observable that upon holding the glass tube, when heated by friction, nigh small pieces of brass-leaf; some of those pieces which by the

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Exp. I.

the attraction had been raised towards the tube, were, before they could reach it, driven back again with great precipitation: and of those which adhered to the tube some were thrown off with a velocity much greater than could possibly arise from the force of gravity in such light bodies, and consequently must have been driven down by some repelling power in the glass. And in the experiments of the glass globe and woollen threads; when the threads were, by the attractive force of the globe, made to extend themselves towards its surface, upon moving one's finger towards them, they were observed to recede and fly off, and that at considerable distances from the finger; which plainly argues a repelling power interceding the finger and threads, when under the circumstances of those experiments. From this power it is that the leaves of the sensitive plant shrink and retire from the touch of an approaching hand. And to the same power we are to attribute the elasticity of the air; as also the shaking off of the particles of light from the sun and other luminous bodies.

Besides the forementioned principles of attraction and repulsion, whereby nature seems to perform most of her operations, and which for that reason are very properly stiled active principles; there is another of a passive nature, commonly called the *vis insita* and *vis inertiae* of matter, a force arising from the inertness or inactivity of matter; which force in any body is proportional to its quantity of matter. From this force result three passive laws of motion, usually called by modern naturalists the three LAWS OF NATURE*.

The

* By virtue of the *vis inertiae* it is, that the motion of a body produced by a force impressed upon it, is measured by the quantity of matter in the body and its velocity, taken together. For the body by its *vis inertiae*, resists the force impressed upon it which causes its motion, in proportion to its quantity of matter; and consequently, to produce a given tendency in the body forward,

The first of these laws is, That every body, in proportion to its quantity of matter, perseveres in its present state, whether it be of rest or uniform motion straight forward in a right line. For as every particle of matter is with respect to itself perfectly unactive, it is utterly impossible it should produce any alteration in its own state; for which reason (setting aside all impressions from external causes) if it be at rest, it must continue so for ever; or if in motion, it must for ever continue its motion without any change either as to direction or velocity: so then the continuation of motion in bodies projected (the cause whereof very much perplexed the naturalists of old) is to be attributed to the passive nature of matter, which makes it as impossible for a body of itself to stop its own motion when once begun, as it is for it to move itself originally, or of itself to change its figure.

As a consequence of this law it follows, that all motion is of itself equable and rectilineal. For first whatever be the velocity wherewith a body begins to move, the same velocity must continue during the motion, unless a change be made therein by some cause from without; wherefore the body

forward, by which it moves at a given rate or with a given velocity, the force impressed must be proportional to the resistance arising from its *vis inertiae*, that is, to its quantity of matter; and if the quantity of matter in the body, and consequently the resistance arising from its *vis inertiae*, be given, the force impressed will be proportional to the tendency forward which it communicates to the body, that is to its velocity; and if neither the quantity of matter in the body nor its velocity be given, the force impressed will be in a ratio compounded of the quantity of matter and velocity; that is, putting F for the force impressed, Q for the quantity of matter in the body, and V for its velocity, F will be as $Q \times V$. But the motion of the body is the effect produced by the force F , and is proportional to it, that is, putting M for the motion of the body, M is as F . And therefore, by proportion of equality, M will be as $Q \times V$; or the motion of the body will be measured by its quantity of matter and velocity taken together.

must

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must in equal times move thro' equal spaces with an uniform velocity; that is, the motion must be equable. And as motion is by virtue of this law in itself equable; so is it likewise rectilinear: for motion cannot otherwise be conceived than as directed and determined towards some place or other; and it must by the foregoing law keep the direction which it had at first, until it be hindered or put out of its way by some extrinsic cause, that is, it must move on in a right line. If therefore a body moves in a curve, that curvature must of necessity proceed from some external force continually acting on the body; and whenever that force ceases to act, the body will move forward in a right line touching the curve in that point wherein the body is at the instant of time when the force ceases to act. Thus for instance, if a stone moved about in a sling, be set at liberty by slipping one end of the sling; it will not continue its circular motion, but go on in a right line touching the circle made by the circumvolution of the sling in that point where the stone is let go. If the circle BCDE be the curve described by the revolution of the sling AB about the center A; and if the stone be let off at the point B, it will move on in the right line BG, which touches the circle in B. For by the law, the natural tendency of the stone in the point B is along the line BG, tho' by the force of the sling it be made to revolve in the curve. And what has been said of the stone in the point B, is in like manner true of the same at any other point, as C, D, or E; for in those points its tendency is along the lines CF, DH, and EK.

Fig. 8.

Another consequence of the foregoing law is that all bodies, which revolve about a center, must endeavour to recede from the center: for since bodies, that are moved round in a curve, do of themselves in every point of the curve tend to move in the tangents to each point; and since all the parts of the

the tangents are more distant from the center of motion than are the parts of the curve, as is evident from the figure; it is manifest that bodies so moved must perpetually endeavour to fly off from the center of motion, which endeavour of receding is commonly called the centrifugal force; and it is opposed to the centripetal force, or that force which by drawing the bodies towards the center makes them to revolve in a curve.

These two forces are by one common name called the central forces: and they are in all cases equal the one to the other. For let us suppose a body to revolve in the orbit EAC , and that being in the point A , the centripetal force ceases to act; it will then move forward in the direction of the tangent AB , and BC will be the space thro' which the body recedes from the orbit by means of the centrifugal forces; and if AB be in its nascent state, the centrifugal force will be as BC ; but if the centripetal force acts at A , it will make the body describe the arc AC in the same time that it would describe the tangent AB , in case it were not acted upon by the centripetal force; consequently, the space BC is described by means of the centripetal force; and the arc AC being in its nascent state, the centripetal force will be as BC , and of consequence equal to the centrifugal.

In treating of these central forces I shall proceed in the following manner. First, I shall consider two equal bodies moving uniformly in two different circles; and thence deduce one general expression for the central forces in the terms of the circle. Secondly, by substituting other proportional quantities in the place of those which constitute the general expression, I shall form other general expressions for the same forces. Thirdly, by a proper application of those expressions I shall determine the laws of central forces in particular cases, and

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Fig. 9.

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and at the same time confirm each law by an experiment.

Fig. 10.
11.

As to the first, if two equal bodies moving uniformly in the circles marked 1, and 2, do in the first proportion of time taken indefinitely small describe nascent arches AC ; and if from the points C draw the lines CB , perpendicular to the tangents AB , those lines will express the proportion of the central forces. For since the time in which the arches AC are described is indefinitely small, the bodies will be carried thro' the spaces BC , by one single impulse of each central force; for which reason the motions of the bodies thro' those spaces will be uniform; consequently, since the time of the motion is the same, and the bodies equal, the motions will be as the spaces described, that is, as the lines BC ; but forces which generate equal motions are to one another as the motions generated; that is, in this case, as the lines BC ; which lines being equal to the versed sines AD of the arches AC , must be equal to the squares of the arches AC , divided by their respective diameters AE . For from the nature of the circle, the versed sine of any arch is equal to the square of the chord divided by the diameter; but as in this case the arches AC are supposed to be nascent, they do not differ from their chords, and therefore in each circle the versed sine of the arch AC (which versed sine expresses the central force) is equal to the square of the arch divided by the diameter: consequently, the central forces are as the squares of the nascent arches applied to their respective diameters and forasmuch as those nascent arches are to one another as any other two arches which are described by the revolving bodies in a given time, the central forces of two equal bodies revolving uniformly in different circles, are to one another as the square of the arches described in a given time applied to their

by an equal respective diameters ; or because the diameters
as the radii, as the squares of the arches applied
owing their respective radii. Wherefore putting A to
in the place of the arch of a circle described in a given
time, D for the radius, and F for the central force.
as $\frac{A^2}{D}$, as it stands in the first place of the first
rank of symbols.

$$F \text{ is as } \frac{A^2}{D}.$$

$$F \text{ is as } \frac{QA^2}{D}.$$

$$F \text{ is as } \frac{V^2}{D}.$$

$$F \text{ is as } \frac{QV^2}{D}.$$

$$F \text{ is as } \frac{D}{P^2}.$$

$$F \text{ is as } \frac{QD}{P^2}.$$

$$F \text{ is as } DN^2.$$

$$F \text{ is as } QDN^2.$$

As the bodies are supposed to move uniformly in
circles, it is evident that the arches described in
a given time are as the velocities of the revolving
bodies ; and therefore in the general expression for
the central force, the velocity of the body may be
substituted in the place of the circular arch ; whence
putting V for the velocity of the body, F is as
as in the second place of the first rank of
symbols, which is a second general expression for
the central force.

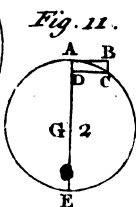
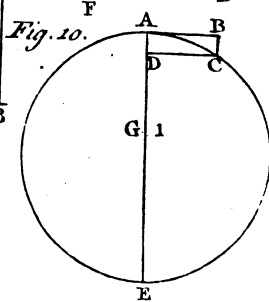
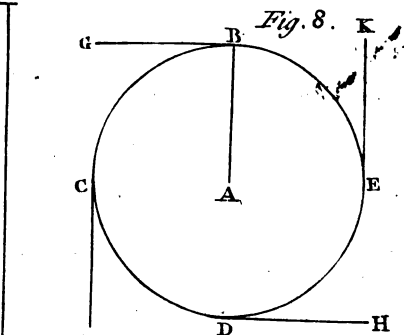
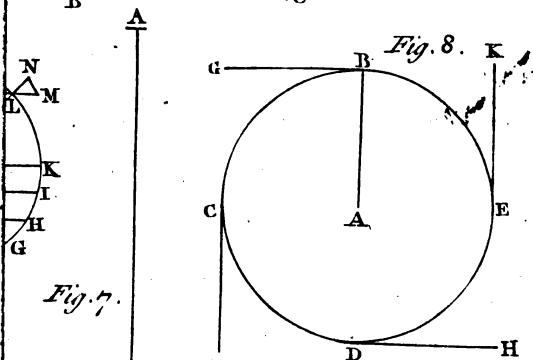
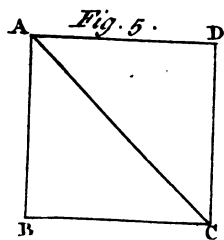
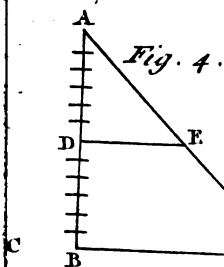
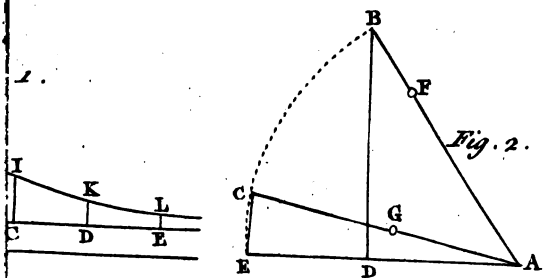
Again, the velocity of a body moving uniformly
in a circle, is as the radius applied to the periodic
time, or the time of one intire revolution. For if
the velocity of the body be given, the periodic time
must be proportional to the circumference of the
circle, inasmuch as a body, which with a given ve-
locity describes a certain space in a certain time,
will require a double or triple time to describe a
double or triple space ; and universally whatever be
the magnitude of the space, the time in which it is
described

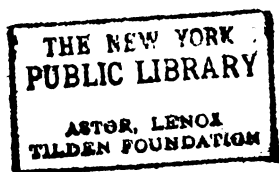
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described will be proportional to it. If the circumference of the circle be given, the periodic time will be inversely as the velocity with which the body moves; for if a body moves thro' a given space with a certain velocity in a certain time, it will with double the velocity move through the same space in half the time, and with a triple velocity in one third of the time; and in general, in whatever proportion the velocity is increased, in the same proportion will the time be lessened; that is, the periodic time will be inversely as the velocity. If therefore neither the circumference of the circle, nor the velocity of the body be given, the periodic time will be directly as the circumference, and inversely as the velocity; that is, as the circumference applied to the velocity; or (because the circumference is as the radius) as the radius applied to the velocity. Wherefore putting P for the periodic time of a body revolving in a circle, P is as $\frac{D}{V}$, and consequently V is as $\frac{D}{P}$. If therefore in the

second general expression $\frac{D}{P}$ be substituted in the place of V , we shall have a third general expression for the central force, wherein F is as $\frac{D}{P^2}$, as in the third place of the first rank of symbols.

Again the periodic time of a body revolving uniformly is inversely as the number of revolutions performed in a given time. For if the periodic time of a body be such, as that in a given time it can perform a certain number of revolutions; if the periodic time thereof be doubled, it will perform but half the number of revolutions in the same time; and if the periodic time becomes thrice as great, it will perform but one third of the number of revolutions in the given time; and so on, as the periodic time is enlarged the number of revolutions will





will be diminished in the same proportion, so that putting N for the number of revolutions in a given time, P will be as $\frac{1}{N}$. Consequently, if in the third

general expression $\frac{1}{N}$ be substituted in the room of P , we shall have a fourth general expression for the central force, wherein F is as DN^2 , as it stands in the last place of the first rank of symbols.

In collecting these general expressions, I have all along supposed the quantity of matter in the revolving body to be given; and for that reason have not made it a part of those expressions, inasmuch as it may be denoted by unity; and as such, whether it be taken in or left out, it will not vary the expressions. But the case will be different, if the quantity of matter in the revolving body varies; because the central forces, and consequently the expressions for those forces will likewise vary; so as to be greater *cæteris paribus* in larger quantities of matter than in smaller. For the whole central force of any body is made up of the forces of each particle whereof the body consists; and therefore the more numerous the particles of matter are in any body, the greater will its central force be; so as to be double in a double quantity of matter, triple in a triple quantity; and so on in proportion to the quantity of matter. In order therefore to render the expressions yet more general, let Q be put for the quantity of matter in the revolving body, and let it be multiplied into each of the four expressions, as in the second rank of symbols.

Before I apply these expressions to the several particular cases, I shall offer an experiment in confirmation of what I just now proved, *viz.* that the greater the quantity of matter in any body is, the greater is the central force.

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Exp. 1.

Let three glass tubes half full, one with mercury and water, and another with water and small leaden bullets, the third with water and a piece of cork, be stopped close, and made fast to an inclined plane; and let the plane be so fixed to a table moveable about its center by means of a wheel and axle, as that the lowermost part of the plane may rest upon the center of the table. As long as the table continues at rest, the liquors and solids contained in the tubes will by reason of their gravity possess themselves of those parts of the tubes which lie next the center of the table, leaving the remoter parts empty: and of the two bodies included in each tube, that which is heaviest will be nearest the center; but upon turning the table about, the several bodies will by reason of their centrifugal forces, whereby they are carried from the center of motion, fly to the uppermost parts of the tubes; and in each tube, the heavier body will possess the uppermost place as being indued with the stronger centrifugal force.

Exp. 2.

If bodies moving in equal circles perform their revolutions in equal times, or in other words, if the velocities of bodies revolving in circles be equal, and their distances from the center likewise equal, their centrifugal forces are as their quantities of matter. For in the second general expression, since V and D are given, F is as Q ; that is, the central force is as the quantity of matter; which is confirmed by the following experiment. Let two small troughs be so fixed to two moveable tables, as that the centers of the troughs may lie upon the centers of the tables, and let the centers of the tables be fixed to two axles, on each of which is a grooved wheel, with equal diameters; let the two wheels be turned by means of one and the same chord going round them: it is manifest, that as the wheels are equal, they, and consequently the tables with their affixed troughs, must perform their revolutions in the same time; and the parts of the
tables

tables and troughs, whose distances from their respective centers are equal, will revolve equally swift; and so likewise must all bodies that are placed in the troughs at equal distances from the centers; so that by this contrivance, if two bodies be placed one in each trough at equal distances from the centers, they will revolve equally swift. Let then two balls, whereof one is double the other, be laid one in each trough, and let each ball be fastened to one end of a chord, whose other end passing thro' an hole in the center of the table is made fast to a weight, which rests upon the floor; and let the lengths of the chords be such, as that being stretched, and the weights not raised, the balls in the troughs may be equally distant from the centers. This being done, if the weights be to one another as the balls, and if the tables be turned about with such a velocity as that the centrifugal forces of the balls may be sufficient to raise the weights, they will be lifted up precisely at the same time. Whence it appears, that in this case the centrifugal forces are as the quantities of matter, inasmuch as they overcome resistances which are in that proportion.

If equal bodies moving in unequal circles perform their revolutions in equal times; or in other words, if the quantity of matter in the revolving bodies be given, as also the number of revolutions performed in a given time, their centrifugal forces are as their distances from the center. For in the fourth general expression since Q and N are given, F is as D ; that is, the force is as the distance. For the confirmation whereof, let two equal balls be placed in the troughs at distances from the centers, which are as one and two, and when the tables are turned about, that ball, whose distance from the center is double, will raise a double weight.

If equal bodies move in unequal circles with equal velocities; or more generally, if the quantity of matter in the revolving bodies be given, as also the

D 3

velocity

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velocity wherewith they revolve; their central forces are inverſly as their diſtances from the center. For in the ſecond general expreſſion ſince Q and V are given, F is as $\frac{1}{D}$, that is, the force is inverſly as the diſtance. Before I mention the experiment whereby this law is confirmed, I muſt obſerve to you, that to the axle of one of the tables is fixed a ſecond wheel, whoſe diameter is but one half of the diameter of the other wheel; and therefore when the chord goes round the ſmaller wheel, the table muſt turn as faſt again as when it goes round the larger wheel; ſo that the table which is moved by means of the ſmaller wheel, will revolve twice in the ſame time, that the other table which is turned by means of the larger wheel, performs one revolution.

Exp. 4.

This being premiſed, let two equal balls be ſo placed in the troughs, as that the diſtance of that ball which is to revolve by means of the ſmaller wheel, may be but one half of the other's diſtance from the center; in which caſe their velocities will be equal: for tho' the peripheries of the circles which the two balls deſcribe, are as one and two; yet will the leſſer periphery be deſcribed twice in the ſame time that the larger is deſcribed once; and therefore the ſpaces thro' which the bodies move in a given time will be equal, and of conſequence their velocities will be ſo too. If then two weights be made faſt to the chords of the balls in the manner of the former experiments; the tables being turned about, the ball whoſe diſtance from the center is as one, will raiſe twice the weight, that is raiſed by the ball whoſe diſtance is as two; ſo that the weights raiſed, and conſequently the forces which raiſe them, will be inverſly as the diſtances of the balls from the center.

If equal bodies revolve in equal circles with unequal velocities, their central forces are as the ſquares of the velocities, or becauſe the velocities are as the
number

number of revolutions in a given time, the forces are as the squares of the numbers of revolutions performed in a given time. For by the fourth general expression since Q and D are given, F is as N^2 ; that is, the force is as the square of the number of revolutions in a given time. To confirm this law, let two equal balls be placed in the troughs at equal distances from the centers; and let that table, whose axle has two wheels, be turned about by means of the smaller, so that it may perform two revolutions in the same time that the other table performs one: in this case the numbers of revolutions performed by the two balls in a given time being as one and two, their squares will be as one and four, in which proportion the weights raised will likewise be.

Exp. 5.

If unequal bodies revolve in equal circles with unequal velocities, their central forces are as the products of their quantities of matter into the squares of their respective velocities; or, which is the same thing, as the products of their quantities of matter into the squares of the numbers of revolutions in a given time. For by the fourth general expression, D being given, F is as QN^2 . Let therefore two balls, whereof one is double the other, be placed at equal distances from the centers; and let the larger revolve twice in the same time that the smaller revolves once. In this case the quantity of matter in the lesser ball, which is as unity, being multiplied into the square of its number of revolutions in a given time, which is likewise as unity, gives one for the product. And the quantity of matter in the larger ball, which is as two, being multiplied into the square of its number of revolutions in the given time, which square is as four, gives eight for the product: so that the weights raised by the two balls will be as one and eight.

Exp. 6.

If unequal bodies revolve in unequal circles with unequal velocities, their forces are as their quantities of matter multiplied into the squares of their re-

D 4

spective

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spective velocities, and that product divided by their respective distances from the centers; or what amounts to the same thing, their forces are as the products arising from the continued multiplication of their quantities of matter into their respective distances from the centers, into the squares of their numbers of revolutions in a given time; or to use the mathematical phrase, their forces are in a ratio compounded of their quantities of matter, of their distances from the center, and of the squares of their numbers of revolutions in a given time. For by

Exp. 7. the fourth general expression, F is as QND^2 . To confirm this law by an experiment, let two balls, whereof one is double the other, be placed in the troughs, so as that the distance of the smaller from the center may be to the distance of the larger as two to one; and let the larger revolve twice in the same time that the smaller revolves once. In this case the quantity of matter in the smaller body, which is as one, being multiplied into the distance from the center, which is as two, and the product being multiplied into the square of the number of revolutions performed by the smaller body in a given time, which is as one, gives two for the product. In like manner the quantity of matter in the larger body, which is as two, being multiplied into the distance from the center, which is as one, and the product of that multiplication being again multiplied into the square of the number of revolutions performed by the larger body in the given time, which square is as four, gives eight for the product; consequently the weights which are raised, as also the forces which raise them, are as two and eight, or one and four.

If equal bodies revolve in unequal circles in such a manner as that the squares of their periodical times are as the cubes of their distances from the center, their central forces are inversely as the squares of their distances from the center. For since the quantity

quantity of matter in the revolving bodies is given, and the cubes of the distances are as the squares of the times; if in the third general expression the cube of D be substituted in the room of the square of P , F will be as D divided by the cube of D , or as one divided by the square of D ; that is, the force will be inversely as the square of the body's distance from the center. To confirm this law, let two equal balls be placed in the troughs, so as that the distance of one from the center may be as two, and the distance of the other as three and one sixth; and let that which is at the smallest distance revolve twice in the same time that the other revolves once; so that their periodical times may be as one and two, the squares of which being one and four, are very nearly proportional to the cubes of the distances; for the cube of the smaller distance is eight, and that of the larger thirty-two very nearly; consequently, the balls must raise weights which are to one another inversely as the squares of the distances from the centers; that is, the weight raised by the ball, whose distance is as two, must be to the weight raised by the ball whose distance is as three and a sixth, as the square of the last distance to the square of the former, that is, as ten to four, or five to two very nearly.

If the squares of the periodical times be proportional to the cubes of the distances, and the revolving bodies unequal, the central forces are directly as the quantities of matter in the bodies, and reciprocally as the squares of their distances from the center. For in the third general expression, if the cube of D be substituted in the room of the square

of P ; F will be as $\frac{Q}{D}$. If therefore all things remain as in the last experiment, excepting that the body which is at the greater distance from the center is to the body less distant, as two to one; the weight which is raised by the former, will be to the weight

10

raised

LECT.
III.

Exp. 8.

$$\begin{array}{r} 2 \quad 2 \\ \hline 3 \quad 16 \\ \hline \end{array}$$

$$p:p:2:2$$

$$4:1:8:$$

$$\begin{array}{r} 3.16 \\ 3.16 \\ \hline 19.00 \\ 31.65 \\ \hline 450 \\ \hline 10.0000 \end{array}$$

LECT.
III.

raised by the latter, as two to two and a half; that is, the weights raised, will be as the products arising from the multiplication of the quantity of matter in one body into the square of the other body's distance.

Among the several laws of central forces, that which obtains in nature, and by virtue whereof the heavenly bodies are made to revolve in their several orbits, is, where the forces are to one another inversely as the squares of the distances of the revolving bodies from the center. For it has been found by observation, that all the planets as well primary as secondary revolve either in circular orbits or such as are nearly so. And that the six primary planets move about the sun as their center in such a manner, as that the cubes of their mean distances from the sun are very nearly proportional to the squares of their periodical times. And the same thing has been discovered with regard to the four secondary planets or satellites that move about JUPITER, as also with respect to the other five that revolve about SATURN. And therefore the forces whereby they are retained in their orbits must be in the inverse ratio of the squares of their distances from the central bodies about which they revolve.

Exp. 9. If two bodies are by means of their mutual attraction made to revolve about each other, and also about a fixed point; and if their distances from that fixed point, be reciprocally proportional to their quantities of matter, that is to say, if as much as one body exceeds the other in quantity of matter, so much is its distance from the fixed point exceeded by the other's distance from the same point; or what amounts to the same thing, if the product arising from the multiplication of one body into its distance from the fixed point, be equal to the product arising from the like multiplication of the other body into its distance from the fixed point, their central forces are equal. For as the two bodies must of necessity

necessity perform their revolutions in the same time ; the number of their revolutions in a given time is given : and therefore by the fourth general expression, F is as QD , that is, the central force is as the product arising from the multiplication of the quantity of matter into the distance from the center, or fixed point ; but by supposition the product of one of the bodies into its distance from the fixed point, is equal to the product of the other into its distance, consequently, their central forces are equal ; for which reason neither of them can fly off from the fixed point so as to draw the other after it ; for however strongly either of them endeavours to recede by virtue of its own centrifugal force, it is with equal strength drawn the contrary way by the centrifugal force of the other. But if the distances of the bodies from the fixed point be not reciprocally proportional to their quantities of matter ; that body, whose distance with regard to the distance of the other is greater than in the forementioned proportion, will fly off and draw the other after it ; for in this case, the product of the former body into its distance from the fixed point is greater than the product of the latter into its distance ; which products being as the centrifugal forces of the bodies, the former body will have a greater centrifugal force than the latter, and of course must recede from the fixed point, and drag the other after it ; all which is fully confirmed by the following experiments. Let two equal balls be tyed together by a small chord ; and let them be laid in one and the same trough, one at each end, so as that the chord being stretched may have its middle point just over the center of the table ; let then the table be turned about, and the balls will revolve about the center without flying off either way ; and continue so to do as long as the motion of the table lasteth. And the same thing will likewise happen tho' one ball be double the other, provided its distance from the

Exp. 19.

center of the table be but one half of the distance of the smaller. But when equal balls are made use of, if one of them be placed at a greater distance from the center than the other, upon turning the table it will fly off and draw the other after it. So likewise when unequal balls are made use of, should that which is double the other be placed at a distance from the center greater than one half of the distance of the smaller, it will fly off and draw the smaller after it. And on the other hand, if the distance of the larger be less than half the distance of the smaller, the smaller will in that case fly off and draw the larger after it.

LECTURE IV.

OF THE COMPOSITION AND RESOLUTION
OF MOTION.LECT.
IV.

THE second LAW OF NATURE, resulting from the inertness of matter, is, that whatever motion, or change of motion is produced in any body, it must be proportional to, and in the direction of the force impressed. For since a body cannot by reason of its inactivity contribute to the production of its own motion, or of any change therein, it is plain, that whatever motion or change of motion is generated in any body, it must intirely proceed from the force impressed on the body; and of consequence, since effects are ever proportionate to their adequate causes, must be proportional thereto. And it must likewise be directed and determined towards the same part with the generating force. Wherefore if the body whereon the impression is made, was in motion before the impulse, that motion will be retarded or accelerated according as the force impressed opposes it, or conspires therewith; or if it acts obliquely to the same, the direction thereof will be changed, and the body will

will move in a direction situated between the direction of its former motion, and that of the impressed force. For instance, if a body moving from A towards B, be impelled at the point A by a force acting in the direction AC, it will move along a line as AD placed between AB and AC, the situation of which may be thus determined. Let the lines AB denote the velocity wherewith the body moves in the direction AB; and let AC denote the velocity wherewith the body would move by virtue of the impulse along the line AC, supposing it had no other motion: that is, let AB be to AC as the space described by the body in a given time in the direction AB, to the space described by it in the direction AC, each of the motions being considered singly and apart; then completing the parallelogram ABDC, and drawing the diagonal AD, that diagonal is the line in which the body moves; for the proof of which, let us suppose a small inflexible wire equal in length to the line AB, to pass thro' the center of a ball, and that whilst the ball moves uniformly on the wire from A towards B, with a velocity which is as AB, the wire is also moved uniformly from AB towards CD, with a velocity which is as AC, and in such a manner as to be always parallel to AB, and with its extremities to describe the lines AC and BD. Then, forasmuch as the spaces described in a given time where the motions are uniform, are to one another as the velocities of the motions; it is evident, that in whatever time the ball moves the length of the wire, in the same time will the wire move the length of AC, to wit, from AB to CD; consequently, at the end of that time the ball will be found in D at the extreame point of the diagonal AD. From any point in the diagonal taken at pleasure as E, let the line EF be drawn parallel to DB, and from the nature of similar triangles, AF will be to FE, as AB to BD, that is, as the velocity

LECT.
IV.Pl. 2.
Fig. 1.

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IV.

velocity of the ball to the velocity of the wire; consequently, in the same time that the ball moves the length of AF along the wire, the wire will move the length of FE , from AB to KL ; and the point F , which is the place of the ball on the wire, will be found in E . And what has been thus proved in relation to the two points D and E of the diagonal, may in the very same manner be demonstrated of any other point in the same line; wherefore the ball will by virtue of its own motion, and that of the wire, whereof it partakes, be carried in such a manner as to be always found in the diagonal AD ; that is, it will by virtue of its compound motion describe the diagonal line. This being so, it plainly follows, that if the wire be taken away, and the ball at A have two motions impressed upon it at once; one in the direction AB , the other in the direction AC ; and if the motions impressed, or, which is the same thing, if the forces impressing those motions be to one another in the proportion of AB to AC , the ball will by virtue of the double impression, move along the diagonal AD . For as to the effect it matters not whether the motion which the ball has in the direction AC arises from a force impressed on it at the point A , or whether it be communicated by a wire supporting the ball, and carrying it along with it in that direction.

Exp. 11. To confirm this by an experiment, let three ivory balls of equal size, be suspended from three pins by strings of equal lengths, and let the middle ball rest over one angle of a wooden square; then let each of the extream balls be let fall separately from the same height, in such manner as to strike the middle ball in the direction of one side of the square, and the middle ball will by each of the strokes made separately, be moved along over that side of the square, which correspondeth to the direction of the stroke; but if the two balls be at the same instant of

of time let fall from equal heights, so as that they may strike the middle ball at once, and in the directions of the two sides of the square, the middle ball will by the double stroke, be driven over the diagonal of the square,

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As a COROLLARY it follows, that a body will in the same time describe the diagonal AD of a parallelogram with two forces conjoined, that are to one another as the sides AB and AC , that it would the respective sides with each of those forces separately. As also, that the velocity wherewith a body moves along the diagonal, is to the velocity wherewith it is carried along the sides when acted upon by each force singly, as the diagonal to each side respectively: consequently, if the two forces be given, the velocity along the diagonal, which arises from the conjunction of both forces, will be so much the greater, by how much the angle BAC is less; for as that angle is diminished, the diagonal which in this case denotes the velocity, is lengthened, till at last the angle vanishing by the coincidence of the sides, the diagonal becomes equal to both the sides taken together; and the velocity of the body equal to the sum of the velocities wherewith the body would move, were each of those forces impressed upon it in the same direction. Thus the lines AB and AC being placed at three different angles, so as to constitute the sides of three different parallelograms, (the diagonals whereof are represented by the pricked lines) it is evident to sight, that as the angle BAC grows less, the diagonal grows longer; and that when the angle vanishes by the coincidence of AB with AC , the diagonal AD becomes equal to AC and CD , that is, to AC and AB ; and the velocity denoted by AD , is in that case a *maximum*, or the greatest that can arise from the conjunction of those two forces. On the other hand, as the angle enlarges, the velocity along the diagonal must decrease, till at length the angle vanishing

Pl. 2.
Fig. 2.

Pl. 2.
Fig. 3.

LECT.
IV.Pl. 2.
Fig. 4.

vanishing by the two sides becoming one right line, the velocity becomes equal to the difference of the velocities, arising from the impression of each force when made singly and separately. Thus the lines A B and A C being as before placed at three different angles B A C, it is evident that the diagonals A D represented by the pricked lines, grow shorter as the angle B A C enlarges ; till at last the angle, and with it the diagonal vanishing, the two sides B A and A C constitute one right line as B A C, wherein the body is, as it were, carried two contrary ways, to wit, from A towards B by the force which acts in the direction A B, and from A towards C by the force acting in the direction A C, and the difference of the velocities, which arise from the impressions of the two forces when they act separately, is the velocity wherewith the body actually moves in the direction of the stronger force, which velocity is a *minimum*, or the least velocity that can arise from the joint action of those two forces.

Pl. 2.
Fig. 6.

As a second COROLLARY it follows, that a body may be moved thro' one and the same line by numberless pairs of forces acting upon it. For if instead of the force, whose direction is A B, we suppose another, the direction whereof is A E ; and if instead of the force acting in the direction A C, we suppose one to act in the direction A F, and that those forces are to one another as A E to A F ; then compleating the parallelogram A E D F, the line A D will be the diagonal of this parallelogram, as well as of the former ; and therefore the body will from the joint action of these two forces describe the same line A D which it did before : and as A D may be made the diagonal of numberless parallelograms, it is evident that it may be described by a body acted upon by numberless pairs of forces in different directions. And not only so, but it may likewise be described by a body, where-
on

on a great number of forces act at the same time; for as the forces acting upon the body in the directions AB and AC make it to move along the diagonal AD, so may the direction along AB arise from the directions of two other forces, and each of those from the directions of two others, and so on without number. Hence we see, that all forces and motions whatever may be resolved into innumerable forces and motions; and any simple direct force or motion may be looked upon as compounded of innumerable oblique forces or motions. For the line and direction of the motion is the same, whether that motion be compounded of two motions arising from forces impressed in the directions AB, AC, or in the directions AE, AF, or arise from the impression of a single force in the direction AD; and therefore the motion along the line AD, tho' it be simple, arising from one single force acting in that direction, yet may it be considered as compounded of two or more motions in other directions, such as AB and AC, or AE and AF, since the very same motion would arise from such a composition.

This composition and resolution of motions and forces is of singular use in mechanicks; for by the help thereof, the effects of powers acting in oblique directions are readily determined, as will appear hereafter.

The third LAW OF NATURE arising from the inertness of matter is, that reaction is always equal to action, and contrary thereto; or, in other words, that the actions of two bodies, one upon another, are constantly equal, and in directions contrary to each other; so that whatever change is made in the state of one body, whether at rest or in motion, by the action of another; the same change is produced in the state of the other by the reaction of the former; but the tendencies or directions of those changes are contrary ways. Thus, when one presses

E

a stone

Pl. 2.
Fig. 6.

LECT.
IV.

Exp. 12.

a stone with his finger directly downward, the finger is equally pressed by the stone, and that directly upward. And when a horse draws a load, he is equally drawn back by the load ; for as much as he promotes the progress of the load, so much is he retarded in his own motion ; that is, he is in effect drawn back ; for the same force of muscles and sinews, which he exerts in order to drag on the load, would, if he was freed from the incumbrance, carry him forward to a distance much greater than what he reaches in the same time whilst tied to the load ; and consequently, as far as his progress falleth short of that distance, so much is he in effect drawn back ; and whatever motion he communicates to the load, so much does he lose of his own, the load reacting upon him with the same force that he acts upon it ; for which reason, if by addition of weight the load be so far increased as to require the whole strength of the horse to move it, no motion will ensue, the whole power of the horse, wherewith he endeavours to go forward, being but just equal to the reaction of the load whereby he is drawn back. This equality of action and reaction obtains in all kinds of attractions whatever. When a loadstone attracts a piece of iron, it is equally attracted by it ; as will appear from the following experiment. Let a piece of iron and a loadstone equal in weight, be suspended by two cords of an equal length, and let the distance between them be so small, as that they may not be out of the reach of each other's attraction ; then will they, from a state of rest, begin to move towards each other, and that with equal velocities so as to meet at the middle point of their first distance : if they be again separated, and the loadstone fixed, the iron being suspended at the same distance from it as before, will move towards it, so as at length to touch it, and adhere thereto. And on the other hand, if the iron be fixed, and the stone moveable,

REACTION EQUAL TO ACTION.

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the stone will approach the iron in the same manner as the iron did the stone, all which plainly shews that the attraction between the loadstone and iron is mutual, the one drawing the other as much as it is drawn by it: so that the reaction of the iron upon the stone is exactly equal to the action of the stone upon the iron.

The equality of action and reaction with respect to attractions is likewise manifest from hence, that if a man placed in a boat, draws another boat by means of a rope fastened thereto; the boat wherein the man is placed will be equally drawn with the other, and the two boats will approach one another with equal quantities of motion; so that if they be equal in weight and of the same size and shape, they will approach with equal velocities, and meet at the middle point; but if one be heavier than the other, then by how much it exceeds the other in weight, by so much will it be exceeded by the other in the velocity of its motion; for instance, if the weight of one be to the weight of the other as one to two, then will the velocity of the former be to the velocity of the latter as two to one; that is, their velocities will be reciprocally proportional to their weights. To confirm this by an experiment, let a cord be made fast to one end of a small boat, and let it pass over a pulley fixed to the end of another small boat of the same shape and size, and let a weight be tied to the end of the cord, and hang in the water; this being done, let the boats be placed at such a distance as that the cord may be stretched, then letting go the boats the weight will descend, and in descending draw the boat to whose end the cord is fastened towards the other, and at the same time the other will move towards it; and when they come together, the space described by the boat whose weight is as one, will be to the space described by the boat, the weight whereof is as two, as two to one; that is, if the

Exp. 13.

E 2

distance

LECT.
IV.

distance between the two boats be divided into three equal parts, that boat which is double in weight to the other, will move thro' one of those parts in the same time that the lighter moves through the other two.

As action and reaction are equal with regard to attractions, so are they likewise in respect of strokes or impulses made by bodies one upon another; the force of two bodies, striking each other equally, affecting the motions of both, and producing equal changes therein towards contrary parts. On this equality of action and reaction do the several laws which have been collected concerning the collision of solid bodies in a great measure depend; which laws, as they relate to bodies void of elasticity, I shall now explain; in doing of which, I shall lay down one general PROPOSITION concerning the collision of such bodies, whence I shall deduce the laws of particular cases, and at the same time confirm each law by an experiment.

The PROPOSITION is as follows: *If two bodies void of elasticity move in one right line, either the same or contrary ways, so as that one body may strike directly against the other; let the sum of their motions before the stroke when they move the same way, and the difference of their motions when they move contrary ways, be divided into two such parts as are proportional to the quantities of matter in the bodies; and each of those parts will respectively exhibit the motion of each body after the stroke.* For instance, if the quantities of matter in the bodies be as two and one, and their motions before the stroke as five and four, then the sum of their motions is nine, and the difference is one; and therefore when they move the same way, the motion of that body, which is as two, will after the stroke be six, and the motion of the other three: but if they move contrary ways, the motion of the greater body after the stroke will be two thirds of one, and of the lesser one third of one.

For

For since the bodies are supposed to be void of elasticity, they will not separate after the stroke, but move together with one and the same velocity ; and of consequence, their motions will be proportional to their quantities of matter ; and from the equality of action and reaction it follows, that no motion is either lost or acquired by the stroke when the bodies move the same way, because whatever motion one body imparts to the other, so much must it lose of its own ; consequently the sum of their motions before the stroke is neither increased nor diminished by the stroke, but is so divided between the bodies, as that they may move together with one common velocity, that is, it is divided between the bodies in proportion to their quantities of matter ; but it is otherwise where the bodies move contrary ways ; for then the smaller motion will be destroyed by the stroke, as also an equal quantity of the greater motion, because action and reaction are equal ; and the bodies after the stroke will move together equally swift, with the difference only of their motions before the stroke ; consequently, that difference is by means of the stroke divided between them in proportion to their quantities of matter.

The several particular cases concerning the collision of bodies may be reduced to four general ones. For, 1st, it may be that one body only is in motion at the time of the stroke. Or, 2^{dly}, they may both move one and the same way. Or, 3^{dly}, they may move in direct opposition to each other, and that with equal quantities of motion. Or, lastly, they may be carried with unequal motions in directions contrary to each other. As the bodies may be either equal or unequal, each of these four general cases may be looked upon as consisting of two branches ; and as such I shall consider them, and treat of them in the order, wherein I have laid them down.

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Exp. 14.

As to the first, if a body in motion strikes another equal body at rest, they will by the proposition move together, each of them with one half of the motion that the body had which was in motion before the stroke; and since the quantity of motion in any body, is as the product arising from the multiplication of its quantity of matter into its velocity; the common velocity of the two bodies, will be but one half of the velocity of the moving body before the stroke. For the confirmation whereof let two equal balls of clay be suspended from two pins of an equal height, by threads of an equal length, and in such a manner, as that when they hang freely they may just touch one another, and that their centers and point of contact may lie in a right line parallel to the horizon. This being done, and one of the balls being at rest, let the other be removed to any distance from it, and then let fall; it will in its descent describe the arch of a circle, and by the time it arrives at the lowest point of the arch, that is, when it comes to touch the quiescent ball, it will have acquired such a velocity as would carry it to the same height from which it fell, as shall be shewn when I come to treat of pendulums; and consequently, if the other ball was removed, would actually ascend to that height; but upon striking the other ball, which is of equal size, it will communicate one half of its motion to it, and they will move together with half the velocity that the moving body had at the time of the stroke, so as to ascend to one half only of the height from which the striking body fell.

That the nature of this and the other experiments relating to the collision of bodies may be more readily comprehended, I shall lay down some things concerning the motion of bodies thro' the arches of circles, the truth whereof shall be demonstrated in my lecture upon pendulums. And first, all the arches of a circle, provided they be not

not large, are described in equal times by bodies descending along them; and therefore if two bodies be let fall at the same time, one from C and the other from E, or from D and F, they will both arrive at the lowest point B, at one and the same time; and the stroke of the subsequent body upon the preceding will be made at B: and for the same reason if one be let fall from C, and the other from D or F, or one from E, and the other from D or F, they will meet and strike one another at B.

LECT.

IV.

Pl. 2.

Fig. 7.

2dly, The velocity which a body acquires in falling thro' the arch of a circle, is as the chord of the arch; that is, the velocity of a body which has fallen from C to B, is to the velocity of a body that has fallen from E to B, as the chord C B to the chord E B. And here I must observe to you, that when in the following experiments I speak of a body falling from, or rising to any height, as four, six, or ten inches, I would be understood to mean it of a body's falling thro' or moving up an arch, whose chord is of such a length.

3dly, The velocity wherewith a body begins to rise up thro' the arch of a circle, is as the chord of the arch which the body describes in its ascent. Thus the velocity wherewith a body begins to move from the point B towards D, if it ascends as high as D, is as the chord B D, but if it rises only to F, the velocity is as the chord B F. So that in the experiments the chords of the arches thro' which the bodies descend, express the velocities of the bodies in the point B at the time of the stroke; and the chords of the arches thro' which the bodies ascend after the stroke express the velocities of the bodies immediately after the stroke.

These things being laid down, I shall now proceed to determine the laws of the four general cases. As to the first, it has been already shewn, that

E 4

where

LECT.
IV.

Exp. 15.

where the moving body is equal to the quiescent, the common velocity of the two bodies after the stroke, is but one half of the velocity of the moving body before the stroke; and of consequence the motion of each body after the stroke, is equal to one half of what the moving body had before the stroke. But if the quiescent body differs in size from the moving body, then the common velocity after the stroke will be so much less than the velocity of the moving body before the stroke, by how much the sum of the two bodies exceeds the body which was first in motion. Thus if the moving body be to the quiescent as two to one, the common velocity after the stroke will be to the velocity of the moving body before the stroke, as two to three; wherefore if a ball of clay, falling from the height of nine inches, strikes another at rest, and of one half the magnitude, they will ascend together to the height of six inches only: and on the other hand, if the larger be quiescent, and the smaller falls from the height of nine inches, they will ascend to the height of three inches only; and the quantity of motion in each body immediately after the stroke will be had, by multiplying each of them into the common velocity.

As to this and all other experiments of this nature, it must be observed, that they do in some measure vary from the theory; and that for two reasons. First, because clay or any other body, wherewith these experiments can be made, is not perfectly void of elasticity. Secondly, because the air resists the motions of the balls, and by so doing diminishes their velocities.

As to the second general case, where both the bodies are in motion before the stroke, and move one and the same way: In order to find their common velocity after the stroke, let the sum of their motions before the stroke, be divided by the sum of the
bodies,

bodies, and the quotient will express the common velocity. Wherefore if two equal balls of clay be let fall at the same time, one from the height of three inches, and the other from the height of six, after the stroke they will ascend to the height of four inches and an half; for as in this case the bodies are equal, their motions are as their velocities, that is, as six and three, the sum of which being divided by two, the sum of the bodies, gives four and an half for the common velocity after the stroke.

LECT.
IV.

Exp. 16.

Where the bodies are unequal, let us suppose the preceding body to be as one, and to fall from the height of three inches as before, so that its quantity of motion will be as three; and let the subsequent body be as two, and fall from the height of six inches, so that its quantity of motion will be twelve; and the sum of the two motions will be fifteen, which being divided by three, the sum of the two bodies, gives five in the quotient; so that in this case, after the stroke, the balls will ascend to the height of five inches, and the motion of the greater will be as ten, and that of the smaller as five.

Exp. 17.

As to the third general case, where the bodies move in direct opposition to each other, if they have equal quantities of motion, they will upon the stroke lose all their motion, and continue at rest; for by the proposition, the bodies after the stroke will be carried with the difference of their motions before the stroke; which difference is supposed to be nothing. Wherefore, if two equal balls of clay be let fall at once from equal heights, upon the stroke they will cease to move; and the same thing will happen where the balls are unequal, provided the heights from which they fall are reciprocally proportional to their quantities of matter; for instance, if the balls be as one and two, let the former fall from the height of six inches, and the latter from the height of three, and upon their meeting

Exp. 18.

Exp. 19.

LECT.
IV.

meeting they will stand still, for in this case, the quantities of motion, wherewith they oppose each other, will be equal.

When two bodies meet with unequal quantities of motion, if the difference of their motions be divided by the sum of the bodies, the quotient will express their common velocity after the stroke; for by the proposition, the difference of their motions before the stroke is equal to the sum of their motions after the stroke; consequently, that difference divided by the sum of the bodies must give the velocity. Wherefore, if two equal balls of

- Exp. 20. clay be let fall at the same time, one from the height of three inches, and the other from the height of six, after the stroke they will ascend together to the height of an inch and an half; for since the balls are equal, their motions will be as their velocities, that is as six and three, the difference whereof is three, which being divided by two, the sum of the bodies, gives one and an half in the quotient. If the balls be unequal in the proportion, for instance, of two to one; and if that, which is as two, falls from the height of six inches, and the other from the height of three; after the stroke they will ascend together to the height of three inches; for the greater ball being as two, and its velocity as six, its motion is as twelve; whereas the smaller being as one, and its velocity as three, its motion is likewise as three, which being subducted from the greater motion leaves a remainder of nine; and this being divided by three, the sum of the bodies, gives three for the common velocity, or the height to which the bodies will rise.
- Exp. 21.

In order to discover the quantity of motion communicated by one body to the other, I shall lay down four rules adapted to the four general cases. And First, if one of the bodies be quiescent at the time of the stroke, let that body be multiplied into the common velocity after the stroke, and the product

duct will express the communicated motion. For since that body had no motion before the stroke, it is manifest, that whatever motion it has after the stroke must be communicated to it by the striking body; but that motion is as the product arising from the multiplication of the quantity of matter in the body into the common velocity; consequently, that product expresses the communicated motion.

Since the body which is at rest before the stroke has no motion but what is imparted to it by the striking body; and since the motion of the striking body is by the proposition to be divided between the two bodies in proportion to their quantities of matter; it follows, that where the striking body is greater than the quiescent, it will communicate less than half its motion, and where it is equal to it, it will impart one half; and where it is less, more than one half; and if the quiescent body be infinitely great with respect to the striking body, which is in effect the case where the quiescent body is fixed, so as not to give way to the stroke, the striking body will impart all its motion to the other; for as the quiescent body is supposed to be infinitely greater than the striking body, the motion, which it receives from the striking body, must bear an infinite proportion to the motion remaining in the striking body; but as the motion communicated is a finite quantity, it cannot bear an infinite proportion to the remaining motion, unless that remaining motion be in its evanescent state, and reduced to nothing.

When both the bodies are in motion before the stroke, and their motions are directed the same way, which was the second general case; the rule for determining the quantity of motion communicated is as follows. Let the preceding body be multiplied into the common velocity after the stroke, and from the product let the motion which it had before the stroke be subducted, and the remainder will

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IV.

will be the motion communicated. For the product arising from the multiplication of the preceding body into the common velocity gives the whole motion of that body after the stroke ; and therefore, if from thence be taken the motion which it had before and independent of the stroke, the remainder must be the motion acquired by the stroke.

When the bodies move towards one another with equal quantities of motion, as in the third general case ; the motion communicated is equal to the motion of either before the stroke. For as in this case, both their motions are destroyed by the stroke ; it is plain, that whichever of the bodies is considered as giving the stroke (and either of them may) it must communicate just as much motion to the other, as the other has at the time of the stroke ; for by this means the motion communicated, as it is directly opposed to the former motion of the body, will be just sufficient to destroy the same, and by so doing cause the body to rest.

When the quantities of motion in two bodies moving directly towards each other are unequal, which is the fourth general case ; the motion communicated is determined by the following rule. Let the body which had the lesser motion before the stroke be multiplied into the common velocity after the stroke, and to the product let the motion which it had before the stroke be added, and the sum will be the motion communicated. For as the body, to which the motion is communicated, does after the stroke move in a direction contrary to what it did before, it is evident, that besides the motion wherewith it is carried in that contrary direction, it must have received as much more in the same direction, as was sufficient to withstand the motion it had before the stroke in an opposite direction ; for till that motion was destroyed by an equal motion opposed thereto, the body could not change its direction, and move backward.

LECTURE

LECTURE V.

OF THE COLLISION OF ELASTICK BODIES.

HAVING given you an account of the collision of bodies void of Elasticity, I come now to consider the effects thereof in such as are elastick; by which I mean bodies that consist of such parts as yield and give way when pressed, and which restore themselves upon the removal of the pressure: if the force wherewith they restore themselves be exactly equal to the pressure whereby they are bent inward, then are the bodies said to be perfectly elastick; and such are all those bodies supposed to be, wherewith experiments are usually made for confirming the theory relating to the collision of elastick bodies; but as there is not perhaps in nature any body perfectly elastick, if among the experiments that are now to be made, any shall be found to vary a little from the theory, such variation must be looked upon as rising rather from the want of perfect elasticity in the bodies, than from any error in the theory itself, or in the calculations grounded thereon.

The method which I shall observe in treating of the percussion of elastick bodies is this; First, to lay down one general proposition concerning such percussion, and then, Secondly, to deduce the laws relating to the four general cases mentioned in my last lecture, and to confirm each of those laws by experiments.

Before I lay down the proposition, I must observe to you, that wherever I mention the striking body, I thereby mean that body which is in motion where one of the two is quiescent, as also that body which moves swiftest when they both move the same way; and lastly, that body which has the greatest quantity

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tity of motion, when they move in opposition to one another, or in this case, if their motions be equal, then either of them may be taken indifferently for the striking body.

This being premised, the PROPOSITION is as follows.

If of two bodies perfectly elastick, one be at rest, and the other in motion; or if they both move either the same or contrary ways, so as that one shall strike the other; let them be considered as void of elasticity, and by the proposition laid down in my last lecture, let the motion of each body after the stroke be found, and by one of the four rules laid down in the same lecture, let the motion communicated by the striking body to the other be likewise found; and let this motion be subducted from the motion of the striking body after the stroke, and added to that of the body which received the stroke, and the residue will be the motion of the striking body, and the sum the motion of the other body after reflection. For, since the bodies are supposed to be perfectly elastick, their parts which are bent in by the stroke will restore themselves with a force equal to that which bends them in; but the force which bends them in, is measured by the quantity of motion communicated by the striking body to the other, and therefore the parts of each body which are bent inward will restore themselves with such a force, as is sufficient to generate a motion equal to that which is communicated; consequently the bodies will by virtue of their elasticity throw one another contrary ways, each with a quantity of motion equal to that which the striking body communicates to the other; for which reason, if that motion be subducted from the motion remaining in the striking body after the stroke, as being contrary thereto, and added to the motion of the other body after the stroke, as conspiring therewith, the residue and sum will give the true motions of the bodies after reflection.

To

To apply what has been said to the four general cases, the first whereof is where one of the bodies is at rest at the time of the stroke. If a body perfectly elastick strikes another of the same kind and of equal magnitude at rest, the striking body will communicate all its motion to the other and remain at rest; for by the first of the four rules laid down in my last lecture, the striking body will upon the stroke, communicate half its motion, and by the proposition now laid down, a quantity of motion equal to that which is communicated, must be subtracted from the motion remaining in the striking body, and be added to the motion of the body which receives the stroke, by which means the striking body will have no motion left; but the other body will have a quantity of motion equal to what the striking body had before the shock. For the confirmation of which, let two equal ivory balls be suspended as were those of clay; and let one of the balls fall from any height, and so as to strike the other at rest, the ball which receives the stroke will ascend to the same height from which the other fell, and will leave the other at rest.

Exp. 1.

If instead of one there be two, three, or more quiescent balls contiguous to one another, that which is farthest removed from the striking ball will fly off with the velocity of the striking ball, and all the intermediate balls together with the striking ball will quiesce; for as the striking ball imparts all its motion to the first of the quiescent balls, so does that in like manner to the ball which lies next beyond it, and that again to a third, and so on; till at length the last ball meeting with none other to resist it, flies off with all the motion of the striking ball, leaving that and the intermediate ones at rest.

Exp. 2.

If two balls be let fall together contiguous to one another, upon the stroke the two farthest will fly off, leaving the others at rest; for as the foremost of

Exp. 3.

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of the two moving balls is carried equally swift w the subsequent, it cannot during its motion rece any impression from the subsequent ball; consequently, when it makes the stroke, it will produ the same effect in the quiescent balls as if the subsequent ball was away; that is, it will by means the intermediate balls communicate all its moti to the last, and make that fly off; but no sooner h it made the stroke, and thereby parted with its o motion, but the subsequent ball impels it, and in parts to it all its motion, and this motion being propagated thro' the several intermediate balls before, makes the last but one to fly off, and th in such a manner as to keep pace with, and closely pursue the other; because in the same instant o time that the foremost of the two moving ball makes its stroke, it likewise receives the stroke from the hindmost ball, and of consequence, the flying off of the two last balls, which is the effect of the double stroke, must happen at one and the same time.

Exp. 4. For the same reason that two balls fly off where the number of striking balls is two, three will fly off when there are three striking balls, and four, where there are four, and so on, whatever be the number of striking balls, an equal number will constantly go off.

Exp. 5. If two elastick balls be unequal, for instance, if one be double the other, and if the greater be let fall from the height of nine inches, and strike the smaller at rest; they will both move forward after the stroke, the striking body with one third of the motion which it had before the stroke, and the other with two thirds; and the striking body will ascend to the height of three inches, and the other to the height of twelve. For since the striking body is to the quiescent as two to one, it will by the first of the four rules laid down in my last lecture, communicate one third of its motion to it, and on account of

If the elasticity a quantity of motion equal to what is communicated, must be taken from the motion remaining in the striking body, and added to the motion of the other; consequently, the striking body will retain one third only of its motion; the other two thirds being communicated to the body which receives the stroke; wherefore since the striking body is as two, and the height from which it falls as nine, its motion must be as eighteen, one third of which, to wit, six, it will retain after the reflection; and the other two thirds, to wit, twelve, will be the motion of the other body, and these motions being divided by the bodies, will give three and twelve for the quotients; which quotients are as the velocities of the bodies after reflection, or as the heights to which they ascend.

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On the other hand, if the larger ball be quiescent, and the smaller be let fall from the height of nine inches, its motion will be as nine, whereof two thirds will by the first of the four rules be communicated by the stroke to the greater, and one third only will remain in the striking ball, from which on account of the elasticity must be taken as much as was communicated to the larger ball, that is, two thirds; but upon subducting two thirds from one third, there will remain one third negative, which shews, that the striking ball will be reflected with one third of the motion it had at the time of the stroke, so as to ascend backward to the height of three inches; and the quiescent ball, to which two thirds of the striking ball's motion was communicated by the stroke, will likewise on account of the elasticity receive two thirds more, so as to be carried forward with a motion equal to what the striking ball had at the time of the stroke, and one third more; that is to say, with a motion which is as twelve, which being divided by two, the quantity of matter in the ball gives six for the velocity, or the height to which that ball must ascend.

Exp. 6.

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From

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From what has been said it follows, that when the quiescent ball is smaller than the striking ball, there can be no reflection, because in that case the striking ball will by virtue of the stroke communicate less than half its motion, and the motion which is to be taken from the striking ball on account of the elasticity being equal to the motion communicated, will upon the subduction always leave some motion in the striking ball to carry it forward, consequently it cannot be reflected. Where the two balls are equal there will likewise be no reflection, but the ball which was quiescent will go forward with all the motion of the striking ball, and the striking ball will become quiescent; as is evident from what was said concerning that case. But where the striking ball is less than the quiescent, it will be reflected, and there will likewise be an augmentation of motion in the greater ball; for the smaller ball must upon the stroke communicate more than half its motion to the greater ball, and there must likewise, on account of the elasticity, as much motion be subducted from the smaller ball, and added to the larger, as is communicated; wherefore, since two equal quantities of motion, each of which exceeds half of the smaller ball's motion, are to be subducted from the smaller ball, and given to the larger; it is plain, that the smaller must lose all its motion and something more, that is, it must be carried backward or reflected; and the greater ball must go forward with more motion than was in the smaller at the time of the stroke, that is, there will be an augmentation of motion; and the excess of motion in the greater ball, above the motion which the smaller ball has at the time of the stroke, is ever equal to the motion wherewith the smaller ball is reflected after the stroke, as is evident from what has been said. If therefore motion be communicated from a smaller elastick body to a larger, by means of several intermediate bodies each larger

larger than the other, the motion will be augmented in each of them, and the motion of the last will greatly exceed that of the first; and this augmentation of motion is greatest when the bodies are in a geometrical progression; for instance, if there be two bodies which are as one and four, and if the smaller communicates motion to the larger by means of one intermediate body, the motion will be greater in the larger body, if the middle body be as two, that is, a geometrical mean between the two, than if it be as one and an half, or two and an half, or three, or in short in any other proportion whatever but that of the geometrical mean. For the proof of which, let the lesser body be expressed by unity, and the larger by the square of a , and the geometrical mean will be expressed by a ; so that the three bodies taken in their order from the least, will be expressed by the symbols in the first step, and the motion produced in the second body by the stroke of the first, will be expressed by the second step; and the motion produced in the third by the stroke of the second, will be expressed by the third step. Again, let another body greater or less than a be substituted in the room thereof, and let the difference between that body and a be called x , in this case, the bodies will be expressed by the symbols in the fourth step, and the motion produced in the second by the stroke of the first, will be expressed by the fifth step; and the motion produced in the third by the stroke of the second, will be expressed by the sixth step; but this fraction of the sixth step is less than that of the third step, for if from the product arising from the multiplication of the denominator of this fraction into the numerator of that be subtracted, the product which arises from the multiplication of the numerator of this into the denominator of that; that is, if from the seventh step the eighth be subducted, there will remain the quantity which is expressed in the ninth step.

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Whence it appears, that the former product is greater than the latter; and therefore, by the 2d COROL. of the 19th PROP. of the 7th book of the elements, the numerator of the former fraction bears a greater proportion to its denominator, than that of the latter fraction does to its denominator; that is, the fraction in the third step which expresses the motion of the greatest body when the intermediate one is a geometrical mean, is greater than the fraction in the sixth step, which expresses the motion of the greatest body when the middle body is not a geometrical mean; consequently, the motion is more augmented when the intermediate body is a geometrical mean, than when it is greater or less in any proportion.

1st. $1, a, a^2.$

2d. $\frac{2a.}{1+a}$

3d. $\frac{a^3}{a^3+2a^2+a} \times 4.$

4th. $1, a \pm x, a^2;$

5th. $\frac{2 \times a \pm x.}{1+a \pm x}$

6th. $\frac{a^3 \pm a^2x}{a^3+2a^2+a \pm a^2x \pm 2ax \pm x+x^2} \times 4.$

7th. $a^5+2a^4+a^3 \pm a^2x \pm 2a^4x \pm a^3x \pm a^2x^2.$

8th. $a^6+2a^5+a^4 \pm a^3x \pm 2a^4x \pm a^3x.$

9th. $----- a^3x^2.$

To give you an instance, how prodigiously motion may be augmented, by being successively communicated to several bodies in a geometrical progression;

gression; if twenty elastick bodies be placed one after another, each succeeding body exceeding the foregoing in the proportion of twenty to one; and if motion be communicated through the several intermediate bodies from the first to the last, it will be so far augmented, as to be two hundred thousand times greater in the last body than in the first; so that if we suppose the first to be a cannon-ball, moving with the same velocity wherewith it flies from the mouth of a cannon, which, from the observations of Mr. DERHAM, I shall suppose to be at the rate of 612 feet in a second, tho' there are pieces of cannon which discharge their balls with double that velocity, the motion of the last body will be so great as if applied to the ball would carry it at the rate of above twenty-three thousand miles in one second of time; which velocity is five thousand times as great as the velocity of a body revolving about the earth, by the force of gravity at a small distance from its surface; for a body so revolving will not come round in less than an hour and twenty-four minutes.

From the increase of motion in elastick bodies, a reason may be drawn for the augmentation of sound in speaking trumpets; for as the speaking trumpet is narrowest at the mouth-piece, and thence widens and enlarges continually to the extremity, the air within it, which is an elastick fluid, as shall be shewn hereafter, may be considered as divided into a great number of cylindrical bodies, of very small but equal altitudes, the basis of the first being equal to the mouth of the trumpet, and the basis of the rest increasing one above another as they are more and more removed from the mouth; upon which account the motion that is impressed by the force of the voice on the first cylindrical body of air, grows larger in the second, and larger still in the third, and so on, till at length at the exit of the tube it becomes so large as to magnify the sound to a great

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Pl. 2.
Fig. 8.

degree; and of the several kinds of trumpets, those magnify the sound most, that are of such a figure as arises from the revolution of the logarithmick curve about its axis; that is, let AG be the logarithmick curve, and HO its axis, the figure arising from the revolution of AG about HO, is such as a speaking trumpet ought to have in order to give it the greatest advantage possible. For from the nature of the curve, if HI, IK, KL, LM, and so on, be taken equal; the ordinates HA, IB, KC, LD, and so on, are in geometrical proportion; wherefore if HI, IK, and so on, be taken very small, they will represent the equal altitudes of the cylindrical bodies of air in the trumpet; and the ordinates HA, IB, and so on, will be the radii of their bases, and the bodies of air being of equal heights will be to one another as their bases, that is, as the squares of their radii; but the radii being to one another in a geometrical proportion, their squares will be so too; consequently, the little cylindrical bodies of air will be in a geometrical progression, the smallest whereof lies next the mouth, and the largest at the exit of the tube; for which reason the augmentation of sound will be greater, *ceteris paribus*, in a trumpet of such a form than of any other form whatever.

But to proceed to the second general case, wherein both the bodies move one and the same way, but the subsequent more swiftly than the preceding.

Exp. 7.

If two equal elastick bodies move in the same direction, and in such a manner as that one may overtake and strike the other, upon the stroke they will change their quantities of motion with each other; for instance, if the motion of the subsequent body before the stroke be double the motion of the preceding body, then will the preceding body after the stroke have double the motion of the subsequent body after the stroke; and the preceding body after the stroke, will move with the same velocity where-

wherewith the subsequent body moved before the stroke; and the subsequent body will after the stroke be carried with the velocity of the preceding body before the stroke; so that upon the stroke the bodies will change their motions and velocities. For since by supposition the sum of the motions is three, and since the bodies are equal, the motion of each after the stroke, setting aside the elasticity, must be one and an half; and by the second rule for determining the quantity of motion communicated by the striking body to the other, the motion communicated in this case will be as one half, and so likewise will the motion arising from the elasticity, which being deducted from the motion which remains in the striking body after the stroke, and added to that of the preceding body, leaves the motion of the former as one, and of the latter as two; so that upon the stroke the motions will be changed. Wherefore if two ivory balls of an equal size be let fall at the same time, one from the height of six inches, and the other from the height of three, after the stroke, the preceding ball will rise to the height of six inches, and the subsequent to the height of three only.

If the bodies be unequal and move the same way, their motions and velocities after the stroke may in like manner be discovered by the help of the proposition. For instance, if the subsequent body be as two, and have twelve parts of motion, and the preceding body as one, and its motion as three; the motion of the subsequent body after the stroke will be as eight, and that of the preceding body as seven, and the velocity of the former will be as four, and that of the latter as seven; for the sum of the two motions before the stroke being fifteen, and the bodies being as one and two, the motion of the lesser body after the stroke, setting aside the elasticity, will be as five, and that of the greater as ten; but the motion of the lesser body before the stroke

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was

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was as three, consequently, the communicated motion is as two; wherefore adding so much on account of the elasticity to the motion of the lesser body, and subducting as much from that of the greater body, which in this case is the striking body, we shall have eight for the motion of the greater, which being divided by two, the quantity of matter in the greater, gives four for its velocity; and we shall have seven for the motion of the lesser body, which because the quantity of matter in the lesser is as one, will likewise express the velocity. Wherefore

Exp. 8. if two ivory balls, one double of the other, be let fall at the same time, the larger from the height of six inches, and the smaller from the height of three, after the stroke the lesser will ascend to the height of seven inches, and the greater to the height of four. On the other hand, if the smaller ball be let

Exp. 9. fall from the height of six inches, and the greater from the height of three; after the stroke the lesser will ascend to the height of two inches, and the greater to the height of five, and the motion of the former will be two, and that of the latter ten; for since the smaller ball is as unity, and falls from the height of six inches, its motion at the time of the stroke is six; and since the larger ball is as two, and falls from the height of three inches, the motion thereof at the time of the stroke is likewise six; and the sum of those two motions, which is twelve, being divided between the bodies in proportion to their quantities of matter, gives eight for the motion of the greater, and four for the motion of the lesser, which motions they would have after the stroke, supposing they were not elastick; and since the motion of the greater body before the stroke was six, the motion communicated to it by the stroke is two, which by reason of the elasticity being subducted from four, the motion of the striking body, and added to eight, the motion of the other body, gives two and ten for the motions of

of the two bodies, which motions being divided by the respective bodies, give two and five for the velocities.

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If two equal bodies meet one another with equal quantities of motion, which is one branch of the third general case, they will rebound with the same motions and same velocities wherewith they approached; for were they void of elasticity they would upon the stroke stand still, because they communicate to one another a quantity of motion equal to that which each of them has at the time of the stroke, and that in a contrary direction; but by the proposition, each of them must on account of the elasticity receive as much motion as was communicated by the stroke; and the motions which are thus received by the bodies being equal, and contrary to the motions whereby the bodies met, and which were destroyed by the stroke, must carry the bodies backward with the same velocities wherewith they approached. Wherefore, if two equal ivory balls be let fall at the same time from equal heights, so as to meet one another, upon the stroke they will be reflected back to the heights from which they fell.

Exp. 10.

If the balls be unequal, for instance, if one be double the other; let the larger fall from one half only of the height from which the smaller descends, by which means when they meet their motions will be equal, and upon the stroke they will be reflected each to the height from which it fell.

Exp. 11.

Where the bodies meet one another with unequal motions, which is the fourth general case, if the bodies be equal, they will both be reflected, and each of them will recede with the motion and velocity wherewith the other approached; that is, they will change their motions and velocities; for let us suppose the motions of the two bodies to be as six and three; if they were void of elasticity the body which has the smallest quantity of motion would

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would upon the stroke be turned back, and the two bodies would be carried with the difference of their motions divided equally between them, that is, the motion of each would be as one and an half, and the motion communicated would by the fourth rule be as four and an half; but a quantity of motion equal to what is communicated must be subducted from the motion remaining in the striking body, and added to the motion of the other, that is, four and an half must be subducted from one and an half, and likewise added thereto; whereby there will be three negative for the motion of the striking body, which shews that it will be carried back with a motion which is as three; and there will be six positive for the motion of the other body, which shews that it will be carried with a motion which is as six, in the direction of the striking body before the stroke; that is, it will be reflected; so that each of them will be carried back with the motion wherewith the other approached. Wherefore, if two equal balls of ivory be let fall at the same time, one from the height of six inches, and the other from the height of three, upon the stroke they will return back; but that which fell from the height of six inches will rise only to the height of three, whereas that which fell from three inches will rise to six.

Exp. 12.

If the balls be unequal, and meet one another with unequal quantities of motion, their motions after the stroke may in like manner be determined by the help of the rule laid down in the proposition; for instance, if two ivory balls which are as one and two be let fall at the same time, the greater from the height of six inches, and the smaller from the height of three; in this particular case, the greater ball will upon the stroke lose all its motion, and the smaller will be reflected with the difference of their motions, so as to rise to the height of nine inches; for since the larger ball which descends from the height

Exp. 13.

height of six inches is as two, its motion is as twelve, whilst the motion of the smaller ball, which is as unity, and descends only from the height of three inches, is as three, the difference of which motions is nine; and this being divided between the bodies in proportion to their quantities of matter, gives six for the motion of the larger, and three for that of the smaller; and with these motions the bodies would be carried after the stroke, supposing they were void of elasticity; but because of the elasticity, a quantity of motion equal to what is communicated by the striking body to the other, which in this case is six, must be taken from the motion of the greater body, and added to that of the smaller, which two motions being six and three, the remainder after subduction, which expresses the motion of the greater body, will be nothing; and the sum arising from the addition, which expresses the motion of the smaller ball, will be nine.

LECTURE VI.

OF THE CENTER OF GRAVITY, BALANCE,
AND LEVER.

MY design in this lecture is to give you an account of the first and second of the mechanick powers, commonly called the balance and the lever; but I shall first take notice of some things relating to heavy bodies, the knowledge of which is in a great measure necessary to the right understanding of what shall be said concerning the mechanick powers in general. And First, in every body there is a certain point, commonly called by the writers of mechanicks, the center of gravity; the nature of which will best appear from its chief properties, which are these.

1st, If a body be suspended by its center of gravity, it will continue in any position whatever wherein

LECT.
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Exp. 1.

wherein it is placed; whereas if it be suspended by any other point, it will not rest in any other position but where the center of gravity is either directly above, or directly beneath the point of suspension; thus, if two beams be supported, the one by an axle passing thro' its center of gravity; the other by an axle which doth not pass thro' the center of gravity, but thro' such a point, as when the beam is parallel to the plane of the horizon, lies directly above the center of gravity; the former will rest in any position, whether it be perpendicular, parallel, or inclined to the horizontal plane; but the latter will rest in the parallel position only; and should it by any force be removed from that position, it will, upon the removal of the force, begin to move in order to recover the parallel position, and after several vibrations will at length settle therein.

A second property of the center of gravity is, that where that is supported the whole body is likewise sustained; for which reason the whole weight of a body may be looked upon as applied to that single point, and as centered therein.

Exp. 2.

A third property of this center is, that it continually endeavours to move downward towards the center of the earth, and where all lets and impediments are removed does actually descend; and therefore if in any case a body seems to move upward by the force of gravity, it will be found that the center of gravity descends notwithstanding any appearance to the contrary. Thus, if two rulers be so placed as to meet in an angle at one of their ends, and there to rest upon an horizontal plane, whilst at their other end they are raised a little above the plane; and if a body consisting of two equal similar cones united at their bases, be laid upon the rulers in such a manner, that the edge of their bases may lie between the rulers, it will when left to itself begin to roll towards the elevated extremities of the rulers, and upon that account appear to ascend, whereas in reality

by reality it moves downward; for if a string be stretched horizontally beneath the rulers so as that it may touch the edge of the bases of the cones at the concurrence of the rulers, it will be found that the edge of the bases descends below the string, and that more and more as the body moves nearer to the higher end of the rulers.

Whilst the body rolls upon the rulers, the parts of the cones which rest thereon, do by reason of the widening of the rulers grow continually smaller; upon which account, at the same time that the body ascends along the plane of the rulers, it is as it were carried down another plane equal in length to the side of the cone, and whose perpendicular altitude is equal to the semidiameter of the bases of the cones; and therefore, if the perpendicular altitude of the rulers in that part where their distance is equal to the length of the double cone, be less than the semidiameter of the bases, the body will move up along the rulers, because, by so doing, it will in reality descend, and the descent thereof will be equal to the difference between the semidiameter of their bases, and the perpendicular altitude of the rulers in that part where their distance is equal to the length of the cones; but if that perpendicular altitude be equal to the semidiameter, the body will rest on any part of the rulers, being carried as much upward on one account as it is downward on the other; and if the altitude of the rulers be a little increased, so as to exceed the semidiameter of the bases of the cones, the body will roll down the rulers, and thereby descend thro' a space equal to that excess.

If a cylinder be so contrived as to have its center of gravity near one of its sides, which may be done by making a wooden cylinder hollow towards one side, and then filling it with lead; when it is placed on an inclined plane in such a manner as that the side

Exp. 3.

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side which is nearest to the center of gravity may lean towards the upper part of the plane, it will ascend, provided the inclination of the plane be not too small, but the center of gravity will at the same time descend; for it will suitably to its nature endeavour to move downward, and thereby cause the cylinder to revolve about its axis; and this revolution will make the cylinder, and consequently its center of gravity, to move up the plane; so that the center of gravity will have as it were two motions, one upward arising from the progression of the cylinder along the plane, the other downward occasioned by the rotation of the cylinder about its axis; but the descent occasioned by the latter motion, will be greater than the ascent arising from the former; as will appear by stretching a line horizontally at the same height with the center of gravity before the cylinder begins to roll, for after the rotation ceases the center of gravity will be beneath the line; so that upon the whole, that center will be found to descend notwithstanding the ascent of the cylinder on the plane.

When the elevation of the plane becomes so great that the ascent arising from the progression becomes equal to, or greater than the descent arising from the rotation, the cylinder will in the former case continue at rest, and in the latter roll down the plane.

A line drawn from the center of gravity of any body, perpendicular to the plane of the horizon, is called the line of direction of the center of gravity, because when the body is carried downward by the force of gravity, if it meets with no let or obstacle, its center of gravity will describe that line. The chief property of this line is, that as long as it falls within the base of the body, so long the body stands, whereas no sooner does it fall beyond the base, but the body tumbles; as will appear from the

the following experiment; let a piece of wood be set on a moveable plane with a plummet hanging from its center of gravity, and let the plane be gradually elevated, till at length the plum-line (which as it is always perpendicular to the horizon, will represent the line of direction) falls beyond the base; the wood will not tumble as long as the plummet line falls within the base, whatever be the elevation of the plane whereon it stands, but the moment that line gets beyond the base the body falls.

The reason why a body stands during the continuance of the line of direction within its base is, that no motion can arise in any body from the force of gravity, unless the center of gravity can by such motion be carried downward; but as long as the line of direction of any body falls within the base, its center of gravity is supported, and therefore cannot descend; and consequently, the body will remain unmoved; whereas upon the removal of the line of direction beyond the base, the center of gravity ceases to be supported, and is therefore at liberty to descend.

From what has been said it appears, why among bodies descending on inclined planes, some, for instance cubes, only slide, whilst others, as globes or cylinders, roll; the lines of direction falling beneath the bases of the former, but not the latter.

The center of motion in any body is a fixed point or axis about which the several parts of a body do move, and in moving describe circular arches.

The direction of any power or weight is, that strait line wherein it moves or endeavours to move. And the moment of any power or weight is, that force wherewith it either moves or endeavours to move, and it is always proportional to the product arising from the multiplication of the power or weight into the velocity wherewith it moves or would move if it were not hindred by some opposite

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posite power or weight; and therefore if the product arising from the multiplication of one weight or power into its velocity, be equal to the product arising from the like multiplication of any other weight or power into its velocity, the moments of those two weights or powers must be equal; and this will always be where the weights or powers are to one another reciprocally as their velocities; consequently, two weights or powers may balance, if as much as one exceeds the other in magnitude, so much must it be exceeded by the other in velocity; and herein consists the whole force and efficacy of all mechanical engines; for they are so contrived as to diminish the velocity of one weight or power and to increase that of the other, by which means a very small weight or power may become a balance to one exceedingly great, as will appear from what shall be said concerning the mechanick powers, which are commonly reduced to six, namely, the *balance*, the *lever*, the *pulley*, the *axle in the wheel*, the *wedge*, and the *screw*, of each of which in their order,

The *BALANCE*, strictly speaking, is a beam supported by an axle whereon it turns; which axle therefore is the center of motion; the parts of the beam which lie on each side of the axle are called its arms, and those parts of the arms to which the weights are applied are called the points of suspension; concerning which it must be observed, that the appending weight, whatever be the length of the cord by which it hangs, acts with the same force and in the same manner as if its center of gravity was applied to the point of suspension; so that it matters not what the distance is between the weight and point of suspension, as will appear from the following experiment: Let a weight appended at one arm of a balance be counterpoised by a weight at the other, and let it by means of a cord be hung at different distances below the point of suspension; the position of

Exp. 5.

of the balance will remain unvaried, and the weights will continue to counterpoise each other at all those distances.

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The moment of any weight appended at the arm of a balance, is proportional to the product arising from the multiplication of the weight into the distance of the point of suspension from the axis of the balance; for as was before said, the moment of a weight is proportional to the product of the weight into its velocity, and in this case the velocity of the weight is as the distance of the point of suspension from the axis; for since the weight acts in the same manner as if its center of gravity was applied to the point of suspension, whatever be the velocity wherewith that point moves round the axis, the same will be the velocity of the weight be; but the velocities wherewith the several points in the arm of a balance move round the axis, are as the spaces, that is, as the circular arches, which they describe in the same time, which arches from the nature of the circle are to one another as their respective radii, that is, as the distances of the points from the axis.

Thus, if A B represents the arm of the balance moving round the axis at A, the velocities of the points B and D, which describe the arches B C and D E, will be as those arches, because they are described in the same time; but from the nature of the circle, those arches are to one another as their radii A B, and A D, that is, as the distances of those points from the axis; consequently, the moment of a weight appended at the arm of a balance, is as the product of the weight into the distance of the point of suspension from the axis. Whence it follows, that if two weights be appended at the arms of a balance in such a manner, as that the distances of the points of suspension from the axis shall be reciprocally proportional to the weights, those weights will counterpoise each other, and the balance will be in *æquilibrio*; for instance, if two equal weights

Pl. 2.
Fig. 9.
Exp. 6.

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Exp. 7.

be applied at equal distances from the axis, the balance will not incline to either side, but remain parallel to the horizon, the weights in this case counterpoising one another. Again, if one weight be larger than the other in any proportion, for instance; in the proportion of three to one, if the point at which the smaller is applied be thrice as far distant from the axis as the point at which the larger is applied, the balance will be in *æquilibrio*.

Exp. 8.

On this *æquilibrium* arising from the suspension of weights at distances reciprocally proportional to the weights, is founded the *Statera Romana*, otherwise called the steel-yard, which consists of two arms very unequal in length, but equally poised by means of a weight annexed to the shorter, from which likewise hangs a scale in order to receive such things as are to be weighed; the longer arm is divided into a number of equal parts beginning from the axis, and sustains a weight which slides from one end to the other; which weight being applied to the second division, will counterpoise double the weight in the scale of the shorter arm, that it will when applied to the first division; and triple when applied to the third division; and so on, whatever be the division to which it is applied, the weight in the scale of the shorter arm must be proportional thereto; otherwise the products arising from the multiplication of the weights into their respective distances from the axis would not be equal, and consequently would not balance each other.

Exp. 9.

On the same *æquilibrium* is likewise founded the deceitful balance, which is so contrived, as tho' one arm be longer than the other, yet is the shorter made so much thicker than the longer, as thereby exactly to poise the same; upon which account the balance appears to be just, and consequently such weights as counterpoise are judged equal, whereas in truth that which is appended at the longest arm is less than the other, and that in the proportion of
the

the length of the shorter arm to that of the longer ; for instance, if the longer arm be to the shorter as ten to nine ; a weight of nine ounces applied at the longer arm, will counterbalance ten ounces appended at the shorter.

Several weights appended at several distances from the axis in one side of a balance, will counterpoise several others appended likewise at several distances on the other side ; provided the sum of the products which arise from the multiplication of the weights on one side into their respective distances from the axis, be equal to the sum of the products arising from the like multiplication of the weights on the other side into their respective distances. Thus, if on one side a weight of one ounce be appended at the distance of two inches from the axis, and another of two ounces at the distance of three inches, and a third of three ounces at the distance of four inches ; and if on the other side be appended one weight of five ounces at the distance of an inch from the axis, and another of three ounces at the distance of five inches ; the two latter will balance the three former ; for the product of five into one, being added to the product of three into five, gives the sum of twenty ; as does likewise the addition of the three products of one into two, two into three, and three into four.

Exp. 10.

The chief use of the balance, commonly called a pair of scales, is to compare the weights of different bodies together ; and that this machine may be as exact and perfect as possible, it is requisite, 1st, that the center of gravity of the beam be placed a little below the axis, because in this case, when there is an *equilibrium*, the beam will not rest in any position but the parallel ; consequently, the weights which are compared together will appear to be equal, as they really are ; whereas if the axis be placed beneath the center of gravity, should the center of gravity be moved out of the perpendicular

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lar line, which can scarcely be avoided, it will not return, but from its tendency downward will be carried lower, so as to give the beam an inclined position; for which reason the weights will appear to be unequal, tho' in reality they are not so; and the same inconvenience will arise if the axis passes thro' the center of gravity, for in that case it has been already shewn, that the beam, notwithstanding the *equilibrium*, will rest in any position.

Secondly, the arms of the beam ought to be exactly equal both as to weight and length, the reason of which is evident, from what was said concerning the deceitful balance.

Thirdly, the points from which the scales are suspended, ought to be in one right line passing thro' the beam's center of gravity; for by this contrivance the weights will act directly against each other, so that no part of either will be lost on account of any oblique direction.

Fourthly, the friction of the beam against the axis ought to be as little as possible; because, should the friction be great, it will require a considerable force to overcome it; upon which account, tho' one weight should a little exceed the other, it will not preponderate, the excess not being sufficient to overcome the friction, and bear down the beam.

That the friction may be as little as possible, the parts of the beam which play upon the axis, as also the axis itself, should be well polished, and the axis should be made as small as the uses of the balance will admit; but as friction cannot be entirely prevented, to remedy the inconveniences arising from it as much as possible, the arms of the beam ought to be made as long as they conveniently can; because the longer the arms are, the less will the weight be that is requisite to overcome the friction; the moments of weights increasing in proportion to their distances from the center of motion, as has been already shewn.

I shall

I shall close what I had to say concerning the balance, by laying before you one property of it, which is somewhat singular and surprising; tho' it has not that I can find been taken notice of by any of the mechanick writers*, namely, that if a man standing in one scale and counterpoised by a weight in the other, lays his hand to any part of the beam, and presses it upward, he will thereby destroy the balance, and make the scale wherein he stands to preponderate.

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Exp. 11.

In order to account for this property, let A B represent the beam of a pair of scales playing on the axis at C, and let a man standing in the scale D, and counterpoised by a weight in the scale E, lay his hand to some part of the beam, either on the same side of the axis with himself as at H, or on the other side as at K, and press the same upward; inasmuch as action and reaction are always equal, it is manifest that with whatever force the hand presses upward against the point H or K, with the same the hand, and consequently the man's whole body, is pressed downward; and therefore the scale D wherein he stands bears the same pressure from his feet that the point H or K does from his hand; but the pressure upon the scale D may be looked upon as applied to the beam at the point A from which the scale hangs; consequently, the same force which presses up the point H or K, presses down the point A; wherefore putting F to denote that force, $F \times HC$ will express the moment wherewith the arm A C is pressed upward when the hand is applied at H, and $F \times KC$ the moment wherewith the arm

Pl. 2.
Fig. 10.

* The property here mentioned, had not been taken notice of by any of the Mechanick Writers, when the Author composed this Lecture; but has been published since, both in the Philosophical Transactions for the year 1729, and in a course of experimental Philosophy, by Dr. DESAGULIERS, to whom our Author communicated it, as he told me and many others, about thirteen or fourteen years ago when he was in London.

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BC is pressed upward, the hand being applied at K; and in both cases $F \times AC$ will express the moment wherewith the arm AC is pressed downward by means of the reaction; if therefore the hand be applied at H, it is manifest that as the arm AC is at one and the same time pressed upward by a force which is as $F \times HC$, and downward by a force which is as the same $F \times AC$, and as HC is ever less than AC, the arm AC must descend with the difference of those forces, that is, with a force equal to $F \times AH$, which is the distance of the hand from the point A; if the hand be applied at K, the arm CB is pressed upward, and consequently AC downward with a force equal to $F \times KC$, and upon account of the reaction AC is likewise pressed downward with a force equal to $F \times AC$; and therefore it must descend with a force equal to the sum of those two forces, that is, with a force equal to $F \times AK$ the distance of the hand from the point A; so that the scale D must preponderate whether the hand be applied to that part of the beam which lies on the same side of the axis with the man, or to that which lies on the other side; and if D be put to denote the distance of that point to which the hand is applied from the point A, the force wherewith the preponderating scale descends will be universally as $F \times D$, that is, as the force which the hand exercises against the beam, multiplied into the distance of the hand from the point A. And if the force wherewith the hand presses the beam be required, it may be discovered by throwing in as much weight into the scale E as is sufficient to balance the force of the hand, and to prevent the descent of the scale D; for putting W to denote that weight, its moment is as $W \times BC$ or AC , which being equal to $F \times D$ the moment of F, F will be found equal to $W \times \frac{AC}{D}$, that is, to the weight multiplied into half the length of the beam, and

divided by the distance of the hand from A. For instance, if the balancing weight be twenty pounds, and the distance of the hand from A be to half the length of the beam as one to two, the force where-with the hand presses the beam is equal to twenty pounds multiplied by two and divided by unity, that is, it is equal to forty pounds; from what has been said it follows, that when the hand is applied to that part of the beam which lies on the same side of the axis with the man, the force of the hand upon the beam is greater than the weight which balances it in the scale E, and less than the same when the hand is applied to that part of the beam which lies on the other side of the axis with respect to the man; for in the first case, $W \times \frac{AC}{D}$ is greater than

W, and in the latter less, inasmuch as A C is in the former case always greater, and in the latter less than D.

The second, and indeed the most simple of all the mechanick powers is the LEVER; an engine chiefly made use of to raise large weights to small heights. By the writers of mechanicks, it is supposed to be an inflexible line void of all gravity; tho' such as are in common use are both flexible and weighty. In every lever there is one immoveable point, about which as a center all the parts of the lever turn; and whatever supports that point is called the prop; and with regard to the different situations of the moving power, and the weight to be moved in respect to the prop, the lever is divided into three kinds; the first of which is where the prop is placed between the moving power and the weight to be raised; which kind of lever is represented, where-in C denotes the prop, B the weight, and A the power. In this lever there will be a balance between the power and the weight, provided they be to one another reciprocally as their distances from the prop; that is to say, if the power at A be to

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the

Pl. 2.
Fig. 11.

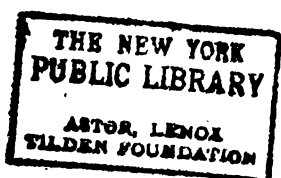
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the weight at B, as CB to CA ; for upon the motion of the lever round its fixed point C , the power at A will describe the arch AD in the same time that the weight at B describes the arch BE ; consequently, the velocity of the power will be to the velocity of the weight, as the arch AD to the arch BE ; that is, because the arches are similar, as is evident from the manner wherein they are generated, as AC to CB . That therefore the product arising from the multiplication of the power into its velocity, may be equal to the product of the weight into its velocity; or in other words, that their moments may be equal, the power must bear the same proportion to the weight, that BC the distance of the weight from the prop bears to AC the distance of the power from the prop. For instance, if BC be to AC as one to two, and if a man's strength be such as that without the help of a machine he can support an hundred weight, he will by the help of this lever be enabled to support two hundred; because as BC is to AC , which by supposition is as one to two, so must the power at A be to the weight at B ; but the power at A is supposed to be equal to one hundred, consequently the weight must be equal to two.

Exp. 12.

As in this lever the prop may be placed either at the middle distance between the moving power and the weight, or nearer to one than the other, it is evident that there may be a balance between the power and the weight, either when they are equal, or when the one exceeds or is exceeded by the other according to the different situations of the prop.

To this kind of lever may be reduced several sorts of instruments, such as scissars, pincers, snuffers, each of which may be considered as made up of two levers, whose prop is the same with the pin which rivets them together. Quarry crows are likewise levers of this kind, concerning which it must



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must be observed, that the larger and more ponderous they are, provided they are not so big as to become unmanageable, the more useful they must be, because the weight of that part of a crow which lies on the same side of the prop with the power, and which usually far exceeds the other part in length, acts in conjunction with the power, and thereby facilitates the raising of the stones.

If the arms of the lever, instead of lying in a right line, meet each other at the prop in a right angle, where AC and BC represent the arms of a lever united at the prop C , in such a manner as to constitute a right angle ACB ; if to one arm as CB placed horizontally, a weight be appended at B , and to the other as AC standing perpendicularly a power be applied at A acting in the direction AD . In order to a balance the power must be to the weight as BC to AC , that is the power and weight must be in the inverse *ratio* of the lengths of the arms to which they are applied. For as the arms turn together upon the prop C , in the same time that the point B describes any arch as BK , the point A must describe a similar arch as AH ; consequently, the velocity of A will be to the velocity of B as AC to BC ; but as the moment of the power at A is supposed equal to the moment of the weight at B , the power must be to the weight, as the velocity of the latter to the velocity of the former, that is, as BC to AC .

To confirm this by experiment, let BC be one fourth of AC , and a weight of twelve ounces be appended at B ; to the cord ADF made fast to the point A and passing over a pulley at D , let a weight of three ounces be hung at F so as to pull the arm AC in the direction AD , and there will be a balance. And if BC be one third or one half of AC , then a weight at F , which in the former case is one third, and in the latter one half of P , will balance the

LECT.
VI.Pl. 3.
Fig. 1.

Exp. 13.

LECT. VI. the same; and if AC and BC be equal, the balancing weights must be so too.

From the experiments, and what has been said concerning them, it is evident, that the greater the proportion is which AC bears to BC, the greater is the force of the lever, or the less the power at A requisite to balance a given weight at B. And forasmuch as the hammer when made use of in drawing nails is a lever of this kind, it is manifest, that the longer the handle is in proportion to that part of the hammer which lies between the handle and that portion of it which gripes the nail, the less will the force be that is requisite to draw the nail.

Pl. 3. The second kind of lever has its prop at one end, where C is the prop, A the power, and B the weight; in this lever, in the same time that the iFig. 2. power at A moves thro' the arch of a circle whose radius is AC, the weight at B moves thro' a similar arch of a lesser circle whose radius is BC; consequently, the velocity of the power is to the velocity of the weight as AC to BC; in order therefore to a balance, the power must be to the weight as BC to AC; that is, as much as AC, the distance of the power from the prop, exceeds BC, the distance of the weight from the prop, so much must the weight exceed the power.

As in this lever the distance of the weight from the prop is always less than the distance of the power from the prop, it is evident that there cannot be a balance in any case but where the weight exceeds the power.

To this kind of lever may be reduced the oars and rudders of ships, cutting-knives fixed at one end, and doors moving upon hinges.

Exp. 15. If in this lever we suppose the power and the weight to change their places, so as that the power may

may be applied at B between the weight at A and the prop at B, it will become a lever of the third kind; wherein in order to a balance, the power at B must so far exceed the weight at A, as BC the distance of the power from the prop, is less than AC the distance of the weight from the prop.

It is evident, that the moving power receives no advantage from this kind of lever, and therefore it is never made use of but in cases of necessity, and where the weights to be raised cannot be managed in a more convenient manner; as is the case of ladders, which being fixed at one end are by the force of a man's arms reared against a wall.

As levers are of service in raising weights, so are they likewise in carrying and supporting the same; concerning which it is to be observed, that when two powers support a weight by help of a lever, the sum of the powers must equal the weight; and the weight being placed between them, their respective distances therefrom must be reciprocally as the powers; thus, if a weight resting on the lever at B be supported by two powers, one at A and the other at C, the distance of A from B must be to the distance of C from B, as the power at C is to the power at A. For in this case the lever is of the second kind, where each of the powers is in its turn to be looked upon as the prop, and then the other power must be to the weight as the distance of the weight from the prop to the distance of the power from the prop; that is, when A is considered as the prop, the power at C must be to the weight at B, as AB to AC; and when C is considered as the prop, the power at A must be to the same weight at B, as CB to CA. Consequently, since the power at A is to the weight, as BC to AC; and since the same weight is to the power at C, as AC to AB, the power at A must be to the power at C, as BC to BA, that is, the powers must be to one another inversely as their distance from the weight; and thus it

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Exp. 16.

Pl. 3.
Fig. 4.

it will appear to be from experiments. For if from the point B of the lever AC a weight as D be suspended, and if two other weights as E and F be suspended from the extreame points A and C by cords passing over pullies, so as that they may draw the lever directly upward; they will support the weight D provided the sum of those two weights be equal to the weight D, and the weight E be to the weight F as BC to BA.

Exp. 17.

The same thing will happen, if the three weights be made to pull the lever horizontally, which may be done by passing the cords over small wheels or pins placed on a level with the lever.

In shewing what the proportion ought to be between two powers which support a weight placed upon a lever, I have supposed the position of the lever to be parallel to the plane of the horizon; what the proportion ought to be, and in what manner such proportion is determined in inclined positions of the lever, shall be shewn, when I come to treat of powers acting in oblique directions.

Pl. 3.
Fig. 5.
Exp. 18.

If instead of a single lever, several be combined together in such a manner, as that a weight being appended to the first lever, may be supported by a power applied to the last, as in the machine, which consists of three levers of the first kind, and is so contrived as that a power applied at the point L of the lever C, may sustain a weight at the point S of the lever A. The power must be to the weight, in a *ratio* compounded of the several *ratios*, which those powers that can sustain the weight by the help of each lever when used singly and apart from the rest have to the weight; for instance, if the power which can sustain the weight P by help of the lever A alone, be to the weight as one to five; and if the power whereby the same weight can be sustained by the help of the lever B alone, be to the weight, as one to four; again, if the power which can support the same weight by the help of the lever C alone,

alone, be to the weight as one to five; the power which supports the weight by means of those three levers joined together will be to the weight in a *ratio* compounded of one to five, one to four, and one to five, that is, it will be as one to an hundred. For since in the lever A, a power equal to one fifth of the weight P pressing down the lever at L, is sufficient to balance the weight; and since it is the same thing whether that power be applied to the lever A at L, or the lever B at S, the point S bearing on the point L, a power equal to one fifth of the weight P being applied to the point S of the lever B, and pressing the same downward, will support the weight; but one fourth of the same power being applied to the point L of the lever B, and pushing the same upward, will as effectually depress the point S of the same lever, as if the whole power was applied at S; consequently, a power equal to one fourth of one fifth, that is, to one twentieth part of the weight P, being applied to the point L of the lever B, and pushing up the same, will support the weight; but it matters not whether that force be applied to the point L of the lever B, or to the point S of the lever C, since if S be raised, L which rests thereon must be so too; but one fifth of the power applied at the point L of the lever C, and pressing it downward, will as effectually raise the point S of the same lever, as if the whole power was applied at S and pushed up the same; consequently, a power equal to one fifth of one twentieth, that is, to one hundredth part of the weight P, being applied to the point L of the lever C, will balance the weight at the point S of the lever A; that is, a power which is to the weight, in a *ratio* compounded of the three *ratios*, which the powers have to the weight in each lever taken separately, will be a balance to the weight, when the three levers are used jointly. And by the same way of reasoning it will

will be found, that in all machines of this kind, the power requisite to sustain the weight, is to the weight, in a *ratio* made up of the several *ratios* of the power to the weight in each lever taken separately, whatever be the number of levers.

In all that has been hitherto said concerning the lever, the power and the weight are supposed to act in direct opposition to each other; and on this supposition, the power must be to the weight in each of the three kinds of levers, in the reciprocal *ratio* of their distances from the prop, as has been fully proved with regard to each kind; but where the directions of the power and weight are inclined to each other, the proportion will vary from what has been here determined, as shall be shewn, when I come to treat of powers acting in oblique directions.

LECTURE VII.

OF THE PULLEY.

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IN this lecture I shall give you an account of the *Pulley*, the *Axe in the Wheel*, the *Wedge*, and the *Screw*. The PULLEY is a small wheel that turns about its axis, and which has a drawing rope passing over it. It is made use of in raising large weights to considerable heights, and is of two kinds, fixed and moveable; the sole use of the fixed pulley, is to change the direction of the moving power; which in all cases where weights are to be raised to great heights, is exceedingly convenient, and very often of absolute necessity; for instance, if the weight P is to be raised by the force of a man's hand to any height as A above the reach of the hand, the man must quit his place and ascend in order to carry up the weight, which for the most part

Exp. 1.
Pl. 3.
Fig. 6.

part is found to be inconvenient, and sometimes impracticable; whereas if to a rope as $P A F$ passing over the fixed pulley at A , the weight be made fast at one end as P , and the hand applied to the other end at F , the man by drawing the rope $A F$ downward, will without moving from his place raise the weight as effectually, as if his hand was applied to it and moved upward from P to A ; so that in raising weights to great heights the fixed pulley is of singular service, in as much as by changing the direction of the power, it takes off the necessity that a man would otherwise lie under of ascending along with the weight, and by so doing lessens his labour; besides, it has this farther convenience attending it, that by means thereof the joint strength of several persons may be made use of to raise one and the same weight, which in many cases cannot be done, at least not so conveniently, where the weight is raised by the immediate application of the hands: but this pulley does not in the least assist the power, by encreasing its moment; because it neither lessens the velocity wherewith the weight rises, nor augments that of the power; for whatever be the space thro' which the power moves by drawing the rope $A F$, the weight must in the same time be drawn up thro' an equal space; the rope $A P$ constantly shortening in the same proportion, that the rope $A F$ is lengthened; and therefore, wherever any power supports a weight by means of a fixed pulley, that power must be equal to the weight.

When a pulley rises and falls along with the weight, as does this pulley, it is said to be moveable, and with regard to its use, it is just the reverse of the fixed pulley; for it adds to the moment of the power, but causes no change in its direction: for if the hand be applied at F to the rope D , in order to raise the weight P appended to the moveable pulley E , it must move directly upward

Pl. 3.
Fig. 7.

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ward in the very same manner, as if it was applied immediately to the weight; consequently, the direction of the hand which raises the weight is no way altered by this pulley, but the moment thereof is doubled, because it is made to rise twice as fast as the weight; for in the same time that the hand moves upward from F to G, thro' the space F G equal in length to the two equal ropes D and C, the pulley, and consequently the weight annexed, will be drawn up thro' the space E H, whose length is equal to one of the ropes only.

Pl. 3.
Fig. 8, 9,
10.

Pl. 3.
Fig. 8.

In machines consisting of several pullies, whereof some are fixed and some moveable, and which have one common rope that goes round them all; if one end of the rope be fixed, as is the case in the machines represented by these figures, in order to a balance, the moving power must be to the weight, as one to twice the number of moveable pullies; because the velocity wherewith the power moves in raising weights by the help of such engines, is to the velocity of the rising weight, as twice the number of moveable pullies to unity; as I shall now shew you in the machine, which consists of one fixed pulley as A, and another moveable as E. Since it is one and the same rope that is continued from G to F, the part A F which lies beyond the fixed pulley, cannot be drawn down and thereby lengthened, unless the two parts D and C, which lie on each side of the moveable pulley, be at the same time drawn up and shortened, and that equally; whence it is evident, that the part A F will be lengthened as fast again as either D or C is shortened, inasmuch as what each of those parts lose of their length is added to the length of A F; but the point F to which the power is applied, descends as fast as A F is lengthened, and the point E to which the weight is fastened, ascends as fast as D or C is shortened; consequently, the velocity of the power is to the velocity of the weight, as two to

to one, that is, as twice the number of moveable pulleys to unity; if therefore a weight appended at F, be to a weight appended at E, as one to two, they will balance each other, as being to one another in the reciprocal *ratio* of their velocities.

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Exp. 2.

In the machines, each of which consists of two fixed and as many moveable pulleys, and which differ only in this, that in one the pulleys of the same kind move upon one and the same axis, and in the other upon different axes; I say, in these machines, the velocity wherewith the power moves is to the velocity wherewith the weight rises as four to one, that is, as twice the number of moveable pulleys to one; for as the part of the rope A F is drawn down and lengthened, the four parts B, C, D, H, which lie on each side of the two moveable pulleys are drawn up and shortened, and that equally; and what each of them loses of its length is added to the length of A F; consequently, A F is lengthened four times as fast as each of the other parts shortens; but the power moves as fast as A F lengthens, and the weight rises as fast as the other four shorten; and therefore, the velocity of the power at F is to the velocity of the weight at E, as four to one, or as twice the number of moveable pulleys to unity: for which reason, if a weight be appended at F, which is to the weight at E as one to four, that is, in the reciprocal *ratio* of their velocities, there will be a balance.

Pl. 8.
Fig. 9. 10.
Exp. 3.

What has been thus proved with regard to the three last machines, namely, that the velocity wherewith a power moves in raising a weight is to the velocity wherewith the weight rises, as twice the number of moveable pulleys to unity, is in the same manner demonstrable with regard to any other machine of the same kind, whatever be the number of pulleys whereof it consists; and therefore, in all machines consisting of several pulleys, whereof some are fixed and others moveable, and round

H

which

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which goes one common rope, fixed at one end, it may be laid down as a general rule, that in order to a balance between the moving power and the weight, the former must be to the latter, as one to twice the number of moveable pullies.

Exp. 4.

If the rope which goes round the pullies, instead of being fixed at one end, be fastened to the weight or to the block which supports the moveable pullies, so as to rise therewith, as in this machine, which consists of five pullies, whereof three are fixed and two moveable, and in which the end of the rope is joined at G to the block which supports the two moveable pullies; the velocity of the power is to the velocity of the weight, as the sum of twice the number of moveable pullies increased by unity to one; for in this case, the parts of the rope, which are equally shortened in order to lengthen the part AF, are more in number by one than the sum of the moveable pullies when doubled; consequently, since the power at F moves as fast as AF is lengthened, whilst the weight at E rises in proportion only to the shortening of the ropes B, C, D, H, K, the velocity of the power bears the same proportion to the velocity of the weight, as the sum of twice the number of moveable pullies increased by unity does to one; and therefore, if the power be to the weight in the inverse *ratio*, that is, as one to twice the number of moveable pullies added to unity, there will be a balance. Thus, if in the machine, a weight appended at F be to another at E, as one to five, they will balance, and remain unmoved.

Pl. 3.
Fig. 11.

Exp. 5.

If to any of the forementioned machines be added a runner, that is, a single moveable pulley, which has its own rope distinct from that which is common to the other pullies, one end whereof is fixed as at L, the other being fastened to the block at E, and the weight appended at M, the force of the former machines will be doubled by this additional

Pl. 3.
Fig. 12.

additional pulley; for since the point E moves with twice the velocity of the point M, as I shewed when speaking of the single moveable pulley, whatever be the proportion which the velocity of the power at F bears to the velocity of the weight when appended at E, it will be doubled if the weight be appended at M; consequently the power will by the help of the runner be able to sustain twice the weight it did before.

If a machine be combined of one fixed and several moveable pulleys, put together in such a manner as that each of the moveable pulleys has a separate rope, one end whereof being fixed, the other either passes over the fixed pulley, as does that of the first moveable pulley E, or is joined to the moveable pulley which lies next above it, as is the case of the ropes B, C, D, which belong to G, H, and I, the second, third and fourth moveable pulleys; B being joined at N to the first moveable pulley, C at K to the second, and D at L to the third; the weight being appended to the last moveable pulley at H. The velocity wherewith the weight rises in such a machine is to the velocity of the power, as one to the last term of a duple progression, whereof the first term is unity, and the number of terms more by one than the number of moveable pulleys.

For as I proved when speaking of the single moveable pulley, the velocity of the power at F is to the velocity wherewith the pulley E rises, as two to one; and so likewise is the velocity of E, to that of G, and that of G, to that of I, and so on, whatever be the number of moveable pulleys, the velocity of each succeeding pulley is but one half of the velocity of the preceding; wherefore, if the velocity of the last pulley, which is the same with the velocity of the weight, be put equal to unity, the velocity of that which immediately precedes it, to wit H, will be as two, and the velo-

H 2

city

Pl. 3.
Fig. 13.
Exp. 6.

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city of G, as four, and of E, as eight, and so on ; if there be more moveable pullies, the velocity will be continually doubled, and since the velocity of the last pulley is expressed by unity, that of the first will be expressed by the last term of a duple progression whose first term is unity, and the number of terms equal to the number of moveable pullies ; and consequently, since the velocity of the power is double that of the first moveable pulley ; if the duple progression be continued to one term more, that term will express the velocity of the power, the velocity of the weight being as unity ; thus, in this machine, the number of moveable pullies being four, the velocity of the weight at M is to that of the power at F, as one to sixteen ; if therefore a weight appended at F be to the weight at M, as one to sixteen, there will be a balance.

Tho' this engine be of greater force than any other wherein there is the same number of moveable pullies, yet inasmuch as it does for that very reason raise weights more slowly ; men for the sake of dispatch choose rather to make use of such combinations of pullies as are represented in the 9th and 10th Figures, and where they have occasion to raise very large weights, they double the force of those machines by the addition of a runner.

The fourth mechanick Power is called the AXLE IN THE WHEEL ; which is a simple engine consisting of one wheel fixed to the end of an axle that turns along with the wheel ; the manner of raising weights by the help of this machine is thus : the power being applied to some part of the wheel's circumference, turns the wheel and together with it the axle, by which means a rope that is tied to the weight at one end, and made fast to the axle at the other, is wound about the axle, and thereby the weight drawn up ; and for as much as the wheel and its axle revolve together, in whatever time the power moves thro' a space equal to the circumference of

of the wheel, the weight must in the same time be raised up thro' a space equal to the circumference of the axle, consequently, the velocity of the power is to the velocity of the weight, as the circumference of the wheel to the circumference of the axle; that is, from the nature of the circle, as the diameter of one to the diameter of the other; if therefore the power be to the weight in the inverse *ratio* of those diameters; that is to say, if the power be to the weight as the diameter of the axle to the diameter of the wheel, there will be a balance; the power in that case being just sufficient to support the weight. For instance, if the diameter of the wheel be five inches, and that of the axle one, a weight of one ounce hanging from any point in the circumference of the wheel, will support a weight of five ounces hanging at the axle; and if the diameter of the axle be but half an inch, then will ten ounces at the axle be supported by one at the wheel.

Exp. 7.

Where the parts of the axle differ in thickness, if weights be hung at the several parts, they may be sustained by one and the same power applied to the circumference of the wheel, provided the product arising from the multiplication of the power into the diameter of the wheel, be equal to the sum of the products arising from the multiplication of the several weights into the diameters of those parts of the axle from which they are suspended. Thus a weight of five ounces hanging from the part of an axle whose diameter is one inch, and another of ten ounces from a part whose diameter is half an inch, will be balanced by a weight of two ounces hanging from the circumference of the wheel whose diameter is five inches; for the sum of the products of five into one, and of ten into one half, which expresses the moments of the weights, is equal to ten, as is also the product of two into five, which expresses the moment of the power.

Exp. 8.

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If to the axle in the wheel be added one or more wheels with teeth, so that motion may be communicated from the first wheel to the last; the weight being hung from the axle of the last wheel, whilst the moving power is applied to the circumference of the first wheel; in order to a balance, the power must be to the weight in a *ratio* compounded of the inverse *ratio* of the diameter of the first wheel to the diameter of the last axle, and of the inverse *ratio* of the number of revolutions made by the first wheel, to the number of revolutions made by the last axle in a given time; for if the first wheel and the last axle revolved in the same time, the *ratio* of the diameter of the wheel to that of the axle, would express the *ratio* of the velocity of the power, to the velocity of the weight; but if the wheel revolves oftener than the last axle in a given time, it is evident, that the *ratio* of the velocity of the power to that of the weight, will be greater in that proportion; consequently the velocity of the power must be to the velocity of the weight in a *ratio* compounded of the *ratio* of the diameter of the first wheel to the diameter of the last axle, and of the revolutions of the first to those of the last axle in a given time; and therefore, that there may be a balance between the power and the weight, the former must be to the latter inversely in the same compounded *ratio*. For instance, in a machine consisting of two wheels with their axles, wherein the diameter of the first wheel is four inches, and that of the second axle a quarter of an inch, and wherein the cogs or teeth of the first axle, by applying themselves successively to the teeth of the second wheel, turn it about, and therewith its axle; but the teeth of the first axle being in number but one fourth of the teeth of the second wheel, that axle, and consequently the first wheel, must revolve four times in order to turn the second wheel and its axle once; so that the revolutions of the first

Exp. 9.

first wheel in a given time are to the revolutions of the second axle, as four to one : in this machine, in order to a balance, the power must be to the weight inversly in a *ratio* compounded of sixteen to one, and of four to one ; that is, it must be to the weight inversly as sixty-four to one ; so that a weight of one ounce at the circumference of the first wheel, will support a weight of sixty-four ounces fastened to the second axle.

Again, in a machine composed of three axles, Exp. 10. the two last having wheels with teeth, and the first a perpetual screw, which in each revolution of the first axle moves one tooth only of the wheel of the second axle ; which wheel having twenty-eight teeth, moves round once in the same time that the first axle turns twenty-eight times ; and there being a small wheel with fourteen teeth at the other end of the second axle, and these teeth applying themselves continually to the teeth of a wheel fixed on the third axle, which are twenty-eight in number ; the wheel of the third axle must revolve but once in the same time that the wheel of the second axle revolves twice, and of consequence the third wheel and its axle move round but once whilst the first axle performs fifty-six revolutions ; and the diameter of the first axle is to that of the last as two to one ; in order therefore to a balance between the power which is applied to the first axle, and the weight which is applied to the last ; the power must be to the weight inversly in a *ratio* compounded of two to one, and of fifty-six to one ; that is, the power must be to the weight as one to an hundred and twelve ; so that one ounce hanging from the first axle will support an hundred and twelve ounces hanging from the last axle.

In order to exhibit the force of the WEDGE, which is the fifth mechanick power, let AD represent the base of a wedge, from whose middle point B let the line BE be drawn perpendicular to the

H 4

side

Pl. 3.
Fig. 14a

LECT.
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side DC, and the line BC at right angles to AD, and consequently, bisecting the angle ACD made by the concourse of the wedge's sides.

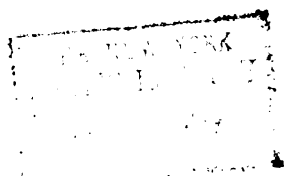
In cleaving timber with a wedge, the force of the mallet which strikes the wedge is to be looked upon as the moving power, and the cohesion of the parts of the timber, as the resistance or weight to be moved; now, whilst the wedge is driven by the repeated strokes of the mallet from B to C, (for I suppose the edge of the wedge to be placed on the top of a piece of timber at B in order to rend it) the space described by the wood as it yields on each side of the wedge in lines perpendicular to those sides, is equal to BE. Consequently, that the moment of the mallet may be equal to the resistance of the wood, the absolute force of the mallet must be to the force wherewith the parts of the wood cohere, as BE to BC, that is, as the sine of the angle BCD to radius; whence it follows, that all similar wedges are of equal force, for in such the angle BCD is given; it likewise follows, that the powers of dissimilar wedges are inversely as the sines of the angles BCD, or in other words, that the forces requisite to rend timber with such wedges, are directly as the sines BE, which is confirmed by the following experiment.

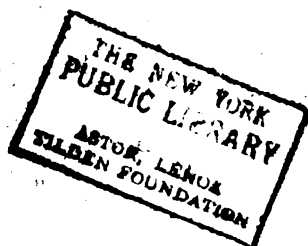
Exp. 11.

Let a machine be so contrived, as to consist of two equal cylinders, rolling upon their axles in an horizontal position along the edges of two rulers, and let them be drawn and kept together by a weight of 2000 grains hanging freely by a rope, fastened at each end to the cylinders, and let the edge of a wedge be placed between the cylinders, so as that when a sufficient weight is hung to it, it may be drawn down between the cylinders; in this machine the force wherewith the cylinders are drawn together, added to the attrition of their axles in rolling upon the rulers, may be looked upon as the resistance of the timber, and the weight of the

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wedge





wedge together with the appending weight whereby it is pulled down between the cylinders, as the force of the mallet upon the wedge; now, if three wedges be made use of, each three inches long, in which the lines B E are as one, two, and three, their weights likewise being in the same proportion, the first will be drawn down by a weight of 300 grains, the second by one of 600 grains, and the third by one of 900 grains.

To the wedge may be reduced the axe or hatchet, the teeth of saws, the chisel, the augur, the spade and shovel, knives and swords of all kinds, as also the bodkin and needle, and in a word, all sorts of instruments which beginning from edges or points grow gradually thicker as they lengthen; and the manner wherein the power is applied to such instruments, is different according to their different shapes and figures, and the various uses for which they were contrived.

The next and last mechanick power is the screw, which consists of two parts, whereof the first is called the male or outside screw, being a cylinder cut in, in such sort as to have a prominent part going round it in a spiral manner, which prominent part is commonly called the thread of the screw; the other part which is called the female or inside screw, and by common workmen the nut, is a solid body that contains an hollow cylinder, whose concave surface is cut in the same manner as the convex surface of the male screw, so that the prominent parts of the one may fit the cavities of the other. The chief design of this machine is to press the parts of bodies closely together, and in some cases to break and divide them; when it is made use of one part is commonly fixed, whilst the other is turned round, and in each revolution the moveable part is carried in the direction of the axis of the cylinder thro' a space equal in length to the interval between two contiguous threads, where-
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by the parts of the body whereon the pressure is made are forced to move towards one another thro' a space equal to that interval; which interval therefore does express the velocity wherewith the several parts of the body give way to the pressure, whilst the circular periphery, which is described by the power whereby the moveable part of the screw is turned round, expresses the velocity of the power; for the moveable part of the screw is usually turned by means of an handle or handspike, to some part of which the power is applied, and by moving round with that part describes the circumference of a circle; if therefore the moving power be to the resistance of the body which is pressed, as the distance between two contiguous threads of the screw to the circular periphery described by the power, there will be a balance; and if the power be ever so little increased beyond that proportion, it must overcome the resistance, and move the screw; and thus it would constantly be, provided there was no resistance from the attrition of the parts of the screw one against another; but as that is very considerable, there is an addition of power requisite to overcome it, over and above what is necessary to overcome the resistance of the body whereon the pressure is made: for which reason such experiments as are made to shew the force of the screw, must vary more from the theory, than those which have been made concerning the other mechanick powers, wherein the attrition is far less considerable; however, it will appear from the following experiment, that small powers are sufficient by the help of the screw to overcome great resistances in the bodies which are pressed.

Exp. 12.

Let a wheel whose diameter is four inches, be fixed at its center to the head of a male screw in an horizontal position, and let the end of a rope, which is wound about the groove of the wheel, pass over a pulley in such a manner as that having

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a weight

A weight fastened to it, it may be drawn in a line, that is a tangent to the wheel, by which means the intire gravity of the weight will be employed in turning the wheel; to one end of a lever, supported by a prop at the middle, let a weight of seven pounds be hung, and let the bottom of the male screw rest on the other end of the lever; and let the distance between the threads of the screw be equal to one fifth of an inch, and a weight of three ounces and 250 grains being hung to the end of the rope which passes over the pulley, will just turn the wheel, and thereby thrust down the screw, and with it the end of the lever whereon it rests, and by so doing raise up the weight at the other end.

In this case the power which moves the screw, is to the weight raised whereby the resistance that is made to the pressure is measured, as one to 24 nearly; whereas it ought not to exceed the proportion of one to 63; for the diameter of the wheel being four inches, the circumference is twelve and an half nearly, but 12.5 is to $\frac{2}{3}$, which is the interval between the threads of the screw, as $62\frac{1}{2}$ to one; consequently, if the power which turns the screw be to the weight that is to be raised in the inverse *ratio* of those numbers, that is, as one to $62\frac{1}{2}$, it ought to balance the weight, and if it be increased ever so little, it should overpower and raise the weight: since therefore the force that is requisite to turn the wheel is nearly three times as great as what is necessary to overcome the resistance of the weight to be raised, it is evident, that almost two thirds of that force is employed in overcoming the resistance arising from the attrition of the parts of the screw one against another; what the nature of this resistance is, and in what proportion it varies, shall be shewn hereafter.

LECTURE

LECTURE VII.

OF COMPOUND ENGINES.

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THE mechanick powers, which for the most part are made use of separately, may in many cases be combined together, and engines thereby formed of such efficacy, as that by the help thereof exceeding great weights may be raised by very small powers. In all such compounded machines the proportion which the moving power bears to the weight when they balance each other, is compounded of the several *ratios* which those powers have to the weight which balance it in each simple machine, whereof the compounded engine consists. Thus when a machine is composed of an axle in the wheel and a pulley, by fastening the drawing rope of the one to the axle of the other; the power which balances the weight in such a machine, must be to the weight, in a *ratio* compounded of the *ratio* which that power has to the weight which balances it by means of the axle in the wheel alone, and of the *ratio* which that power has to the weight, which balances the weight by means of the pulley alone. For instance, if the nature of the pulley be such, as that a power equal to one tenth part of the weight balances it; and if the axle in the wheel be such, as that a power equal to one fifth part of the weight can support it; the power which balances the weight in the compounded machine, will be to the weight in a *ratio* compounded of one to ten, and of one to five, that is, it will be to the weight as one to fifty; for, since the weight is in effect fastened to the axle of the wheel by means of the rope which goes round the pullies, it is evident that the axle will be drawn by a force equal to that, which

Exp. 1.

which when applied to the drawing rope of the pulley is requisite to sustain the weight by means of the pulley, which force is by supposition equal to one tenth part of the weight; but that force at the axle is balanced by a fifth part thereof applied to the wheel; consequently, the power requisite to balance the weight in this machine, is equal to one fifth of one tenth part of the weight, that is, the power is to the weight, as one to fifty. So that one ounce at the wheel will support fifty ounces at the pulley.

If a machine be composed of the lever, the axle, and the perpetual screw; the lever being thirteen inches long, and fixed at its center to an axle, whereon is a perpetual screw, the tooth whereof adapts itself to the teeth of the wheel of an axle, the teeth of that wheel being twenty-four in number, and the diameter of the axle belonging to that wheel equal to six tenths of an inch; in such a machine the power being applied to one end of the lever, and the weight to the axle of the toothed wheel, the former will balance the latter, if it be in proportion thereto, as one to 520; for if the lever to which the power is applied, moved round in the same time with the axle of the toothed wheel whereunto the weight is fastened, the power would be to the weight, as the diameter of the axle to the length of the lever, that is, as six tenths of an inch to thirteen inches, or in whole numbers, as six to an hundred and thirty; but as there are 24 teeth in the wheel of that axle which sustains the weight, and as the endless screw moves but one of those teeth in each revolution of the lever, the lever must go round 24 times in order to turn the axle, which sustains the weight, once; upon which account the power must be to the weight, as one to 24, which ratio of one to 24 being combined with the former of six to 130, gives a ratio of six to 3120, or of one to 520; so that an ounce weight being made to act with all its gravity at one end of the

Exp. 2.

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the lever in order to turn it round, which may be done by fixing a wheel to the lever, will balance a weight of 520 ounces at the axle of the toothed wheel.

Exp. 3.

If to the last machine one moveable pulley be added, it will constitute a machine of double the force; for the *ratio* of the power to the weight in the foregoing machine, being as one to 520, and in a single moveable pulley, as one to two; the *ratio* compounded of both, will be as one to 1040; so that in this machine an ounce will balance 86 pounds 8 ounces; and if the strength of a man's hand be such, as that it can without the assistance of an engine support an hundred pounds, it will by the help of this machine sustain 104000 pounds.

In all that has been hitherto said concerning the mechanick powers, the moving force and the weight or resistance have been supposed to act in direct opposition to one another. I shall now consider the effects of powers acting obliquely, and shew in what cases they balance each other.

Pl. 4.
Fig. 1.
Exp. 4.

And first, if three powers acting in oblique directions, be to one another, as the respective sides of a triangle formed by the concurrence of three lines drawn parallel to the directions of the powers; those powers will balance one another. For instance, if three powers drawing the point A in the directions AB, AC, and AE, be to one another, as the sides of the triangle ADE, or ADC, made by the concurrence of the lines AD, AE, and ED; or AD, CD, and AC, which lines are parallel to the directions of the powers; they will balance one another, and the point A will remain unmoved.

For if the line AD be supposed to denote a power equal to that which acts in the direction AB, but contrary thereto; the power denoted by AC will draw the point A as forcibly towards D, as it is drawn by the opposite power towards B; consequently, there will be a balance between the two powers;

powers; but the power denoted by AD may be resolved into two powers denoted by AE or CD , and AC or ED ; which two powers acting together upon the point A in their proper directions AE and AC , will draw it as strongly towards D , as it is drawn by the single power denoted by AD ; as is evident from what has been said concerning the resolution and composition of motions and forces; consequently, two powers which are as AE or CD , and ED or AC , acting in the directions AE and AC , will balance the third power which is as AD acting in the direction AB ; that is, two powers, which are as the two sides of a triangle, acting in directions parallel to those sides will balance a third power, which is as the third side, and which acts in a direction parallel thereto; and what has been thus proved in particular of two of the powers with regard to the third, is in like manner demonstrable of any two of the powers with respect to the other; consequently, any three powers which are to one another respectively as the sides of a triangle, and which act in directions parallel to those sides, will destroy each the other's effect, and remain in *equilibrio*. To confirm this by an experiment; let the sides of a triangle ABC drawn on an horizontal plane be as two, three, and four; and let CE be parallel to the side AB , and the side AC continued towards D . Let three small cords be joined together at C , and stretched over three pulleys in such a manner, as that one of them may cover the line CD , another the line CE , and the third the line CB ; this being done, if a weight of four ounces be hung to the cord which passes over CD , and one of three ounces to that which covers CB , and one of two ounces to that which covers CE , there will be a balance; the weights, which in this case are the moving forces, being to one another as the sides of the triangle to which the direction of the weights are parallel.

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Pl. 4.
Fig. 2.
Exp. 5.

If

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Pl. 4.
Fig. 3, 4.
Exp. 6.

If the weight *A* hangs freely from one end of a balance, so as to have its line of direction *DA* perpendicular to the arm of the balance; and if another weight as *B*, be hung at the other end *E*, in such a manner, as that its line of direction *EC* by passing over a pulley at *C* may be oblique to the arm of the balance, the weight *B* must be to the weight *A* when it counterbalances it, as *EC* to *FC*, that is, as radius to the sine of the angle *CEF* made by the oblique direction of *B* with the arm of the balance; for if the whole force of gravity in the weight *B* acting in the direction *EC*, be denoted by the line *EC*, it may be resolved into two forces denoted by *EF* and *FC*, acting in the directions of these lines, of which two forces, the latter only which acts in the direction *FC* perpendicular to the arm of the balance withstands the force of gravity in the weight *A*, the other force which acts in the direction *EF* being entirely employed in pressing the balance against the axis of its motion; since therefore, that part of the weight *B* which acts in opposition to the weight *A*, is to the whole weight *B*, as *FC* to *EC*; it is manifest, that in order to make the weight *B* balance the weight *A*, it must exceed the weight *A* in the same proportion that the line *EC* exceeds the line *FC*; and thus it is found to be from experiments; for if the pulley be so ordered as that *EC* may be to *FG* as three to two, then a weight of three ounces appended at *E*, will balance one of two ounces appended at *D*.

As a COROLLARY it follows, that the perpendicular distances of the lines of direction from the center of motion, are to one another inversely as the weights; for, if from *G* the center of motion be let fall *GH* perpendicular to *EC*, that line will be the perpendicular distance of the direction *EC* from *G*; and *EG*, equal to *DG*, is the perpendicular distance of the direction *DA*; but the triangles *EFC*

$EF C$ and $EH G$ are similar, because their angles at E are equal, and they have each a right angle; consequently, as EC is to CF , so is EG to HG ; but the weight B is to the weight A , as EC to FC , that is, as EG or DG to HG ; so that wherever two powers, which act in oblique directions, are to one another in the inverse *ratio* of the perpendicular or shortest distances of their lines of direction from the center of motion, they must balance one another; whence it follows, that if two weights as A and B , be suspended from two points as D and E in the plane of a wheel placed in a vertical position; and if the line DE which is drawn thro' the two points of suspension, passes thro' C the center of motion, the weights will balance, provided they be to one another inversely as the distances of their points of suspension from the center of motion, that is, if A be to B , as CE to CD ; for since the weights hang freely, their lines of direction DA and FB , will be perpendicular to the horizon, and of consequence, parallel to each other; wherefore, if the line HCF be drawn thro' the center of motion perpendicular to the two lines DA and FB , the triangles DHC and ECF will be similar, consequently, DC will be to EC , as HC to FC ; but by supposition, the weight A is to the weight B , as CE to CD ; that is as CF to CH ; so that the weights are to one another inversely as the perpendicular distances of their lines of direction from the center of motion; consequently, they must balance; and tho' the wheel should be turned upon its axis, and the distances of the lines DA and EB from C be thereby altered, yet will the similarity of the forementioned triangles continue, and of consequence the balance between the weights will be preserved; as will appear from the following experiment. Let a weight of one ounce be suspended from the point D , and another of two ounces from the point E ; DC be-

Pl. 4.
Fig. 5.

Exp. 7.

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ing to EC , as two to one, that is, inverſly as the weights, there will be a balance, and the wheel will continue at reſt. And if by the force of the hand it be turned about its axis either to the right from I towards K , or to the left towards M , the balance will ſtill continue, and the wheel will remain unmoved when the hand quits it, whatever be its poſition.

Pl. 4.
Fig. 6.

If the points of ſuſpention D and E be ſo poſited, as that the right line DE which joins them, does not paſs thro' C the center of motion; let that line be divided any where as in G by another line as IL paſſing thro' the center C , and there will be a balance, if the appending weights be to one another inverſly as the parts of the line DE , that is, if A be to B as EG to DG , provided the poſition of the wheel be ſuch, as that the line IL may be perpendicular to the horizon; for ſince the lines EF , GC , and DH are parallel, FC is to HC , as EG to DG ; but by ſuppoſition, as EG is to DG , ſo is A to B ; wherefore A is to B , as FC to HC , that is, the weights are inverſly as the perpendicular diſtances of their lines of direction from the center of motion, conſequently, their moments are equal; but if by turning the wheel about its axis the line IL be put out of its perpendicular poſition, the balance will be deſtroyed; becauſe, in that caſe, one of the lines of direction will approach nearer to the center of motion, whiſt the other recedes; and of courſe their perpendicular diſtances will not continue in the inverſe *ratio* of the weights; for if the wheel be moved upon its axis from I towards K , ſo as to have the line SCR perpendicular to the plane of the horizon; the line of direction DA will approach towards the center ſo as to become DP , and its perpendicular diſtance from the center of motion will be NC , whiſt the other line of direction recedes as far as EQ , and its perpendicular diſtance from

from C becomes equal to OC; for which reason the weight B must preponderate, and move the wheel about its axis in the direction I K L. And as the wheel continues to move in that direction, the direction of the weight A will approach nearer and nearer to the center of motion, and at length pass beyond it, so as to be on the same side with the direction of the weight B; so that the wheel will then be moved by the joint force of both weights, and continue so to be, till such time as the direction of the weight B getting on the other side of C, B begins to act in opposition to A, and at length the point I being brought into the place of L, the weights do again balance each other, the line BE being divided by the perpendicular line IL in the reciprocal *ratio* of the weights. To confirm what has been said by an experiment, let the line DE in the plane of a wheel be divided in G by the line IL in such a manner, as that DG may be double of EG; then setting the line IL perpendicular, let a weight of one ounce be hung from D, and another of two ounces from E, and the wheel will remain unmoved; let then the wheel be turned a little upon its axis, either to the right hand or to the left; in the former case, the two ounce weight will prevail, and carry the wheel downward to the right hand, but in the latter the smaller weight will preponderate, and make the wheel to revolve towards the left.

Exp. 8.

If the line DE be divided in another point as T, by the line SR, so as that DT may be one third of ET; and if a weight of three ounces be suspended at D, and another of one ounce at E, the same things will happen as in the former experiment; for the line SR being placed vertical there will be a balance; and upon moving it out of that position the balance will be destroyed.

Exp. 9.

If the crooked lever FCD be so placed on its prop at C, as that the arm CF may be parallel to

Pl. 4.
Fig. 7.

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VIII.

the plane of the horizon, and the arm CD inclined thereto, if two weights as B and A , appended at D and F , be in the reciprocal proportion of the perpendicular distances of their lines of direction from the prop; that is, if B be to A as FC to EC there will be a balance; for as long as the arm CF continues parallel to the horizon, the weight B hanging from the point D acts in the same manner in opposition to the weight A , as if it hung from E the extremity of the strait lever FC continued on to E , in which case the weight B that balances the weight A must bear the same proportion to it that FC does to EC ; if therefore the arm DC be bent in such a manner, as that EC may be one half or one third of FC , in the former case a weight of two ounces, and in the latter one of three ounces hanging from D , will be counterpoised by one ounce hanging from F .

If by moving the lever, the arm FC be put out of its parallel position, the balance will be destroyed; for that cannot be preserved, unless the distance of B 's direction from the prop continues to bear the same proportion to the distance of A 's direction, that EC does to FC ; which in this case is impossible; for first, if the point F be moved upward towards H , and of course the point D downward towards G , it is manifest, that the distances of both directions will be lessened; but the decrease of EC in a given time will bear a greater proportion to the decrease of FC than EC does to FC ; for by that time the point D has moved from D to G thro' the arch DG , which measures the angle of CD 's inclination, EC will vanish; whereas FC cannot vanish till such time as the point F has moved from F to M thro' the quadrantal arch FM ; but in the same time that the point D moves from D to G thro' the arch DG , the point F can move only from F to H thro' the arch FH similar to DG ; which arch being always less than the quadrant, the

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perpen-

perpendicular distance of A's direction from the prop, to wit FC, will not vanish upon the arrival of the point F at H, that is, it will not vanish so soon as EC; consequently, the decrease of EC in a given time must bear a greater proportion to the decrease of FC, than EC does to FC: wherefore EC as diminished in any given time, will be to FC as diminished in the same time, in a less proportion than that of EC to FC; or in other words, the perpendicular distance of B's direction from the prop will bear a less proportion to the perpendicular distance of A's direction, than EC does to FC; and therefore the weight A will preponderate. If the point F be moved downward, and consequently D upward, it is manifest from the inspection of the figure, that the distance of A's direction from the prop continually diminishes, at the same time that the distance of B's direction increases; and therefore the weight B must in that case overbalance the weight A.

If FCD be a crooked lever placed as the last, and if a weight, instead of being hung from the arm DC, be laid thereon at D, and by a vertical plane, as HK, set close to it, be hindered from falling off; from the point D, whereon the weight rests, let the line DE be drawn perpendicular to the arm FC continued on towards G; the weight at D will be balanced by the weight A hanging freely from F, provided the weight D be to the weight A, in a *ratio* compounded of EC to CD, and of FC to CD; that is, as a rectangle under EC and FC the perpendicular distances of the directions of the two weights from the prop, to the square of CD the inclined arm of the lever. For whatever be the moment wherewith the weight A presses down the arm FC, the arm DC must with an equal moment be pressed upward, and with it the weight D in the direction DG perpendicular to

I 3

CD;

Pl. 4.
Fig. 8.

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CD; and forasmuch as the same weight presses perpendicularly against HK the vertical plane, it must be pressed backward by the same in an horizontal direction; and at the same time it must have a tendency downward from the force of gravity in the direction ED; so that it is acted upon by three forces in the directions DG, GE, and ED; in order therefore to a balance, the forces must be as the sides of the triangle DGE; and the force of gravity which presses it in the direction ED, must be to the force pressing it in the direction DG, as ED to DG, or, because the triangles DGE and CDE are similar, as EC to CD; but as the force which presses it in the direction DG is of equal moment with the weight A, that force must be to the weight A, as FC to CD; consequently, the force of gravity in the weight D must be to the force of gravity in the weight A, that is, the weight D must be to the weight A, in a *ratio* compounded of EC to CD, and of CF to CD, or as the rectangle under CE and CF to the square of CD. To confirm this by experiment, let a crooked lever as FCD consist of equal arms, and let it be bent in such a manner, as that EC may be to CD, as one to two; and let a weight of one ounce be laid on at D, and another of two ounces be hung from F, and they will balance each other; for in this case the product of EC which is as one, into CF which is as two, will be two; and CD being as two, the square thereof will be four; so that the rectangle under EC and CF, is to the square of CD, as two to four, or as one to two, in which proportion therefore the balancing weights must be.

Exp. 11.

All things remaining as in the last experiment, excepting that the arm CF is as long again as CD, so that EC, CD, and CF, are as one, two, and four; a weight of one ounce at D will be balanced by one ounce hanging freely from F; for CD be-

ing as two, its square is four; and the product of EC, which is as one into FC, which is as four, is likewise four.

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Exp. 12.

In wheels turned by the force of water falling upon them from an height, and which on that account are commonly called overshot wheels, the moving power is partly the percussive force of the water which falls into the uppermost bucket, and partly the gravity of the water contained in the other buckets, which are lodged on the rim of the uppermost quarter of the descending part of the wheel; and the effects which these forces have upon the wheel, are greater or less in proportion to their absolute quantities, and the distances of their lines of direction from the center of the wheel. Thus, where A IOP represents an overshot wheel, C its center, K, L, M, N four buckets fixed on the uppermost quarter of the descending part of the wheel; AB the direction of the water flowing into the uppermost bucket K, CB the perpendicular distance of that line from the center C; DE, FG, and HI, the lines of direction of the centers of gravity of the several portions of water contained in the buckets L, M, N; CE, CG, and CI, the perpendicular distances of those lines from the center C. The force of the water flowing into K is proportional to the quantity flowing in a given time, as also to the velocity wherewith it flows, and the distance of its line of direction from the center; and therefore, where the quantity and velocity are given, the force will be as BC, the perpendicular distance of AB, the line of direction, from C, the center of motion; consequently, the nearer AB approaches to the tangent in the point A, or the more obliquely the water flows in upon the wheel, the greater will its force be. The portions of water contained in the buckets L, M, N, have different forces according to their different quantities, and the different distances of their lines of direction from the center

Pl. 4.
Fig. 9.

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ter C, their quantities being greatest, when the distances of their directions are least, for the buckets empty as they descend ; so that their force lessens as they descend, by reason of the diminution of their quantities, but at the same time it likewise increases on account of the increase of the distance of their lines of direction from the center of motion ; so that upon the whole, the force in each bucket may be looked upon as invariable ; but whether this be so or not, certain it is, that if the wheel be truly centered, and the buckets be equal and alike, and if the water flows in uniformly, the whole moving force must continue the same as long as the wheel continues to move ; and since it acts incessantly, the motion of the wheel must be continually accelerated, and that uniformly ; and thus it would be, were it not that when the wheel arrives at a certain degree of velocity, the resistance which is given becomes so great as to destroy the increments of motion as fast as they are generated by the moving force ; by which means the wheel is made to revolve with one uniform velocity, which is the greatest that can be given it by that moving power.

Pl. 4.
Fig. 10.

A plane as AB placed obliquely to BC, which represents an horizontal plane, is called an inclined plane ; the angle ABC is called the angle of elevation, and its complement BAC the angle of inclination ; the line AC perpendicular to BC is called the height of the plane, and AB its length. If a weight as P be laid on an inclined plane as AB, and be thereon sustained by a power acting in a direction, as PF, parallel to the inclined plane ; in order to a balance, the sustaining power must be to the weight as the height of the plane to the length thereof, that is as AC to AB, or, putting BA for the radius, as the sine of the angle of elevation to radius ; for the weight P is acted upon by three powers in different directions, the first of which is the force of gravity,

gravity, which presses it downward in the direction PD perpendicular to BC ; the second is the power which draws it in the direction PF parallel to BA ; and the third is the plane BA , which does as it were press it upward in the direction PH perpendicular to BA ; for as the weight P presses the plane in a direction perpendicular thereto, it is reacted upon by the plane in a contrary direction. If therefore the line EG be drawn parallel to PD , the sides of the triangle PEG will be proportional to the three powers, and the force which supports the weight on the inclined plane, and which acts in the direction PF , will be to the absolute weight of the body acting in the direction PD parallel to GE , as PG to GE ; but inasmuch as the triangles PEG and CBA are similar, as PG is to GE , so is AC to AB ; consequently, the power necessary to support a weight on an inclined plane must bear the same proportion to the weight sustained, that the height of the plane does to its length; which is confirmed by experiments; for if a weight of four ounces be laid on a plane whose length is to its perpendicular height, as two to one, it will be counterbalanced by a weight of two ounces, provided the whole gravity thereof be made to act in drawing the other weight in a direction parallel to the inclined plane, which may be done by fastening one end of a cord to the greater weight, and then stretching the cord along the plane, so as to keep it parallel thereto, and passing it over a pulley at the top of the plane; for the smaller weight being tied to the end of the cord which lies beyond the pulley will hang freely, and for that reason act with all its gravity in a direction parallel to the plane.

Exp. 13.

The same weight of four ounces being laid on an inclined plane, whose length is to its height as four to one, will be sustained by a weight of one ounce hanging freely as before.

Exp. 14.

The

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The force wherewith a body resting on an inclined plane presses the same, is to the weight of the body, as the sine of the angle of inclination to radius; for in the triangle PEG, PE denotes the force wherewith the body presses the plane, and GE the weight of the body; but from the similitude of the triangles, as PE is to GE, so is BC to BA; and putting BA for the radius, BC is the sine of BAC the angle of inclination; wherefore as BC the sine of the inclination is to the radius AB, so is the force wherewith the body presses the plane to the absolute weight of the body. Hence, if upon an inclined lever as AB, resting on the two props A and B, a weight be laid any where, as at P, it will be easy to determine what proportion of the weight each prop bears; for drawing the horizontal line AE equal in length to AB, and from the point P whereon the weight rests, letting fall PD perpendicular to AE, if AE be supposed to denote the whole weight of the body, AD will denote that part of it which is sustained by the uppermost prop, and DE that part which is supported by the lower; for if the lever was horizontal, so as that the body might press it with all its gravity, the whole weight of the body would be to that part of it which presses the prop B, as BA to PA, as is evident from what has been said concerning the second kind of lever; but as in the inclined position of the lever the whole weight of the body does not press upon it, that part of the weight which the prop B sustains in the horizontal position, must be to the part sustained in the inclined position, in the same proportion with the absolute weight of the body to the force wherewith it presses the inclined plane, that is, as PA to AD; for putting PA for the radius, AD is the sine of the inclination of the lever; consequently, the whole weight of the body must be to that part which presses on the prop B in the inclined position of the lever, in a *ratio* compounded

Pl. 4.
Fig. 11.

compounded of BA to PA, and of PA to AD, that is, it must be as BA to AD, or because AB and AE are equal, as AE to AD; and of consequence the part supported by the other prop A must be as DE.

Hence it follows, that if two persons carry a load fixed upon a lever, the load being placed between them, which is the case of chairmen, upon descents the foremost man will bear the greatest burthen, and upon ascents the hindermost. It likewise follows, that in coaches and all other four-wheel carriages, which have the foremost wheels smaller than those behind, the load must be thrown more upon the former than the latter; what effect this has upon the draft, shall be shewn in my next lecture.

LECTURE IX.

OF F R I C T I O N.

IN my last lecture I shewed you what force is requisite to sustain a body on an inclined plane. If a body be laid on a plane parallel to the horizon, it does not stand in need of any force to support it; for as the direction of gravity is perpendicular to the plane of the horizon, the whole weight of the body must be sustained by the horizontal plane whereon it rests: whence it follows, that if any power endeavours to move a body resting on an horizontal plane in a direction parallel to the plane, it will meet with no resistance from the weight of the body, that being entirely taken off by the reaction of the plane whereon the body presses; but a resistance will arise from the attrition of the body against the plane; for the surfaces of all bodies whatever, even such as are of the finest polish, being in some measure rough and unequal, (as is evident from the obser-

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observations that have been made by the help of microscopes) when a body is moved upon a plane, the prominent parts both of the body and plane must of necessity fall into each others cavities, and thereby create a resistance to the motion of the body, inasmuch as the body cannot be moved unless the prominent parts thereof be continually raised above the prominent parts of the surface whereon it slides; and this cannot be done unless the whole body be at the same time lifted up, and as it were raised on an inclined plane equal in height to the forementioned protuberant parts; upon which account the moving power must sustain some part of the weight of the body, even in moving it along an horizontal plane. But as this is occasioned by the inequalities in the surface, if those were intirely taken off, so as to leave the surface perfectly smooth and even, the resistance arising from friction would likewise be removed; and setting aside the resistance of the medium, the smallest force would be sufficient to move the most ponderous body along an horizontal plane. But since there are not in nature any bodies, whose surfaces are perfectly equal, there will ever be some resistance arising from friction; which resistance will remain unvaried, whatever be the magnitude of the surfaces that rub one against the other, provided the weight which presses those surfaces together, as also the roughness of the surfaces, continue the same; for the same weight will ever require the same force to raise it over prominences of a given height, whatever be the magnitude of the surface whereon the weight rests; consequently, the quantity of resistance will not be varied by varying the magnitude of the surface; which may be confirmed by the following experiment. Let four pieces of polished box, be laid on a polished horizontal plane, and let each piece be so loaded as that its own weight, together with that

of

Exp. I.

of its load may be 6685 grains, and let the basis of one be two inches long, and half an inch broad, and those of the other three be each four inches in length, but let their breadths be half an inch, an inch, and an inch and an half, so that the magnitudes of the bases may be as one, two, four, and six; let then a small cord be fastened to the end of each piece, and by passing over a pulley be kept in a position parallel to the plane, and a weight of 2030 grains hanging from the end of the cord which lies beyond the pulley will just suffice to move each piece along the plane; so that the resistance arising from friction is the same in each piece, notwithstanding the different magnitudes of the surface whereon they rest.

If the roughness of the surfaces whereon the bodies move be given, the resistance arising from friction will vary with the weights of the bodies, and be proportional thereto; for if a certain force be sufficient to raise a certain weight over prominences of a given height, it is manifest that a double or triple weight will require a double or triple force to raise it to the same height. If therefore the pieces of box be so loaded, as that each of them with its load may weigh 13370 grains, that is, as much again as in the last experiment, a weight of 4060 grains, that is, twice as much as before, will be necessary to move them along the same plane.

Exp. 2.

If the roughness of the surface whereon a body moves be increased, the resistance will likewise increase tho' the weight of the body remains the same; but as the degree of roughness in any surface cannot otherwise be determined than by experiment, so neither can the resistance arising therefrom: if the plane made use of in the last experiments be thinly covered with fine sand, the resistance will thereby become greater in the proportion of about five to four; for the same pieces of box which were set a going by 2030 grains when the plane

Exp. 3.

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Pl. 4.
Fig. 12.

plane was free from sand, will in this case require 2500 grains, that is, about one fourth more.

To avoid as much as possible the resistance arising from friction, which in rough and uneven roads must needs be very great, WHEEL CARRIAGES have been contrived; the advantages whereof I shall endeavour to explain to you, but I shall first shew you from what cause it is that wheels turn round during their progressive motion along a plane. If a wheel as ACB playing freely on the axis at A, be lifted off the plane BD by a power applied to the axle, and be carried in any direction whatever, it will not revolve about the axle; for since in all wheels that are truly made, the axle passes thro' the center of gravity, it is evident, that in this case the wheel is suspended by its center of gravity, and of consequence will not of itself change its position, but each point thereof will describe a line parallel to the direction of the moving power without any rotation about the axle, in the very same manner as if the wheel was fixed to the axle; but if one point of the wheel as B rests upon the plane BD, and if a power applied to the axle draws the wheel in any direction as AP, so as to move it along the plane BD; the motion of the point B will be retarded by the resistance arising from friction, whilst the point C which meets with no resistance is carried forward without any retardation of its motion, and consequently must move forward faster than the point B; but as all the parts of the wheel cohere, the point C cannot move forward faster than the point B, unless the wheel revolves about its axis from C towards E; and as the several points of the wheel's circumference, which are successively applied to the plane, suffer a retardation in their motion whilst the opposite points move freely, the wheel during its progressive motion along the plane must continue to revolve about its axle.

By

By this rotation of wheels about their axles, the resistance arising from friction is very much diminished, and draughts thereby rendered more easy; for in plain roads, where the height of the prominent part is inconsiderable with respect to the diameter of the wheel; the parts of the revolving wheel which apply themselves successively to the road, may be looked upon in some measure as descending upon the minute prominencies, and of course must pass over them without any considerable friction. And so much is the resistance arising from friction diminished in wheel carriages, that if upon the same plane whereon the pieces of box were drawn, a carriage be laid with four equal wheels, each three quarters of an inch in diameter, and loaded in such a manner, as that the weight of the carriage and load may amount to 6685 grains, which was the weight of each piece of box with its load; it will be set a going by a weight of 420 grains drawing it horizontally, whereas 2030 grains were requisite to move the pieces of box along the same plane.

Exp. 4.

From this experiment it appears, that the friction is very much lessened by means of wheels; which diminution is not to be attributed to the wheel's touching the plane in a few points, as may possibly be imagined, but to the rotation of the wheels; for if the wheels of a carriage loaded as before be made fast to the axle, so as not to revolve in their motion, 2030 grains will be necessary to set the carriage a going, that is, just as much as was requisite to move the pieces of box.

Exp. 5.

As wheel carriages in general meet with less resistance in their motion than any other, so those of larger wheels, *cæteris paribus*, are less resisted than those of smaller; for the proof whereof, it will be necessary to premise two LEMMAS; the first of which is, *that the secants of angles are to one another inversely as the sines of their complements, that is, AD, which is the secant of BAD, is to AC, which is the*

Pl. 4.
Fig. 13.*secant*

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secant of BAC, as AF, which is the sine of the complement of BAC, to AH, which is the sine of the complement of BAD. For from the nature of similar triangles AD is to AC, as AE to AK, that is, as AE to AG; but AE is to AG, as AF to AH; consequently, AD is to AC, as AF to AH.

The second LEMMA is, *that if two arches of unequal circles have their versed sines equal, the arch of the lesser circle is greater in proportion to the whole periphery, than the arch of the greater circle; or in other words, the angle measured by the arch of the lesser circle, is greater than the angle measured by the arch of the greater circle.* Let HF and DB be two arches of unequal circles; whose versed sines FG and BC are equal; I say, the angle HEF is greater than the angle BAD; for since EF is less than AB, and GF and CB are equal, EF is to GF in a less proportion than AB to CB; consequently, EG is to EF in a less proportion than AC to AB, that is, the sine of the angle GHE is less than the sine of the angle CDA, and of course the angle GHE is less than the angle CDA; consequently, the angle HEG, which is the complement of the lesser angle GHE, is greater than the angle DAC, which is the complement of the greater angle CDA.

Pl. 4.
Fig. 14.

Pl. 4.
Fig. 15.

These two LEMMAS being premised, let HM represent a plane whereon move the two wheels ABH and KLR, which are of different magnitudes, but equal in weight, and let BC and LM be two obstacles of equal heights, and of such a nature as that the wheels cannot otherwise pass than by surmounting those obstacles; the force requisite to draw the larger wheel over the obstacle BC, is less than what is requisite to draw the lesser wheel over the obstacle LM equal in height to the former; for since the wheels revolve in passing over the points B and L, their centers of gravity A and K may be looked upon as revolving about the fixed points B and L, and describing the arches AF and KP; consequently,



consequently, the forces which move the wheels may be looked upon as drawing them upon inclined planes, whose directions coincide with the directions of the curves in the points A and K, that is, they coincide with the tangents A E and K O; which tangents being parallel to the tangents of the wheels in the points B and L, that is to say, to D B and N L; the centers of gravity of the two wheels, and consequently the wheels themselves may be looked upon as drawn up the inclined planes D B and N L; but since the wheels are supposed to be equal in weight, the forces which support them on the inclined planes D B and N L, the height whereof is given, must be to one another inversely as the lengths of the planes; that is, the force which supports the larger wheel on the plane D B, must be to the force supporting the smaller wheel on the plane N L, as N L to D B; that is, putting B C or L M for the radius, as the secant of the angle N L M to the secant of the angle D B C; or, because K S and L M, as also A I and B C are parallel, as the secant of the angle K S L to the secant of the angle A I B; but, from the nature of similar triangles, the angle K S L is equal to the angle K L Q, as is also the angle A I B to the angle A B G; and therefore the force which sustains the greater wheel on the inclined plane D B, is to the force sustaining the lesser wheel on the inclined plane N L, as the secant of the angle K L Q to the secant of the angle A B G; but, by the first *Lemma*, the secant of K L Q is to the secant of A B G, as the sine of B A G to the sine of L K Q; and, by the second *Lemma*, the sine of B A G is less than the sine of L K Q; consequently, the force which raises the greater wheel over the obstacle B C, is less than the force which raises the lesser wheel over the obstacle L M equal in height to the former; but the forces requisite to make the wheels surmount the obstacles are the measures of the resistances, and therefore,

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ceteris

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ceteris paribus, the greater wheel must meet with less resistance from the same obstacle than the smaller. To confirm this by an experiment, let an obstacle one tenth of an inch in height be fixed on an horizontal plane, and close behind it let there be placed the carriage with four equal wheels, each three quarters of an inch in diameter, and if it be loaded in such a manner as that the weight of the carriage and load may amount to 6685 grains, it will not be raised above the obstacle by less than 2850 grains drawing it in a direction parallel to the plane; whereas if four wheels, each an inch and an half in diameter be fitted to the same carriage, the weight of the whole being the same as before, it will be raised above the obstacle by 2050 grains, that is, by 800 grains less than were requisite to raise it with the smaller wheels.

From this experiment it appears, that the resistance which larger wheels meet with in surmounting obstacles is less than the resistance given to smaller wheels by the same obstacles; and from what has been demonstrated it is evident, that the resistance given to the greater wheel is to the resistance given to the smaller, as the sine of an angle measured by an arch of the greater wheel, to the sine of an angle measured by an arch of the smaller wheel, the versed sine of each angle being equal to the height of the obstacle; so that putting R and r for the radii of the two wheels, and x for the versed sine or the height of the obstacle, it follows from the

nature of the circle, that as $\frac{\sqrt{2Rx-xx}}{R}$ is to $\frac{\sqrt{2rx-xx}}{r}$, so is the resistance given to the larger

wheel to the resistance given to the smaller, or dividing by $x^{\frac{1}{2}}$, as $\frac{\sqrt{2R-x}}{R}$ to $\frac{\sqrt{2r-x}}{r}$; but as the

proportion

proportion of these lines is not fixed, but varies with the height of the obstacle, so likewise must the proportion, which the resistance given to the greater wheel bears to the resistance given to the smaller; and all that can be determined in this case is, that larger wheels ever meet with less resistance in surmounting obstacles than smaller; and that the disproportion between the resistances suffered by each wheel, increases with the height of the obstacle. Indeed where the obstacle vanishes, which is the case when wheels move upon planes, the expressions for the resistances, and consequently the resistances

themselves are, as $\sqrt{\frac{1}{R}}$ and $\sqrt{\frac{1}{r}}$, that is, the resist-

ances are inversely as the square roots of the semidiameters of the wheels; so that where the heights of the wheels are as one and two, the forces requisite to draw them along the same horizontal plane, are as fourteen and ten, that is, inversely as the square roots of one and two, which is confirmed by experiments; for whereas the carriage whose wheels are three quarters of an inch in diameter, required 420 grains to move it along the horizontal plane, the weight of the carriage and load being 6685 grains, the carriage whose wheels are $1\frac{1}{2}$ inch in diameter, when loaded in the same manner, will be set a going by 300 grains; but 420 is to 300, as 14 to 10, that is, the forces requisite to move the two carriages along the same plane are inversely as the square roots of the heights of the wheels.

If the nature of the obstacle be such as to be bore down by the pressure of the wheel, the larger wheel will in this respect likewise have the advantage over the smaller, and depress the obstacle with greater force. For let L K be continued to T, so that T L may be equal to A B; and since the wheels are supposed to be equally weighty, let A B and

K 2

T L ex-

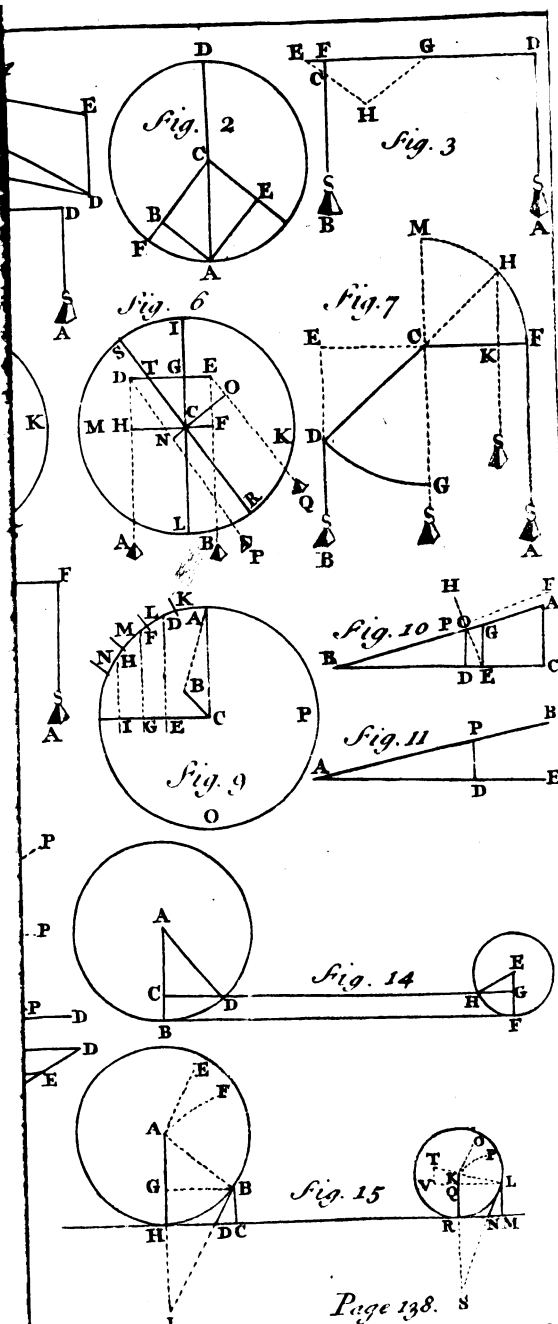
Pl. 4.
Fig. 15.

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T L expresses the absolute forces of the two wheels acting against the obstacles in the directions **AB** and **KL**; it is evident from what has been said concerning the resolution of forces, that the force denoted by **AB** may be resolved into two forces; one whereof may be denoted by **AG**, and the other by **GB**, whereof **AG** alone acts in depressing the obstacle **BC**, inasmuch as it bears directly down upon it; whereas the other force denoted by **GB**, inasmuch as its direction is perpendicular to the obstacle, may thrust it forward, but can contribute nothing towards pressing it downwards from **B** towards **C**. In like manner the force denoted by **T L**, is resolvable into two forces, which may be denoted by **TV** and **VL**, whereof **TV** alone acts in depressing the obstacle **LM**; consequently, the force wherewith the greater wheel depresses the obstacle, is to the force wherewith it is depressed by the lesser, as **AG** to **TV**, or as the sine of the angle **ABG** to the sine of the angle **TLV** or **KLQ**; but by the second *Lemma*, the angle **BAG**, which is the complement of **GBA**, is less than the angle **LKQ** the complement of **KLQ**; consequently, the angle **ABG** is greater than **TLV**, and **AG** the sine of the former greater than **TV** the sine of the latter; but as **AG** is to **TV**, so is the depressing force of the greater wheel to the depressing force of the lesser; consequently, the same obstacle is more easily depressed by the larger wheel than the smaller, and of course must give less resistance to the former than to the latter.

If the obstacle be such as that it can neither be surmounted nor depressed, but must be driven forward, then indeed the smaller wheel has the advantage of the larger; for the forces of the wheels being resolved as before, the lines **GB** and **VL** will express the forces which act in driving the obstacle forward; but it has been demonstrated, that **GB** the sine of the angle **GAB**, is less than **VL** the sine of the angle



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angle VTL equal to QKL ; and therefore the force wherewith the greater wheel propels the obstacle, is less than the force wherewith the smaller wheel propels the same; besides, as the greater wheel presses the obstacle directly downward with a greater force than the smaller, the resistance made by the same obstacle to the propelling force of the larger wheel, will be greater than what is made to the propelling force of the smaller; so that where the obstacle is to be propelled, the smaller wheel is preferable to the larger; but as in drafts this is rarely if at all the case, the obstacles which are commonly met with in roads being such as must either be surmounted or depressed by the wheels, such wheels are to be preferred as best serve both those purposes, and those I have shewn to be the larger wheels; which likewise are attended with other advantages besides what have been already mentioned; for first, it frequently happens in rough and uneven roads that two obstacles are placed so near each other, that before the wheel has quitted one it meets with the other, and resting upon each, hangs between them; in which case the smaller the wheel is, the lower it descends between the obstacles, and thereby renders the draft more difficult; inasmuch as it must be raised to a greater height in order to pass over the foremost obstacle, than when the wheel is larger: For the illustration of which, let FE and HG represent two obstacles placed at so small a distance, that the wheel having surmounted the first but not quitted it, may meet with the second so as to hang between them; it is manifest, that as the arch FDH of the lesser wheel, which lies between the obstacles, has a greater curvature than FBH , the arch of the greater wheel, which lies between the same obstacles, the point D must descend lower than the point B ; consequently, the smaller wheel must be raised to a greater height than the larger, in order to pass over the same ob-

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stacle;

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Fig. 1.

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stacle, and therefore a greater force will be necessary to pull up the smaller wheel, than what is requisite to raise the larger; which is confirmed by the following experiment. Let a carriage with four wheels, each an inch and an half in diameter, and so loaded as that the weight of the carriage and load may amount to 6685 grains, be so placed on the plane before made use of, as that the two foremost wheels may hang between two obstacles whose distance is half an inch, and their height likewise half an inch, and a weight of 1150 grains drawing the carriage horizontally, will move the wheels from between the obstacles; whereas if four smaller wheels be made use of each three quarters of an inch in diameter, a weight of 2700 grains will be requisite to draw them from between the obstacles.

As wheels cannot always run upon the nail, but must frequently meet with heavy roads, they will sink down, and thereby render the draft more difficult; but the larger the wheels are, the less *ceteris paribus* will the depth be to which they sink. For if ABC denotes the plane of the road, and if it be of such a nature as to suffer the smaller wheel to sink down as far as E; it is manifest that the gravity of the wheel must overcome the resistance of as much of the earth whereon it presses, as is equal to the segment HED; for it cannot otherwise sink, than by forcing such a quantity of the earth out of its place; and should the larger wheel sink to the same depth, the gravity thereof must overcome the resistance of as much earth as is equal to the segment AEC, that is, it must overcome a greater resistance in order to sink to the same depth with the smaller; but it cannot possibly overcome a greater resistance, because it is supposed to have the same gravity with the smaller; consequently, it will not sink as deep as the smaller, and for that reason will make the draft less troublesome.

As

As large wheels have the advantage of small ones with regard to the resistance arising from the obstacles and impediments in the roads, so have they likewise in relation to the resistance occasioned by the friction of the box against the arm of the axle; not that this resistance is less in greater wheels than in smaller; for since it is not varied by varying the magnitude of the surface, as has been shewn, if the boxes and arms are truly fitted and of an equal smoothness, and the weights whereby the arms and boxes are pressed together be equal, the quantity of resistance will be given, whatever be the magnitude of the wheels, as also of the arms of the axle whereon they play; but where the arms of the axles are of equal diameters, (which is commonly the case in one and the same carriage, tho' the wheels be unequal) a less force is requisite to overcome the given resistance in a larger wheel than in a smaller; for in this case the semidiameter of the wheel may be looked upon as a lever, whose prop or fixed point is at the center of the arm, and the impediment arising from the friction of the box against the arm may be looked upon as a weight placed upon the lever at the distance of the arm's semidiameter from the prop, whilst the moving power is applied to the extremity of the wheel's semidiameter; and therefore in order to a balance, the power must be to the resistance, as the semidiameter of the arm to the semidiameter of the wheel; since then the impediment is given, as also the distance thereof from the prop, it is evident, that the larger the lever is, and consequently, the larger the wheel, the less is the force requisite to overcome the resistance. Thus, if BEF represents the circumference of the arm of an axle, whereon the wheels AGH and DIK revolve, C the center of the arm, BC its semidiameter, DC the semidiameter of the smaller wheel, and AC that of the larger; in the

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bigger

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Fig. 3.

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bigger wheel the length of the lever is AC , and the smaller DC ; since therefore the same impediment is in both levers placed at the same distance from the prop C , to wit at B , it will be balanced by a less force at A than at D ; and the force at A to the force at D , as DC to AC , that is, inversely as the semidiameters of the wheels; for the force at A is as BC applied to AC , and the force at D as the same BC applied to DC ; that is the force at A , which balances the resistance at B , is to the force at D which balances the same resistance, as BC divided by AC , to BC divided by DC , that is, multiplying crosswise, and throwing out BC as DC to AC . Whence it follows, that when the semidiameter of the arm is given, the more the wheel is enlarged, the less will the force be that is requisite to overcome the resistance arising from the friction of the wheel against the arm; so that upon this account as well as the former, large wheels are to be preferred to small ones.

In order to lessen the resistance arising from the friction of the box against the arm of the axle, there has been a late contrivance, whereby the axle, contrary to what is usual in most carriages, is made to revolve, and its arms, instead of pressing against the boxes, are made to bear on the circumferences of moveable wheels, which wheels from their use in diminishing the friction, are by the author of this contrivance called *friction wheels*. Now that such wheels, where they can be made use of, do take off much of the resistance occasioned by friction, will

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appear from the following experiments; from the axle of the machine called the axle in the wheel, in which the diameter of the wheel is to the diameter of the axle, as nine to one, let a weight of 23163 grains be hung, and a weight of 2770 grains hanging at the circumference of the wheel, will turn the machine, provided the axle turns on the circumfe-

rences

ances of two moveable wheels ; whereas, if it turns on the pivots it will be necessary to add 600 grains more, so as to make the whole 3370 grains ; consequently the resistance occasioned by friction in the latter case, is more than four-fold what it is in the former ; for since the diameter of the wheel is nine times as great as that of the axle, a weight of 2574 grains at the wheel is requisite to balance the weight of 23163 at the axle, which balancing weight being deducted from 2770, and likewise from 3370 grains, leaves 196 grains for overcoming the resistance in one case, and 796 in the other ; but 796 is to 196, as four and a little more to one.

Again, let a small cart with friction wheels be so loaded, as that its own weight added to that of the load, may amount to 20000 grains : a weight of 54 grains drawing horizontally, will move it along a smooth level table : whereas, if the friction wheels be taken off, 322 grains will be necessary to set it a going. If the cart be so loaded, as that the weight of the whole may amount to 40000 grains, then in each case, a double force will be requisite to move it, that is to say, 108 grains with the friction wheels, and 644 without them ; so that in this cart the friction wheels take off five parts in six of the resistance ; for 54 is but a little more than a sixth part of 322, as is likewise 108 of 644. And from these experiments it does again appear, that under like circumstances the resistance arising from friction is proportional to the weight, whereby the surfaces which rub one against the other are pressed together.

Seeing then that great wheels have in so many respects the advantage over small ones, it will not be improper in this place to shew you, on what account it is that the wheels of common carts, as also the foremost wheels of coaches, chariots, and most other four-wheel carriages are commonly made so small as seldom to exceed two feet and an half in diameter ; and the first reason of this contrivance

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is for the convenience of turning; for as in most roads, but more especially such as are narrow, there are windings of such a nature as to allow but a small space for carriages to turn in, it is necessary to make use of such wheels as can turn in the narrowest compass, and such are small ones; for it is a thing well known to carters and all others who are used to drive wheel carriages, that the larger the wheels are, the greater compass do they require in order to turn with ease and safety; and should they at any time attempt to turn carriages with large wheels as short as those which have smaller, the wheels will drag, and thereby render the draft very difficult, and sometimes endanger the upsetting of the carriage.

But the second, and indeed the principal reason for the use of small wheels is, that upon ascents, and in passing over obstacles in rough and hilly roads, as little of the horse's force may be lost as possible; if roads were level and smooth without risings or impediments, the most convenient size for wheels, setting aside the necessity of turning, would be where the axle is upon a level with the breast of the horse; for since the whole force of the horse in drawing is applied to that part of the tackle which lies upon the breast, and to which the traces are joined; and since the traces are fastened to the carriage in such a manner as that being continued they must pass thro' the axle of the foremost wheels, it is manifest, that if that axle be of an equal height with the chest of the horse, the traces, in whose plane the line of direction lies, will be parallel to the road whereon the carriage is drawn; consequently, the whole force of the horse will be employed in drawing the carriage directly forward, without any loss or diminution; whereas if the wheels be of such a size as that the height of the axle is either greater or less than that of the horse's chest, the whole force of the horse will not be employed in the direct draft; but in the former

former case, some part of the force will be spent in pressing the carriage directly downward, and in the latter, in lifting the same directly upward. For the proof and illustration whereof, let the first of the three wheels be of such a size, as that its axle A may be of an equal height with the horse's breast at B; and let the second wheel be so large as that its axle A may stand higher than the horse's chest at B; and in the third, let the axle be lower than the breast of the horse; and in each wheel let the lines of direction of the horse's draft, to wit, A B be taken equal, and let each of those lines express the force of the horse; it is manifest, that in the first wheel, the whole force denoted by A B, is employed without any loss in drawing the wheel forward, because the line of direction A B, wherein the force draws, is parallel to E F, the road whereon the wheel moves; whereas, in the second and third wheels the lines A B, wherein the forces draw, being inclined to E F, whereon the wheels move, some part of each force must be lost; for if each force denoted by A B be resolved into two, to wit C B and A C, whereof C B is parallel to E F, and A C perpendicular thereto; it is evident, that that force alone which is denoted by C B, acts in moving the wheel forward along E F, whilst the force denoted by A C does in the second wheel press it directly downward against the road, and in the third lifts it directly upward; whence it follows, that if the force of a horse be just sufficient to move the first wheel, it will not suffice to stir the second or third. It likewise follows, that if the wheel be so far enlarged, as that the angle which the line of direction A B makes with the plane E F, approaches nearly to a right one, the line C B will bear a very small proportion to A B, whilst A C becomes nearly equal thereto; so that almost the whole of the horse's force will be spent in pressing down, and thereby increasing the load; whence it appears, that

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that notwithstanding the several advantages arising from the largeness of wheels, yet may they be so far increased, as even upon account of their magnitude to render the draft impossible. By the use of small wheels whose axles lie below the level of the horse's chest, provision has been made against the inconvenience last mentioned, and the loss of force (which by reason of the roughness and inequalities of roads cannot wholly be avoided) has been rendered as little as possible, and made to obtain chiefly in level smooth roads, where there is least occasion for the whole force; whereas upon ascents, and in passing over obstacles in rough roads, where the stress is greatest, there little of the force is *lost*; for the proof of which, let the wheel be of such a size, that its axle A may be below the horse's breast at B, and let A B, as before, denote the force of the horse; if the wheel be drawn along a smooth level road as E F, C B will express that part of the force which draws the wheel along the road, and A C that part of the force which is employed in lifting up the wheel, which part is lost as to the draft, but however, is not intirely useless; because, by pulling the wheel directly upward, it eases the load, and thereby renders the draft less difficult; tho' at the same time the draft is by no means as easy as it would be, if the force of the horse was applied at C, so as to draw in the direction A G parallel to E F. If the wheel instead of moving along a smooth road, be to pass over the obstacle D H, or which is the same thing, if it be to be drawn up the ascent E H L; and if the force of the horse be applied at G, so as that the direction of the draft A G may be parallel to E F, and consequently, inclined to E H L; it is manifest upon resolving the force A G into two forces, to wit A K and K G, whereof A K is parallel, and K G perpendicular to E H L; that force alone which is expressed by A K, acts in drawing the wheel up E H L; whereas the force expressed by K G

Pl. 5.
Fig. 5.

KG acts in pressing the wheel directly against EHL, and thereby adds to the weight of the wheel; so that in this case, some part of the horse's force is lost, and the load at the same time increased, both which inconveniencies are avoided where the breast of the horse is so far elevated above the axle of the wheel, as that the line of direction AB may be parallel to EHL; for then no part of the horse's force will be lost, but the whole will be employed in drawing the wheel directly over the obstacle, or up the ascent; so that a less force will be requisite to draw the wheel over the obstacle DH in the direction AB, than in the direction AG; and this is fully confirmed by experiments. For whereas the little carriage with four wheels, each three quarters of an inch in diameter, being so loaded as that the weight of the carriage and load amounted to 6685 grains, was not drawn over the little obstacle one tenth of an inch in height, by less than 2850 grains acting in an horizontal direction, it will be drawn over by 2450 grains, provided the direction be made parallel to the tangents of the wheels in those points which touch the obstacle; and 1950 grains will be sufficient to draw the carriage with the larger wheels over the same obstacle, if the direction of the draft be made parallel to the forementioned tangents, whereas 2050 grains were necessary when the direction was parallel to the horizontal plane. And if the direction be still farther removed from the parallelism of the tangents, which may be done by depressing it below the horizontal plane, the force of 2350 grains will be but just sufficient to surmount the obstacle, and draw the carriage over.

Exp.

Tho' in four wheel carriages, the contrivance of small wheels before has its advantages, yet is it not intirely free from inconveniencies; for by this means the load must of necessity be thrown forward, and
a greater

a greater stress laid on the foremost wheels ; whereby the resistance that arises from the friction of the axle against the wheels will become greater in the foremost than in the hindmost wheels, in proportion to the greater weight which they sustain. Besides, as the spaces described by wheels in each revolution are nearly equal to the peripheries of the wheels, it is manifest that the foremost wheels must revolve oftener than the hindmost, in order to rid the same ground. And this frequency of turning requisite in the foremost wheels joined to the greater stress upon them from the load, as also the greater resistance which they meet with from obstacles in the road, is the true reason why they are more frequently out of order, and stand in need of repair much oftener than those behind.

LECTURE X.

MOTION OF BODIES DOWN INCLINED PLANES.

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MY design in this lecture is to explain the chief properties of the PENDULUM ; and in order thereto, I shall lay down the following PROPOSITIONS concerning the motion of bodies down inclined planes and curve surfaces.

Pl. 5.
Fig. 6.

PROP. I. *The force wherewith a body descends upon an inclined plane, as AC, is to the absolute force of gravity wherewith the same body falls freely and perpendicularly, as the height of the plane to the length thereof, that is, as AB to AC.*

For it has been proved, that the force requisite to sustain a body upon an inclined plane, is to the absolute weight of the body, as the height of the
6 plane

plane to its length ; but the force wherewith a body endeavours to descend upon an inclined plane, must be equal to the force which is necessary to support it upon that plane ; consequently, the proposition is true.

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COROL. I. Hence it follows, that the motion of a body descending on an inclined plane, is uniformly accelerated; for since the force which carries a body down an inclined plane, has every where, and in all parts of the plane, the same proportion to the absolute weight of the body, and since the absolute weight remains unvaried, the other force must do so too; consequently, as it acts incessantly in equal times, it makes equal impressions on the descending body, so as to generate equal degrees of velocity in the motion thereof; that is, in other words, the motion of a body descending on an inclined plane is uniformly accelerated.

COROL. II. On account of this uniform acceleration of the motion, the times of descending, as also the velocities acquired at the end of the descent, are as the square roots of the spaces described, as in the case of bodies falling freely; that is to say, the time wherein a body descends upon the inclined plane from A to D, is to the time of the descent from A to C, as the square root of AD, to the square root of AC; and the velocity of the body when it has descended as far as D, is to the velocity thereof when it arrives at C in the same proportion of the root of AD to the root of AC.

PROP. II. *The velocity acquired in any given time by a body descending on an inclined plane, is to the velocity acquired in the same time by a body falling freely and perpendicularly, as the height of the plane to its length, that is, as AB to AC.*

For, by the first *corollary* of the foregoing *proposition*, the motion of a body down an inclined plane

is

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is uniformly accelerated, in the same manner as the motion of a body falling freely; consequently, at the end of any given time, the velocities acquired must be as the accelerating forces; but by the foregoing *proposition*, the accelerating force of a body moving down an inclined plane as AC , is to the accelerating force of a body falling freely and perpendicularly, as the height of the plane to its length; and therefore the velocities acquired in any given time, must be in the same proportion.

PROP. III. *The spaces described in a given time by two bodies moving from a state of rest, whereof one descends on an inclined plane, and the other falls freely, are in the same ratio of the height of the plane to its length; that is, the space described by a body moving along AC , is to the space described by a body falling down the perpendicular AB , as AB to AC .*

For where the motions are equable, the spaces described in a given time, are as the velocities where-with they are described; if therefore the velocities be increased in a constant uniform manner, the spaces described will likewise increase in the same manner; but by the second *proposition*, the velocities are augmented in such a manner as in a given time to bear the same proportion to one another, as the height of the plane, does to its length; consequently, the spaces described in a given time must be in that proportion.

COROL. I. If from B , the line BD be drawn perpendicular to AC , AD will be the space described by a body moving down the plane AC , in the same time that a body falls freely down the height of the plane from A to B .

For, from the nature of similar triangles, AC is to AB , as AB to AD ; but by the *proposition*, as AC is to AB , so is the space described in a given time by a body falling freely, to the space described by

by a body descending upon the inclined plane AC; consequently, since AB is the space described by the body falling freely, AD must be the space described in the same time by a body descending along AC.

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COROL. II. All the chords of a circle are described in the same time by bodies running down them. For if a circle be described with the diameter AB, which is the height of the inclined plane AC, the point D, which determines the space AD thro' which a body descends upon the inclined plane, whilst another falls freely from A to B, will be in the periphery of the circle, because the angle ADB in the semicircle is always a right one; and for the same reason, if the height of the plane continuing the same, the inclination thereof be varied, so as that it may become AG, the point E which determines the space AE, thro' which a body moves along the plane AG, during the time of a body's fall from A to B, will likewise be in the periphery of the circle; consequently, in the semicircle ADB all the chords as AD and AE will be described in the same time; and as in the semicircle AFB, whatever chords as BF and BH are drawn thro' the point B, other chords as AD and AE may be drawn in the other semicircle parallel thereto and equal; it follows, that whether a body falls freely down the diameter AB, or whether it descends along a chord as HB or FB, it will in the same time arrive at the lowest point of the circle; or in other words, all the chords of a circle will be described in equal times by bodies running along them.

Pl. 5.

Fig. 7.

PROP. IV. *The time wherein a body moves down an inclined plane as AC, is to the time wherein a body falls freely down AB the height of the plane, as the length of the plane to its height, that is, the times are as the spaces described.*

Pl. 5.

Fig. 6.

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For by the second *Corol.* of the first *Prop.* the time of a body's motion along the inclined plane from A to C, is to the time of its motion from A to D, as the square root of AC to the square root of AD; but by the second *Corol.* of the third *Prop.* the time of a body's motion along the inclined plane from A to D, is equal to the time of the fall from A to B; and therefore the time of the motion along the plane from A to C, is to the time of the perpendicular fall from A to B, as the square root of AC, to the square root of AD, that is, because from the similarity of triangles AC, AB, and AD are in continued proportion, as AC to AB, or as the length of the plane to its height.

COROL. Hence it follows, that if several inclined planes have equal altitudes, the times wherein those planes are described by bodies running down them, are to one another as the lengths of the planes.

Pl. 5.
Fig. 7.

For the time of the descent along AC, is to the time of the fall down AB, as AC to AB, and the time of the fall down AB, is to the time of the descent along AG, as AB to AG; consequently, the time of the descent from A to C, is to the time of the descent from A to G, as AC to AG, that is, the times are as the lengths of the planes.

Pl. 5.
Fig. 6.

PROP. V. *The velocity acquired at the end of the fall by a body falling down the perpendicular height of an inclined plane as AB, is equal to the velocity acquired at the end of the descent by a body moving down the inclined plane, from A to C.*

For by the first *Prop.* the accelerating force of a body falling freely from A to B, is to the accelerating force of a body moving along the plane AC, as AC to AB; and by the fourth *Prop.* as AB is to

to AC, so is the time of the fall from A to B, to the time of the descent from A to C; so that the forces which accelerate the bodies during their motions, are to one another, reciprocally as the times that they continue to act; consequently, at the end of those times, the velocities generated must be equal. For instance, if AB be one half of AC, the force which accelerates the body in its fall from A to B, is to the force which accelerates the body in its descent from A to C, as two to one; but the time that a body takes to fall from A to B, is but one half of the time that a body takes to descend from A to C; so that the accelerating force which acts upon the body during its motion from A to C, tho' it be but one half of the accelerating force which acts upon the body during its fall from A to B, yet does it continue to act twice as long; and therefore must in the end produce the same velocity.

COROL. Hence it follows, that the velocities acquired by bodies in falling down inclined planes, are equal where the heights of the planes are equal.

For, the velocity acquired in falling from A to C, is equal to the velocity acquired in falling from A to B, as is also the velocity acquired in falling from A to G; consequently, the velocities acquired in falling from A to C, and from A to G, are equal.

Pl. 5.
Fig. 7.

PROP. VI. *If a body descends along several contiguous planes as AB, BC, and CD, the velocity which it acquires in its descent from A to D, is equal to the velocity acquired by the perpendicular fall from H to D, on supposition that the body is not retarded by the shocks it suffers in the angles B and C.*

Pl. 5.
Fig. 8.

For drawing the horizontal lines HE and DF thro' the points A and D, and producing the planes CB and DC as far as G and E; by the Corol. of the

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last *Proposition*, the same velocity is acquired in the point B, by a body in descending from A to B, as in descending from G to B; consequently, the same velocity is acquired in the point C, by a body descending from A thro' B to C, as in descending from G to C; but by the same *Corollary*, the velocity acquired in descending from G to C, is equal to the velocity acquired in descending from E to C; wherefore, the velocity in the point D acquired by the descent along the three planes AB, BC, and CD, is equal to the velocity acquired by the descent from E to D, which velocity by the foregoing *Proposition*, is equal to the velocity acquired by the perpendicular fall from H to D.

Pl. 5.
Fig. 9.

COROL. Hence it follows, that if a body descends along the arch of a circle as AB or of any other curve, the velocity acquired at the end of the descent, is equal to the velocity acquired by falling down CB, the perpendicular height of the arch.

For curves may be looked upon as composed of an infinite number of right lines inclined one to another.

Pl. 5.
Fig. 10.

PROP. VII. *If two planes as AB and BD joined together at B, have equal degrees of elevation with two other planes as EF and FH joined together at F, and if AB be to EF as BD to FH; the time of a body's fall down the planes ABD, will be to the time of the fall down EFH, as the square root of AB and BD taken together, to the square root of EF and FH taken together.*

Let AB and EF be produced till BC becomes equal to BD, and FG equal to FH. Since AB is to BC, as EF to FG, AB is to AC, as EF to EG; and since those four quantities AB, AC, EF and EG are proportional, their square roots will be so too. Again, since the planes AC and EG are equally

equally elevated, they may be looked upon as parts of one and the same plane, and therefore, by the second *Corol.* of the first *Prop.* the time of a body's fall from A to C, is to the time of the fall from E to G, as the square root of AC to the square root of EG, or as the square root of AB to the square root of EF; but the time of a body's fall from A to B is to the time of the fall from E to F, as the square root of AB to the square root of EF; so that the time of the fall from A to C, is to the time of the fall from E to G, in the same proportion of the time of the fall from A to B, to the time of the fall from E to F; consequently, the time of the fall from B to C, supposing the motion to begin from A, must be to the time of the fall from F to G, supposing the motion to begin from E, in the same proportion of the root of AB to the root of EF; if the bodies after their fall from A to B, and from E to F, instead of moving along BC and FG continue their motions along BD and FH, since those two planes are equally inclined to AB and EF, and since BD is equal to BC, and FH equal to FG, whatever proportion the time of the body's motion along BD bears to the time of its motion along BC, the same will the time of the motion along FH bear to the time of the motion along FG; but it has been already proved, that the time of the motion along BC, is to the time of the motion along FG, as the square root of AB to the square root of EF; wherefore the time of the motion along BD is to the time along FH, as the square root of AB to the square root of EF, that is, in the same proportion with the time along AB to the time along EF; and therefore, the sums of those times will be in the same proportion; that is to say, the time of the motion along AB, added to the time of the motion along BD, is to the time of the motion along EF, added to the time of the motion along FH, as the square root of AB to the square root of EF; but

LECT.
X.

it has been proved, that the square root of AB is to the square root of EF, as the square root of AB and BD taken together, to the square root of EF and FH taken together; and therefore the time of a body's fall from A thro' B to D, is to the time of the fall from E thro' F to H, as the square root of ABD to the square root of EFH, which was to be proved. And what has been thus proved with regard to two planes on each side, is in like manner demonstrable with regard to any number of planes, provided those on one side be proportional to those on the other, and that the corresponding planes have equal degrees of elevation.

Pl. 5.
Fig. 11.

COROL. Hence it follows, that if bodies descend thro' the arches of circles, the times of describing similar arches similarly posited, are as the square roots of the arches. For instance, if bodies move down the similar arches AB and CD, which are similarly posited with regard to the horizontal plane ED, the time of describing AB is to the time of describing CD, as the square root of AB to the square root of CD.

For all circles whatever may be considered as similar polygons, consisting of an indefinite number of sides indefinitely small; and therefore, similar arches must consist of an equal number of sides proportional the one to the other; and forasmuch as the angles which those sides contain are equal, if the arches be similarly posited, the corresponding sides in each arch must have equal degrees of elevation; and consequently, the times of describing the arches will be as their square roots.

In my lecture upon gravity, I shewed you, that if a body be thrown directly upward, it will rise to the same height, whence, if it fell from a state of rest, it would by the end of the fall acquire the same velocity wherewith it is thrown up; I likewise shewed you, that the time of the rise is equal to that of the fall. I now say,

PROP. VIII. *That the same things do likewise obtain with regard to bodies thrown up obliquely, whether they ascend upon inclined planes or along the arcs of curves.*

LECT.
X.

Because the same forces which accelerate the motions of bodies descending on such planes or curves, do in the very same manner retard the motions of such bodies as ascend thereon; and therefore, whatever be the time requisite for a body to descend upon an inclined plane or thro' the arc of a curve, in order to acquire any velocity, the same must the time be, wherein that velocity is destroyed in a body ascending upon the same plane or curve, and whatever be the length of the plane or curve, thro' which a body descends in order to acquire any velocity, the same must the length of the plane or curve be, thro' which it must ascend in order to have that velocity destroyed.

COROL. Hence it follows, that if by any contrivance a body be made to descend thro' the arch of a circle as from C to A, and with the velocity acquired by the descent to ascend along the arch AD of the same circle, the arch AD which it describes in its ascent, will be equal to the arch CA described in the descent; and the times in which those arches are described will be equal.

Pl. 5.
Fig. 12.

And this is the case of the PENDULUM; which is a heavy body as A, hanging by a small cord as BA, and moveable therewith about the point B, to which the cord is fixed. If when the cord is stretched the weight be raised as high as C, and thence let fall, it will by its own gravity descend thro' the circular arch CA; and by the *Corol.* of the sixth *Prop.* it will have the same velocity in the point A, that a body would acquire in falling perpendicularly from E to A; and by the first LAW OF NATURE, it will endeavour to go off with that velocity in the tangent AF; but being by the force of

Exp. 1.

LECT.
X.



the cord made to move in the periphery CAD , it will rise thro' the arch AD as high as D , where losing all its velocity, it will be turned back by its gravity, and descending thro' the arch DA , will, upon its arrival at A , have the same velocity as before, with which it will ascend to C ; and thus it will continue its motion forward and backward along the curve CAD , which motion is called an *oscillatory* or *vibratory* motion; and each swing from C to D , as also from D to C , is called a *vibration*; and if the pendulum suffered no retardation in its motion from the resistance of the air, nor from the friction of the cord against the center about which it moves, the arches described in each vibration would be exactly equal, and the motion of the pendulum would continue for ever; but whereas the motion of the pendulum is continually retarded by the forementioned causes, the arches described in each vibration must grow less and less continually, and at last vanish together with the motion of the pendulum.

Pl. 5.
Fig. 13.

The vibrations of one and the same pendulum vibrating in unequal circular arches are performed very nearly in equal times, provided the arches are but small. Thus, in the pendulum AB , the vibration thro' the arch CAD , is performed very nearly in the same time wherein the pendulum vibrates thro' the arch EAF , on supposition that the arches CA and EA are but small.

For, drawing the chords CA and AD , as also EA and AF , inasmuch as the arches are supposed to be small, they will not differ much either as to length or declivity from their respective chords; consequently the times of describing the arches CA and EA , by a heavy body running along them, will be nearly equal to the times of describing the chords; but by the second *Corol.* of the third *Prop.* the times of describing the chords are equal; wherefore the times of describing the
arches

arches C A and E A, must be nearly equal ; and so likewise must the double of those times, or the times wherein the pendulum vibrates thro' the unequal arches C A D and E A F. And this is confirmed by experiment. For if two pendulums of an equal length, be set going at the same instant of time, so as to vibrate thro' small but unequal arches, they will for a long time keep pace together ; and continue to begin and end their swings without any sensible difference as to point of time, during a great number of vibrations.

If a pendulum as B A vibrates thro' the circular arches C A D and E A F, the velocity which it acquires by that time it arrives at the lowest point A, is as the chord of the arch which it describes in its descent ; that is, the velocity which it acquires in descending from C to A, is to the velocity acquired in its descent from E to A, as the chord C A to the chord E A.

For, drawing the horizontal lines E K and C H, the velocity acquired in falling from H to A, is to the velocity acquired in falling from G to A, in the subduplicate *ratio* of H A to G A, as I proved in my lecture upon gravity ; that is, because, from the nature of the circle H A, C A, and G A, are in continued proportion as C A to G A ; for the same reason the velocity acquired in falling from G to A, is to the velocity acquired in falling from K to A, as G A to E A ; consequently, the velocity acquired in falling from H to A, is to the velocity acquired in falling from K to A, as C A to E A ; but by the *Corol.* of the sixth *Prop.* the velocity acquired in falling from H to A, is equal to the velocity acquired in the descent from C to A, and the velocity acquired in falling from K to A, is equal to the velocity acquired in the descent from E to A ; wherefore, the velocity acquired in descending thro' the arch C A, is to the velocity acquired in descending thro' the arch E A, as the chord C A to the chord E A.

Hence

LECT.
X.

Exp. 2.

Pl. 5.
Fig. 14.

$$\begin{aligned} HA &: \sqrt{GA} \\ HA &: AC \\ HA^2 &: AC^2 \\ HA &: AC \end{aligned}$$

LECT.
X.

Hence it appears, that if the arch of a circle wherein a pendulum vibrates, be so divided in the points 1, 2, 3, 4, and so on, beginning from the lowest point A, as that the chords drawn from A to the several points of division, may be to one another, as those numbers, the velocities acquired by a pendulum in the lowest point A, when let fall successively from the several points of division, will be as the numbers affixed to the respective points; and it was upon this account, that in the experiments relating to the collision of bodies, the balls were constantly let fall from such heights, as that the chords of the arches which they described in their descent, might be to one another in the same proportion with the velocities wherewith the balls were supposed to meet at the lowest point.

Pl. 5.
Fig. 15.

The times wherein pendulums of unequal lengths vibrating in similar arches, perform their vibrations, are to one another, as the square roots of their lengths; for instance, the time wherein the pendulum BA vibrates thro' the arch FG, is to the time wherein the pendulum BC vibrates thro' the arch DE similar to FG, as the square root of BA to the square root of BC.

For, by the *Corol.* of the seventh *Prop.* since the arches FA and DC are similar and similarly posited, the time of the descent thro' FA, is to the time of the descent thro' DC, as the square root of FA to the square root of DC; but by the *Corol.* of the eighth *Prop.* the time of the descent thro' FA, is one half of the time of the vibration from F to G, and the time of the descent thro' DC, is one half of the time of the vibration from D to E; consequently, the time of the vibration thro' FG, is to the time of the vibration thro' DE, as the square root of FA, to the square root of DC; that is, because the arches FA and DC are similar, as the square root of BA to the square root of BC, that is, the times of the vibrations are

as

as the square roots of the lengths of the pendulums. And forasmuch as the times wherein pendulums perform their vibrations are to one another inverſly as the number of vibrations performed in a given time; the numbers of vibrations performed by pendulums in a given time, are to one another inverſly as the ſquare roots of the lengths of the pendulums. For inſtance, if the length of the pendulum B A, be to the length of the pendulum B C, as one to four, the number of vibrations performed in any given time by the ſhorter pendulum, is to the number of vibrations performed in the ſame time by the longer, as the ſquare root of four to the ſquare root of one, that is, as two to one; which caſe is experimentally confirmed by two pendulums, whereof the longer being 39,125 inches, vibrates in one ſecond of time; and the ſhorter being 9.781 inches, vibrates in half a ſecond, and performs two vibrations in the ſame time that the longer performs one.

Exp. 3.

Inches.

Length of a pendulum vibrating	} in a 2d.	{ 39.125 <i>Halley.</i>
		{ 39.207 <i>Newton.</i>
	} in $\frac{1}{2}$ a 2d.	{ 9.781 <i>Halley.</i>
		{ 9.801 <i>Newton.</i>

The time of a pendulum's vibration is no way altered by varying the weight thereof; for ſince the gravity of every body is proportional to its quantity of matter, as I proved in my lecture upon gravity, all bodies in the ſame circumſtances are moved by the force of gravity with the ſame velocity; and therefore if the length of a pendulum continues the ſame, it will perform its vibrations in the ſame time, whatever be the magnitude of the appending weight; which may be confirmed by the following experiment. Let two unequal weights be hung by two threads ſo as to conſtitute two pendulums equal in length, and let them at the ſame inſtant of time fall

Exp. 4

LECT.
X.

fall from equal heights, they will keep pace together so as to perform their vibrations in equal times.

In the foregoing part of this lecture I shewed you, that the vibrations of one and the same pendulum vibrating thro' unequal but small circular arches, are performed in times that are very nearly, but not precisely equal. Whence it follows, that however useful such a pendulum may be in measuring time, where great exactness is not requisite, yet can it by no means be admitted as an accurate measure of time, unless by some contrivance it be made to perform all its vibrations in equal arches, which, considering the unavoidable imperfections of all machines, is extremely difficult, if not impossible; for it has been found by experience, that the best regulated pendulum clocks, wherein the greatest care has been taken to make the pendulums vibrate in equal arches, have notwithstanding varied in a course of time, so as to stand in need of a new regulation, which they could not possibly do in case the pendulums, whereon the regularity of all the other movements depends, continued constantly to vibrate in equal arches.

In order therefore to obtain an exact unerring measure of time, it is necessary to make a pendulum vibrate in such a manner, as that all its swings, whether they be thro' larger or smaller arches, may be performed in times exactly equal; and this may be done by making a pendulum vibrate in the curve of a cycloid, as I shall now demonstrate; but I shall first shew you the manner wherein that curve is generated, and what its chief properties are, as also by what contrivance a pendulum is made to vibrate in such a curve.

Pl. 5.
Fig. 16.

If a circle as $C E F$, which touches the right line $A B$ in the point C , be moved along that line in the manner of a wheel from C to D , so as to perform an intire revolution; the point C will by virtue of its double motion describe the curve line $C I D$,

CID, which curve line is called a *cycloid*; and the right line CD is called the *base*, the line IK perpendicular to the base at its middle point is called the *axis of the cycloid*, and the point I the *vertex*, and the circle CEF or KLI is called the *generating circle*.

LECT.
X.
}

From any point in the cycloid as H, let a right line as HL, be drawn parallel to the base CD, and continued till it meets the generating circle KLI, described about the axis IK; and let the line HM touch the cycloid in the point H; this being done, the chief properties of the cycloid are these three.

First, The arch IPL of the generating circle, intercepted between the vertex of the cycloid and the point L, wherein the right line HL meets the generating circle, is equal in length to the right line HL.

Secondly, The chord IL of the circular arch IPL, is parallel to the right line MH, which touches the cycloid in the point H.

Thirdly, The cycloidal arch IH intercepted between the vertex and the point H, is double the chord IL.

The demonstrations of these properties may be seen in HUYGENS, WALLIS, COTES, and others who have wrote of the cycloid.

The contrivance whereby a pendulum is made to vibrate in the curve of a cycloid, is thus. A cycloid as AVB, being described on the base AB, let the axis VD be produced towards C, till DC becomes equal to VD; thro' the points G and A, and C and B, let two semi-cycloids CA and CB be drawn, each equal to half of AVB, their vertices being at A and B; if then we suppose CA and CB to be two plates of some breadth, and an heavy body to hang from the point C by a string equal in length to CV, and to vibrate between the plates CA and CB, the upper part of the string will constantly

Pl. 5.
Fig. 17.

LECT.
X.

stantly apply itself to that plate towards which the body moves, and by so doing cause it to move in the cycloid AVB as has been proved by HUYGENS the author of this contrivance; and likewise by COTES in his treatise *de motu pendulorum*, where he has delivered the whole doctrine of pendulums in four THEOREMS, which I shall here lay down and explain.

Pl. 5.
Fig. 17.

THEOREM I. *If a pendulum vibrating in a cycloid as BVA , begins its motions downward towards V , from any point taken at pleasure as L , and if upon a radius as VL , equal in length to the cycloidal arch VL , a circle be described; the velocities of the pendulum in the several points of the cycloidal arch, will be as the right sines in the circle which are raised from the corresponding points in the radius; for instance, if in the radius LM be taken equal to LM in the cycloid, and from the point M in the radius corresponding to the point M in the cycloid, be raised the right sine MX , the velocity of the pendulum in the point M , after it has descended from L , will be as the sine MX .*

For the proof of which, from the points L and M in the cycloid, let the right lines LOR and MQS be drawn perpendicular to the axis, cutting the generating circle in O and Q , from whence to the vertex, let the right lines OV and QV be drawn. By the *Corol.* of the sixth *Prop.* the velocity which the pendulum acquires in descending along the cycloid from L to M , is equal to the velocity acquired by a body in falling perpendicularly from R to S ; but the velocity which a body acquires in falling perpendicularly, is in the subduplicate *ratio* of the space described, as I proved in my lecture upon gravity; consequently, the velocity acquired by the pendulum in its descent from L to M , may be expressed by the square root of RS ;

RS; but RS, being equal to the difference between RV and SV, the velocity in the point M may be expressed by the square root of the difference between RV and SV; or, because, RV multiplied into the axis DV, is to SV multiplied into the same DV, as RV to SV, the velocity may be expressed by the square root of the difference between the product of RV \times DV and SV \times DV; but from the nature of the circle, the product of RV \times DV is equal to the square of VO; and the product of SV \times DV is equal to the square of VQ; wherefore, the velocity at M may be expressed by the square root of the difference between the square of VO and the square of VQ; but, by the third property of the cycloid, VO is equal to one half of the cycloidal arch VL, and VQ to one half of the arch VM; wherefore, as VO square, is to VQ square, so is VL square, to VM square; consequently, the velocity of the pendulum at M may be expressed by the square root of the difference between the square of VL and the square of VM; but the cycloidal arches VL and VM are by supposition equal to VL and VM in the *radius* of the circle; and, from the nature of a right-angled triangle, the difference between the square of VX, which is equal to VL, and the square of VM, is equal to the square of MX; wherefore, the velocity of the pendulum at the point M, is as the square root of MX square, that is, as MX, as was asserted in the *Theorem*. And what has been thus proved with regard to the velocity at the point M, is in like manner demonstrable with regard to the velocity at any other point as N; namely, that it is as the right line NY raised from the point N in the *radius* corresponding to the point N in the cycloid; so that the velocities of a pendulum descending in a cycloid, are in the several points of the cycloidal arch, as the right lines in a circle which are raised from the corresponding points of the *radius*, the *radius* being equal

LECT.
X.

equal in length to the cycloidal arch intercepted between the vertex and that point from which the pendulum begins its motion. Thus, VM and VN in the *radius* of the circle, being taken equal to VM and VN in the cycloid, so as that the points M, N, V , in the *radius*, may correspond to the points M, N, V , in the cycloid, the velocities of the pendulum in those points are to one another, as the sines MX, NY , and VZ , the *radius* VZ expressing the greatest velocity at the vertex V .

Pl. 5.
Fig. 17.

THEOREM II. *If a body be supposed to move uniformly in the curve of the circle, with a velocity equal to the velocity acquired by the pendulum in its descent from L to V , which velocity is, as was just now shewn, expressed by the radius VZ ; any arch of the circle as XY taken at pleasure, will be described by the body moving along it in the forementioned manner, in the same time that the pendulum, which begins its motion from the point L in the cycloid, describes the cycloidal arch MN , corresponding to and equal in length to MN , that part of the radius, which lies between the sines MX and NY , which terminate at the extremities of the circular arch XY .*

Let the sine FGH , be drawn indefinitely near to the sine MX , and let XG be drawn parallel to MF : and let MF in the cycloid be equal to MF in the *radius* of the circle. By the foregoing *Theorem*, the velocity of the pendulum in the point M , is as MX ; and therefore, since F is supposed to be indefinitely near to M , the little cycloidal arch MF , equal to MF in the *radius*, is to be looked upon as described by the pendulum with a velocity which is as MX ; and the little circular arch XH , is by supposition described with a velocity which is as VZ , equal to VX ; and the triangles MXV and GXH being similar, inasmuch as the angles at M and G are right ones, and the angle MXV equal to GXH ,

recompensate XH , because $G XV$ is the complement of each
with them to a right one; XH is to XG equal to
1 axis MF , as VX or VZ to MX ; that is, XH and MF
1 equal to one another, as the velocities wherewith they
the are described; consequently, they must be described
nd in the same time. And what has been thus demon-
located of MF and XH , is in like manner demon-
not strated of the several corresponding parts in the cy-
V cycloidal arch MN , and circular arch XY ; conse-
V quently, the whole cycloidal arch MN , will be de-
scribed by the pendulum in the same time, that the
circular arch XY is described by a body moving
along it uniformly with the velocity expressed by
s VZ ; and by the same way of reasoning, the time
of describing any other cycloidal arch as LV , is
b equal to the time of describing the corresponding
d circular arch LZ .

COROL. As a *Corollary* it follows, that the time
wherein a pendulum describes any arch of a cy-
cloid as MN , may be expressed by the correspond-
ing circular arch XY .

For, as the motion along the curve of the circle
is supposed to be uniform, the time of describing
any arch as XY , must be as the length of the arch;
but by the *Theorem*, the times of describing the cir-
cular arch XY , and the cycloidal arch MN , are
equal; consequently, the time in which the pendu-
lum describes the cycloidal arch MN , is as the
circular arch XY .

THEOREM III. *The time of one entire vibration
of a pendulum moving in a cycloid, is to the time
wherein a body falls perpendicularly thro' a space equal
in length to the axis of the cycloid, as the periphery of
a circle to its diameter.*

All things being supposed as before, the time of Pl 5.
describing the semicircular periphery LZP with the Fig. 17.
M velocity

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velocity expressed by VZ , is to the time of describing the semidiameter LV with the same velocity, as the semicircular periphery to the semidiameter, or as the whole periphery to the diameter; but the time of describing the semicircular periphery LZP with the velocity VZ , is equal to the time of an intire vibration; for, by the eighth *Prop.* the time wherein the pendulum describes the cycloidal arch LV , is one half of the time wherein it performs an intire vibration; and by the second *Theorem*, the time wherein a pendulum describes the cycloidal arch LV , is equal to the time wherein the quadrantal arch of the circle, to wit LZ , is described with the velocity expressed by VZ ; consequently, the time of an intire vibration, is equal to the time of describing the semicircular periphery LZP ; and the time of describing the semidiameter LV with the velocity VZ , is equal to the time of a body's fall down the height of the axis DV ; for, by the second *Corol.* of the third *Prop.* the fall down the axis DV , is performed in the same time with the descent along the chord OV ; and by the eighth *Prop.* the velocity acquired at the end of the descent along the chord OV , will in the same time with that of the descent describe a space equal to twice OV ; but by the third property of the cycloid, twice VO is equal to the cycloidal arch LV , which, by supposition is equal to the semidiameter VL ; and consequently, the velocity acquired at the end of the descent along the chord OV , is such, as will in a time equal to that of the fall down the axis DV , describe the semidiameter LV ; but, by the *Corol.* of the sixth *Prop.* the velocity acquired at the end of the descent along the chord OV , is equal to the velocity acquired by the pendulum in its descent along the cycloidal arch from L to V , which by the first *Theorem*, is as VZ ; wherefore, the time of describing the semidiameter LV with the velocity VZ , is equal

equal to the time of the fall down the axis DV ; but it has been already proved, that the time of describing the semicircular arch LZP with the velocity VZ, is to the time of describing the semidiameter LV with the same velocity as the periphery of the circle to its diameter ; and it has been likewise proved, that the time of describing the semicircular arch with the velocity VZ, is equal to the time of an intire vibration of the pendulum ; consequently, the time of such a vibration, is to the time of the fall down the axis, as the periphery of the circle to its diameter.

COROL. From what has been proved it follows, that the time of a vibration of a pendulum moving in a given cycloid is given ; or in other words, that all the vibrations of such a pendulum, whether they be in larger or smaller arches, are performed in times exactly equal.

For, as it has been proved, that the time of the vibration which begins from the point L, is to the time of the fall down the axis, as the periphery of the circle described on a *radius* equal to the cycloidal arch VL, to its diameter ; it may in like manner be demonstrated, that if the vibration begins from any other point as M, the time thereof will bear the same proportion to the time of the fall down the axis, that the periphery of a circle described on a *radius* equal in length to the cycloidal arch VM, does to its diameter ; but the *ratio* of the periphery to the diameter in any one circle, is the same with that in any other ; wherefore, the times of the vibrations thro' unequal arches, have all the same *ratio* to the time of the fall down the axis, and of consequence must be equal.

From this equality in the times of the swings it is, that this kind of pendulum is preferable to such as vibrate in circular arches, as being a more exact and just measure of time ; a minute of mean or equal time being precisely measured by sixty swings

LECT.
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of a pendulum of this kind, whose length is equal to three horary feet, which answers to 39 inches and one eighth of our measure, according to Doctor HALLEY; or to 39 inches and one fifth, according to Sir ISAAC NEWTON; and now that I have mentioned *mean* or *equal*, otherwise called *true time*, it will not be improper in this place, to shew you wherein it differs from that time, which by astronomers is called *unequal* and *apparent time*.

As time in itself does not fall under the notice of our senses, and as the parts thereof go on in a continued succession one after another, no two existing together, it is impossible to discover the equality or inequality of any two portions of time, by an immediate comparison of one with the other; and therefore, it was necessary for those who first thought of distinguishing the parts of time, to have recourse to something sensible, and of a different nature from time, as a measure thereof. And as nothing seems better fitted to serve this purpose, than such natural appearances as fall under every man's notice, and at the same time have frequent returns, it is highly probable, that in the first ages of the world, men observing the frequent risings and settings of the sun, took the one or the other for their first measure of time, calling that portion of time which passed between two risings or settings, which immediately succeeded each other, by the name of a *day*; in like manner it is rational to suppose, that upon observing the frequent returns of the full and new moons, they made the one or the other their second measure of time, calling that space which passed between two successive new or full moons by the name of a *moon* or *month*. And it is likely, that for some time they contented themselves with these measures, without knowing or considering whether they were exact or not: but in process of time, as men became better acquainted with the motions of the heavenly bodies, they discovered

covered some irregularities in the apparent motion of the sun, and of consequence, an inequality in the natural days which depend on that motion; inasmuch as the portion of time, which passes between the sun's departure from the plane of any meridian and its next return thereunto, is not always the same. By considering the causes of this inequality, they were led into a method of making such corrections in the natural days, by adding to some, and taking from others, as reduced them all to a mean equal length; each day being made to consist of 24 equal *hours*, each of which is divided into sixty equal parts called *minutes*, and each of these into sixty others called *seconds*, and these again into *thirds*, and so on in a *sexagesimal* progression, the parts of each denomination being constantly equal among themselves. And these parts of time thus reduced to an equality constitute the mean or equal time, as it stands distinguished by astronomers, from the unequal or apparent time, which is measured by the apparent motion of the sun.

In order to have a constant measure of equal time, HUYGENS contrived a method of adapting pendulums to clocks, whereby their motions are so exactly regulated, as that in a clock whose movements are rightly adjusted, the seconds, minutes, and hours, are for some time pointed out with the greatest exactness; I say, for some time only, because it is not possible that any clock whatever should continue exactly true for a long course of time; for as the pendulums of clocks according to HUYGENS's first contrivance, and by the general practice of clock-makers at this day are made to vibrate in circular arches, where the times of the vibrations are not precisely equal, unless the arches thro' which the pendulum moves be so too. If the wheels on account of the thickening of the oil by frosty weather, or from any other cause grow more

M 3

sluggish,

LECT.
X.

sluggish, so as to give the weight, which in clocks is the moving power, greater resistance than according to the first adjustment, the force of the crown-wheel, upon the plates of the pendulum will likewise be diminished, and of consequence, the pendulum being thrown less forcibly will move thro' smaller arches than before, and by so doing, will measure out smaller portions of time, the time of sixty swings not amounting to a minute, upon which account the clock must gain, and go too fast. On the other hand, whenever the parts of the movement which rub one against another do, by reason of the thinning of the oil by the heat of the weather, grow more slippery, or from their constant friction become more smooth, so as to give less resistance to the moving power than according to the first adjustment, the crown-wheel acts more forcibly on the pendulum, and causes it to vibrate in larger arches, by which means the time of each swing is enlarged, and of course the clock loses and goes too slow. To remedy these inconveniences, HUYGENS thought of a second method of adapting pendulums to clocks, so as to make them perform their vibrations in cycloidal arches; by which means, tho' the force of the crown-wheel upon the pendulum should vary, so as to cause it to vibrate sometimes in larger and sometimes in smaller arches, yet will not any variation arise from thence in the times of the vibrations; as is evident from the *Corollary* of the third *Theorem*; so that in clocks whose motions are governed by pendulums vibrating in cycloidal arches, the irregularities arising from the variation of the force of the crown-wheel upon the pendulum are wholly avoided; and yet a clock of this kind will not always go true; for as the pendulum cannot vibrate in the curve of a cycloid, unless the uppermost part of the string does as often as it moves from the perpendicular

pendicular towards either side, form itself into a cycloidal arch; and as this cannot be done unless that part of the string be made of silk, or some other soft and pliable substance, which as such is apt to imbibe the moisture of the air; whenever the weather becomes remarkably moist, the watery particles which float in the air, will insinuate themselves into the pores of the string, and by so doing cause it to contract and shorten; upon which account, the vibrations of the pendulum will be quickened, as will appear from the next *Theorem*, and the clock will gain. So that neither a clock of this, nor of any other kind, can go exactly true for any long course of time, which is a thing well known to clock-makers, who have frequently experienced the best regulated clocks to vary, in the compass of a few months, some seconds from the equation table, so as to stand in need of a new regulation.

THEOREM IV. *The times wherein pendulums of different lengths as CV and AB perform their vibrations, are to one another in the same proportion with the square roots of the lengths of the pendulums.*

Pl. 5.
Fig. 17.
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For, by the third *Theorem*, the time wherein the pendulum CV performs its vibrations, is to the time wherein a body falls down the axis DV, as the circumference of a circle to its diameter; and by the same *Theorem*, as the circumference of a circle is to the diameter, so is the time wherein the pendulum AB performs its vibrations, to the time wherein a body falls down the axis EB; consequently, the time wherein the pendulum CV performs its vibrations, is to the time wherein the pendulum AB performs its vibrations, as the time of the fall down DV, is to the time of the fall down EB; but, as I proved in my lecture upon gravity, the time of the fall down DV, is to the time of the

Pl. 5.
Fig. 17.

Pl. 5.
Fig. 18.

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fall down EB, as the square root of DV, to the square root of EB; or, because DV is one half of CV, and EB one half of AB, as the square root of CV, to the square root of AB; wherefore, the time wherein the pendulum CV performs its vibrations, is to the time wherein the pendulum AB performs its vibrations, as the square root of CV, to the square root of AB, that is, the times are as the square roots of the lengths of the pendulums; so that if one pendulum be four times as long as another, the shorter will vibrate in half the time, so as to perform two vibrations in the same time that the longer performs one.

Pl. 5.
Fig. 17,
18.

In this *Theorem*, as also in every thing else that has been hitherto said concerning the pendulum, the force of gravity is supposed to be given; whence it follows, that if pendulums of different lengths, as CV and AB, perform their vibrations in equal times, the force of gravity in such pendulums must vary, and that in proportion to the lengths of the pendulums, that is to say, the force of gravity in the pendulum CV, must be to the force of gravity in the pendulum AB, as CV to AB. For, as the times of the vibrations are supposed to be equal, the times of the perpendicular falls down the axes DV, and EB must likewise be equal, inasmuch as they have been proved to be proportional to the times of the vibrations; since therefore, forces which act constantly and uniformly are to one another as the velocities which they generate in any given time, the force of gravity which carries a body down DV, must be to the force of gravity which in the same time carries a body down EB, as the velocity acquired at the end of the fall down DV, to the velocity acquired at the end of the fall down EB; but I proved in my lecture upon gravity, that the velocity acquired in falling down DV, is such as will in a space of time equal to that of the fall carry a body thro' a space equal to twice DV, that is, thro' a space equal

equal to the length of the pendulum CV; and therefore, the time being given, the velocity may be expressed by the length of the pendulum; and for the same reason, the velocity acquired in falling down EB, may be expressed by the length of the pendulum AB; consequently, the force of gravity which moves the pendulum CV, is to the force of gravity, which acts upon the pendulum AB, as the length of the former to the length of the latter. Since therefore, it has been found by experience, that a pendulum which vibrates in a second of time under the line must be lengthened as it is removed from the line, and that more and more as its distance therefrom increases; it is manifest, that the force of gravity is less in the æquatorial parts of the earth, than in any other, and that it increases continually as the distance from the line increases, so as to be greatest under the poles; in what proportion this increase of gravity is made, and from what cause it proceeds, I shewed in my lecture upon gravity.

As the several parts of the cycloidal arch of LV, have different inclinations to the plane of the horizon, it is evident, from what has been said concerning the motion of bodies upon inclined planes, that the force which accelerates the motion of a pendulum in its descent from L to V, must continually vary; it being greatest in the point L, and thence continually lessening as the cycloidal arch shortens, till at length in the point V it intirely vanishes: and what is particularly remarkable in this case is, that the accelerating forces in the several points of the cycloid, are to one another in the same proportion with the cycloidal arches intercepted between the vertex and the respective points; for instance, the force which accelerates the pendulum in the point L, is to the force which accelerates the same in the point M, as the arch LV, to the arch MV,

For,

Pl. 5.
Fig. 17.

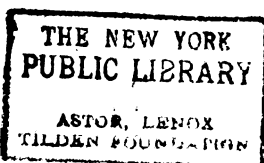
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For as the points L and M have the same directions with their tangents, the accelerating forces in those points must be the same with the forces which accelerate the motions of bodies descending along the tangents ; or because the chords OV and QV in the generating circle are, by the second property of the cycloid, parallel to the tangents at L and M, as the forces which accelerate bodies in their descent upon the chords OV and QV ; but forasmuch as those accelerating forces act constantly and uniformly, they must be to one another, as the velocities which they generate in a given time ; and therefore, since it has been proved, that the chords OV and QV are described in the same time, the accelerating forces are as the velocities acquired at the end of the descent along those chords ; but it has likewise been proved, that those velocities are as the lengths of the chords ; consequently, the force which accelerates a body descending along the chord OV, is to the force which accelerates a body descending along the chord QV, as OV to QV ; but forasmuch as by the third property of the cycloid, OV is one half of LV, and QV one half of MV, as OV is to QV, so is LV to MV ; and therefore, the accelerating force along OV, is to the accelerating force along QV, as the cycloidal arch LV to the arch MV ; but it has been proved, that the accelerating force along QV, is the same with the accelerating force in the point L, and that the accelerating force along QV, is the same with the accelerating force in the point M ; consequently, the force at L, is to the force at M, as LV to MV ; as what has been thus demonstrated of the forces at the points L and M, is in like manner demonstrable of the forces at any other points, so that in a pendulum descending in the arch of a cycloid, the accelerating force is in every point as the length of the cycloidal arch intercepted between the point and the

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the vertex; or in other words, the force is every where proportional to the space to be described.

This then being the law of the accelerating force, and it having been proved, that the pendulum, whether it begins its motion from L or M, or any other point in the cycloid, will arrive in the same time at the lowest point V; it follows, that if several bodies, placed at different distances from any point or center, begin to move towards it at the same instant of time, with forces that are every where proportional to the distances from the center, they will all arrive at the center at the same instant of time; which I thought fit to mention in this place, in order to avoid the trouble of demonstrating the same, when I come to treat of the motions of musical strings, towards the explaining of which this property will be of use.

LECTURE XI.

OF THE MOTION OF PROJECTS.

AS the DOCTRINE OF PROJECTS, whereof I intend to treat in this lecture, cannot be rightly apprehended without some knowledge of the *Parabola*; I shall by way of introduction shew the manner wherein that curve is generated, and point out such of its properties as I shall have occasion to make use of in explaining the motion of projects, referring you for their demonstrations to those authors who have wrote of the *conick sections*.

If a cone as ABC, be touched by a plane in the right line AB, and be cut by another plane parallel to the former, the curve which arises from the intersection of the plane with the surface of the cone is called a *Parabola*; being such as is represented in *Fig. 2*, in which the highest point P is called the *principal*

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Pl. 6.
Fig. 1.

Pl. 6.
Fig. 2.

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principal vertex; the right line CAP passing thro' the point P , and perpendicular to the tangent at that point, is called the *axis*; a right line as DA , drawn from any point in the curve perpendicular to the axis, is called an *ordinate to the axis*; PA the part of the axis intercepted between the vertex and the ordinate is called the *abscisse to that ordinate*; a right line, being a third proportional to the abscisse and its respective ordinate is called the *principal parameter*, or the *parameter to the axis*; a right line as DEH , drawn from any point in the curve parallel to the axis, is called a *diameter*; a right line as PE , intercepted between any point in the curve and the diameter, and parallel to BD which touches the curve in the point D , is called an *ordinate to that diameter*; DE the part of the diameter lying between the vertex D and the point E , is called the *abscisse to the ordinate PE* ; and a right line, being a third proportional to the abscisse DE and the respective ordinate EP , is called the *parameter to the diameter DH* , or *to the vertex D* .

The square of any ordinate divided by the respective abscisse, is equal to the respective parameter; thus the square of DA divided by PA , or the square of OQ divided by PQ , is equal to the principal parameter; and the square of EP divided by DE , as also the square of LM divided by DL , is equal to the parameter belonging to the vertex D . The squares of the ordinates to the axis, or to one and the same diameter, are to one another in the same proportion with their respective abscissa's. Thus, the square of DA , is to the square of OQ , as PA to PQ ; and the square of PE is to the square of ML , as DE to DL .

In one and the same *parabola*, the principal parameter is the least of all the parameters; and the other parameters increase, as the distance of their vertices from the principal vertex increases, tho' not in the same proportion.

If

If from any point in a *parabola* as D, an ordinate be drawn to the axis, and if from the same point a tangent be drawn upward, it will meet the axis when produced; and AB, the part of the axis intercepted between the ordinate DA, and the tangent DB, will be bisected by P the principal vertex.

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These things being premised; if a body be thrown into any direction whatever that is not perpendicular to the plane of the horizon, it will in its motion describe a *parabola*.

For the proof of which, let AE be the direction of the projection, which in the 3d Fig. is parallel to the horizon, and in the 4th and 5th inclined thereto; and let AE be the space which the project would describe in any given time by means of the force impressed, supposing it had no motion downward from the force of gravity; likewise let AB be the space thro' which it would descend in the given time by virtue of its own gravity, supposing it had no other motion; then completing the parallelogram ABCE, it is manifest from what was formerly said concerning the composition of motion, that at the end of the given time, the project must by virtue of its double motion, be found in the point C; but, forasmuch as the motion impressed in the direction AE is uniform, the space described, that is AE, must be as the time in which it is described; consequently, AE square, or BC square, is as the square of the time; but AB or EC, which is the space described in the same time by the force of gravity, is likewise as the square of the time, as I proved in my lecture upon gravity; consequently, AB is as the square of BC; and therefore, from the nature of the *parabola*, the point C thro' which the project moves, must be in the curve of a *parabola*, whose diameter is AB, the vertex A, the point from whence the project begins its motion, and the parameter

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Fig. 3, 4,
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parameter belonging to that vertex, BC square divided by AB , or AE square divided by EC ; and what has been thus demonstrated of the point C , is in like manner demonstrable of all the other points thro' which the project moves; consequently, the line which it describes is a *parabola*.

The velocity of a project in any point of the *parabola* as A , is such as a body acquires in falling down the fourth part of the parameter belonging to that point. For the velocity of the project in the point A is such, as would carry it from A to E in the same time that a body descends from E to C ; and the velocity acquired in the descent from E to C is such, as in the same space of time with that of the fall, would carry a body through a space equal to double EC ; consequently, that velocity is to the velocity of the project in the point A , as twice EC to AE , or as EC to $\frac{1}{2}AE$; but as EC is to $\frac{1}{2}AE$, so is the velocity acquired in falling from E to C , to the velocity acquired in falling down the fourth part of the parameter belonging to the vertex A ; for, by the nature of the *parabola*, the parameter belonging to the vertex A , is equal to $\frac{AEq}{EC}$; wherefore the velocity acquired in falling from E to C , is to the velocity acquired in falling down the fourth part of the parameter, as the square root of EC to the square root of $\frac{1}{4}AEq$, which square roots are to one another, as EC to $\frac{1}{2}AE$, as may appear by multiplying each into the square root of EC ; so that the velocity acquired in falling thro' a fourth part of the parameter belonging to the vertex A , and the velocity of the project in the point A , have one and the same proportion to the velocity acquired in falling from E to C ; consequently, from the nature of proportionals, those two velocities must be equal.

Hence it follows, that if projects move through the same or different *parabolas*, the squares of their velocities in the several points of the *parabolas*, are to one another, as the parameters belonging to the respective points; for, since the velocities in the several points are equal to the velocities acquired in falling down the fourth part of the parameters belonging to those points, and since the squares of the velocities acquired in falling down the fourth part of the parameters, are to one another as the spaces described, as I proved in my lecture upon gravity, it is evident that the squares of the velocities wherewith projects move through the several points of the *parabolas* which they describe, are to one another in the same proportion with the quarter parts of the parameters belonging to those points; but the quarter parts of the parameters being to one another as the whole parameters, the squares of the velocities in the several points of the *parabolas* must bear the same proportion to one another, that the parameters do which belong to those points.

Since this is the case, and since by the nature of the *parabola* the principal parameter is less than any other, and that the other parameters grow larger as the points to which they belong are more distant from the principal vertex; if a project be cast obliquely upward, as in *Fig. 4.* from A towards E, its velocity must continually decrease as it rises and approaches the uppermost point P, wherein the velocity being least must thence increase continually as the project descends and recedes from the point P; and as in one and the same *parabola*, where the distances of any two points as A and K, from the principal vertex P, are equal, the parameters belonging to those points are likewise equal; it is manifest, that a project must have equal velocities in those points; and of consequence, setting aside any difference which may arise from the resistance of the air,

Pl. 6.
Fig. 4.

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air, the project will, *ceteris paribus*, strike a mark as forcibly in the point K as it does at its first setting out in the point A.

The velocity wherewith a project is thrown being given, the velocity thereof in any point of the curve may be thus determined. In the parabola of Fig. 3. let the axis B A be continued upward to D, so as that D B may equal the height from which a body must fall, in order to acquire the same velocity wherewith the project sets out from G; then from any point in the curve taken at pleasure as K, let the horizontal line K L be drawn, and the velocity of the project in the point K, will be to the velocity wherewith it began its motion from G, as the square root of D L, to the square root of D B. For, in my lecture upon gravity, I proved, that if a body be thrown directly upward from B towards D, with the same velocity that it acquires in falling from D to B, it will in any point of its ascent as L, have the same velocity that it would acquire in falling from D to that point; but the velocity acquired in the descent from D to L, is to the velocity acquired in the descent from D to B (which velocity is by supposition equal to the velocity wherewith the body is thrown up) as the square root of D L, to the square root of D B; and by the eighth *Prop.* of my last lecture, the velocity of the project at K, is the same with the velocity at L; consequently, the velocity thereof at K, is to the velocity wherewith it set out from G, as the square root of D L, to the square root of D B. Whence it follows, that if D B be equal to 1600 feet, and D L to 400, the velocity of the project at K, is but one half of the velocity which it had at its setting out from G; and if D L be equal to 900 feet, then is the velocity at K, three fourths of the velocity at G; so that a project being thrown obliquely upward with such a velocity as would carry it to the height of 1600 feet if thrown directly upward,

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will lose a fourth part of its velocity by the time it has risen to the perpendicular height of 700 feet, and one half of its velocity when it has risen 500 feet more.

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The velocity wherewith a project is thrown from any given place being given, as also the position of a mark, the directions wherein the project must be thrown, in order to hit the mark, may be determined in the following manner.

Let A be the place from whence the project is thrown, C the mark situated in the line AC, whose length is given, as also the angle CAB, which it makes with the horizontal line AB; at A erect the perpendicular AP, equal to the parameter belonging to the point A, which parameter is given, inasmuch as the velocity wherewith the project is cast from the point A is given; for it is equal to four times the height from which a body must fall in order to acquire that velocity. Let AP be bisected by the line KH, cutting it perpendicularly in G; at A erect AK perpendicular to AC, and let it be continued till it meets KH. From the point of concurrence K, with the radius KA, let the circle AHP be described. This being done, let a right line as BCEI be erected perpendicular to the horizontal line AB, so as to pass thro' the mark C, and if possible to cut the circle in two points as E and I; AE and AI are the two directions, in either of which, the project being cast with the given velocity, will hit the mark.

Pl. 6.
Fig. 6.

For, drawing the lines PE and PI, the angles CAE and APE are equal, from the nature of the circle; and from the nature of parallel lines, the angles CEA and EAP are equal; consequently, the triangle AEC is similar to the triangle PAE; and therefore PA is to AE, as AE to EC; wherefore, multiplying the extremes and means, and dividing by EC, PA is equal to $\frac{AE^2}{EC}$. In like man-

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ner, the triangles PAI and CAI being fimilar, PA is equal to $\frac{AIq}{IC}$. Wherefore, since PA is equal

to the parameter at the point A, it follows, from the nature of the *parabola*, that those *parabolas* which the project describes when thrown in the directions AE and AI, must pass thro' the point C; consequently, the mark will be hit by a project thrown in either of those directions.

Whenever the mark is placed at such a distance from A on the line ACM, suppose at M, as that the perpendicular NMH, which passes thro' the mark, becomes a tangent to the circle at H, the mark is then at the utmost limit on the line AM, to which a project thrown with the given velocity can reach, and there is but one direction, to wit AH, where-with the mark can be hit; for it is evident, that any other direction must terminate in some point of the circumference above or below the point H; whence if a perpendicular be let fall to the horizontal line AN, it must of necessity fall on this side of HN with respect to A, and of consequence cut the line AM in a point less distant from A than is the point M.

The line AH, which denotes the direction of the project, when thrown to the greatest distance possible on the line AM, bisects the angle PAM, which measures the visible distance between the zenith or vertical point P and the mark M. For, by the nature of the circle, the angle MAH, is equal to the angle HPA; and forasmuch as in the triangles HPG and HAG, the sides PG and AG are equal by construction, and GH common to both, and the angles at G right ones, the angle HPG is equal to HAG, consequently, HAG or HAP is equal to MAH, that is, the angle PAC is bisected by the line AH.

If

If the line ACM be situated below the horizontal line AN, which is the case when the mark is seated on a descent, let all things be constructed as before, and the same things will obtain; to wit, AI and AE, will be the directions necessary to hit the mark at C; and the line AH will bisect the angle PAM, which measures the apparent distance between the zenith and the mark; and the point M will be the utmost limit on the line AM, of a project thrown with the given velocity; the demonstrations of which are exactly the same as in the foregoing case.

If the mark be placed on a level, the line ACM will coincide with the horizontal line ABN, and the parameter AP, will pass thro' the center of the circle and become a diameter, the points K and G coinciding.

In this case, the horizontal distance of the mark, to wit AC or AB, is as the sine of the doubled angle of elevation; or in other words, the horizontal range, or the distance to which a project is thrown on the plane of the horizon with a given velocity, is as the sine of the doubled angle of elevation.

For AC, the horizontal range of a project thrown with a given velocity in the direction AE, is equal to DE, the sine of the angle AKE, but, from the nature of the circle, the angle AKE is double the angle APE, which is equal to CAE, the angle of elevation; consequently, AC, the horizontal distance of the mark, or the distance to which a project is thrown on the plane of the horizon with a given velocity, is as the sine of the doubled angle of elevation.

Hence it follows, that in order to throw a project with a given velocity, to the greatest distance possible on the plane of the horizon, the direction of the projection must be elevated in an angle of 45 degrees; for, since the sine of twice 45 or 90 de-

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grees

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Pl. 6.
Fig. 7.

Pl. 6.
Fig. 8.

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grees is equal to the *radius*, and of consequence, the greatest of all the sines; and since the horizontal ranges at the several angles of elevation are to one another, as the sines of the doubled angles of elevation, it is manifest, that the greatest range, or as it is usually called by gunners, the greatest random, must be when the project is cast in a direction whose elevation is 45 degrees; moreover, the greatest random is ever equal to one half the parameter at the point from which the projection is made; for the line *AM*, which expresses the greatest random, is equal to the *radius* *KH*, or half the diameter *AP*, which by the construction, is equal to the parameter belonging to the point *A*; so that where the velocity with which a project is thrown is given, the utmost distance which that project can reach on the horizontal plane, is likewise given; for it is equal to twice the height, from which a heavy body must fall in order to acquire the velocity wherewith the project is thrown; the parameter belonging to the point *A*, having been already proved equal to four times that height.

A second consequence of the horizontal ranges being as the sines of the doubled angles of elevation, is, that if two projects be thrown with equal velocities, in directions whose elevations are equally distant from 45 degrees above and below, for instance, if the elevation of one be 60 degrees, and that of the other 30, whereof the former exceeds 45 degrees, and the latter falls short thereof by 15 degrees, the horizontal ranges will be equal, or, in other words, the two projects will fall on the plane of the horizon, at the same distance from the place of projection; for as the sum of any two arches of a quadrant, whereof one exceeds 45 degrees as much as the other is exceeded thereby, is equal to a quadrant, it is manifest, that two such arches are complements to each other; wherefore, since by the nature of the circle, the sine of a doubled arch is equal
to

to the sine of its doubled complement, the sines of the doubled angles of two elevations equally distant from 45 degrees above and below, must be equal; and so of consequence must the horizontal ranges which are proportional to those sines. And thus it would constantly be, were it not for two causes which do in some measure disturb this law of projects, so as to make the horizontal ranges of the higher elevations to fall short of those of the lower.

The first of these disturbing causes is the *air*, which as it resists, and thereby retards the motions of projects, must, *cæteris paribus*, cause a greater retardation in those motions which are of longest continuance; consequently, since the higher the elevation of the direction is, the longer is the time of the projects motion, as shall be shewn hereafter; if the directions wherein two projects are cast with equal velocities, be equally distant from 45 degrees, the one above and the other below, the project which is thrown in the higher direction, will be more retarded than that which is thrown in the lower; and of course, will fall on the plane of the horizon at a less distance from the place of projection.

The second disturbing cause, obtains with regard to such projects only as are thrown by the force of *gun-powder*. As the force of the powder acts upon the ball during its continuance in the barrel, so does it likewise to some distance beyond the muzzle; and by so doing makes the ball to move forward in a right line, which line is commonly called the *line of impulse of fire*; at the end of which, the ball quitting the blast of the powder, begins to move in the curve of a *parabola*.

Now tho' the air gave no resistance to projects, yet must the horizontal ranges of a ball shot out of the same piece with equal charges, in two directions equally distant above and below 45 degrees, be different on account of the different directions of the

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Pl. 6.

Fig. 9.

line of impulse of fire; for let us suppose a gun at A, to discharge two equal balls with equal quantities of powder, one in the direction AB, and the other in the direction AC, AB being as far above 45 degrees, as AC is below it; and let AB and AC denote the lines of impulse of fire, so that at B and C the balls will begin to move in *parabolick* curves; from which points let fall the perpendiculars BD and CE; it is manifest, that AD, which is the sine of the complement of the higher elevation, will denote that part of the horizontal range which is owing to the line of fire, when the project is thrown according to the higher elevation; and AE, the sine of the complement of the lower elevation, will be that part of the horizontal distance, which is owing to the line of impulse when the project is thrown according to the lower elevation; consequently, since the sine of the complement of a lesser angle is ever greater than that of a larger angle, the horizontal range of the lower elevation must exceed that of the higher, so that where projects are thrown with the same velocity by the force of powder, in directions equally distant above and below 45 degrees, those must range farthest which are thrown according to the lower elevations, as well on account of the line of fire, as of the resistance of the air.

Pl. 6.

Fig. 8.

The altitude to which a project rises, is as the versed sine of the doubled angle of elevation; for the proof of which, let AE be the direction of the projection, and let AC or AB be bisected in T, and from the point of bisection erect the perpendicular TR; since the point T is equally distant from A, where the project begins its motion, and from B, where the motion of the project ceases, TR will be the axis of the *parabola* which the project describes; and, from the nature of the *parabola*, will be bisected in V by the principal vertex; where,

wherefore, TV will be the height to which the project rises; but from the nature of similar triangles, since AT is one half of AC , TR is likewise one half of CE , and consequently, TV one fourth of CE ; wherefore, from the nature of proportionals, TV is as CE ; but CE is equal to AD the versed sine of the angle AKE , which by the nature of the circle, is double APE ; and APE is likewise, by the nature of the circle, equal to BAE the angle of elevation; wherefore TV , or the height to which the project rises, is as CE the versed sine of the doubled angle of elevation; hence it follows, that the greater the elevation is, the higher the project will rise, inasmuch as the versed sines of the doubled angles of elevation increase continually with the elevation, till at length the elevation becoming perpendicular, the versed sine of the doubled elevation becomes equal to the diameter, which being the greatest of all the versed sines, the altitude of the perpendicular projection must likewise be greatest; and it is equal to one fourth of the parameter; for, I shewed you in my lecture upon gravity, that if a body be thrown up with any velocity, it will rise to the same height, from whence if it fell from a state of rest, it would by the end of the fall acquire the same velocity wherewith it is thrown up; and in this lecture I proved, that the velocity wherewith a project moves in any point of the *parabola*, is equal to the velocity acquired by a heavy body in falling down the fourth part of the parameter belonging to that point; consequently, a project thrown up with a given velocity from the point A , will rise to a height equal to the fourth part of the parameter belonging to that point. Hence it appears, that the greatest height of the perpendicular projection, is equal to half the greatest random, inasmuch as the greatest random has been proved equal to half the parameter belonging to the point A .

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The time of the flight of a project thrown with a given velocity, is as the sine of the angle of elevation: for instance, the time of the flight of a project thrown with a given velocity in the direction AE , is as the sine of CAE , the angle of elevation; for since the project moves thro' the curve of a *parabola* from A to C , by virtue of its uniform motion in the direction AE , and of its accelerated motion in the direction EC , it is evident, that the time of its flight thro' the *parabola*, must be equal to the time of its uniform motion from A to E ; but as the velocity is given, the time of the motion from A to E must be as the space described, that is, as AE ; or by the nature of proportionals, as one half of AE ; but AE being the chord of the arch AE , which measures AKE , the doubled angle of elevation, one half of AE is equal to the sine of half the arch AE , that is, to the sine of the arch which measures CAE , the angle of elevation; consequently, the time of the flight is as the sine of the angle of elevation. Hence it follows, that the greater the elevation is, the longer the time of the flight will be; as also that the time of the perpendicular flight is greatest of all, the sine of the perpendicular elevation being equal to *radius*.

If the velocity wherewith a project is thrown be required, it may be determined from experiments in the following manner; by the help of a *pendulum* or any other exact *chronometer*, let the time of the perpendicular flight be taken; then, forasmuch as the times of the ascent and descent are equal, the time of the descent must be equal to one half of the time of the flight, consequently, that time will be known; and, forasmuch as a heavy body descends from a state of rest at the rate of 16 feet in the first second of time, and that the spaces thro' which bodies descend are as the squares of the times; if we say, as one second is to sixteen feet, so is the square of

of the number of seconds which express the time of the descent of the project, to a fourth proportional, we shall have the number of feet thro' which the project fell, which being doubled, will give us the number of feet which the project would describe in the same time with that of the fall, supposing it moved with an uniform velocity, equal to that which it acquired by the end of the fall; which last found number of feet, being divided by the number of seconds which express the time of the project's descent, will give a quotient, expressing the number of feet thro' which the project would move in one second of time with a velocity equal to that which it acquired in its descent, which velocity is equal to the velocity wherewith the project was thrown up; consequently, the velocity wherewith the project was thrown up is discovered. To illustrate this by an instance, let us suppose half the time of the perpendicular flight to be 8 seconds; then, as one is to 16, so is 64, the square of 8 seconds, to 1024; which being doubled, and then divided by 8, gives 256 in the quotient; which shews that the project was thrown upward with such a velocity as would carry it, supposing it moved uniformly, at the rate of 256 feet in one second of time.

Perhaps it may be objected, that the method here laid down for discovering the velocity of projects, is founded on experiments in which projects are supposed to move freely without any let or impediment, whereas the air resists and retards all projects in their motions, so as not to suffer them to rise to the same height, or to return with the same velocity, that they would in case they moved *in vacuo*; in answer to which it must be confessed, that in the experiments here made use of, the air does resist and impede the motions of projects, so as to shorten their ascent, and to lessen the velocity of their return; but then this does very little affect the truth of the

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the conclusions which are gathered from these experiments concerning the velocities, wherewith projects begin their motions; for as in the method laid down, the only thing necessary to be known from experiment, is the true time of the flight of a project, supposing it to move *in vacuo*; if that time can be had from these experiments, the velocity wherewith the project sets out may be rightly determined, notwithstanding the resistance of the air; but the time of the flight of a project thrown directly upward, is very nearly the same *in vacuo*, as in the air; for, as much as the time of a project's ascent is shortened by the resistance of the air, so much very nearly is the time of its descent lengthened by the same resistance, consequently, the whole time of the flight in air must be very nearly equal to the time of the flight *in vacuo*, and therefore, the time of the flight *in vacuo* is got, by taking the time of the flight in air.

Degrees.	Sines.	Versed sines.
30 — —	50 — —	13
45 — —	70 — —	29
60 — —	86 — —	51
90 — —	100 — —	100
120 — —	— — —	148

Experiments

Experiments concerning projects made with a small mortar, the length of whose chase was $5\frac{1}{2}$ inches; the diameter of the ball and chase $3\frac{1}{2}$ inches; weight of the hollow ball 23000 grains; length of the chamber 2 inches, and its diameter $\frac{3}{4}$ inch.

Quantity of powder in grains.	Degrees of elevation.	Horizontal ranges in feet.	Times of the flights in half seconds.
60	30	135	5
60	45	150	6
60	60	120	8
60	90		11
90	30	200	$6\frac{1}{2}$
90	45	220	8
90	60	200	$9\frac{1}{2}$
90	90		11
120	30	420	9
120	45	450	11
120	60	300	$12\frac{1}{2}$
120	90		16
140	45	660	15
140	90		18
180	30	1000	13
180	45	1100	17
180	60	900	21
180	90		26
240	45	1750	20
240	60	1390	25
240	90		32

LECTURE XII.

OF HYDROSTATICKS.

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IN this lecture I shall give you an account of the gravitation and pressure of WATER, and such other FLUIDS, as are commonly called LIQUIDS.

A *fluid* in general is a body, whose parts yield to any force impressed, and in yielding are easily moved one among another.

The minute particles of fluids do not seem to differ from those of solid bodies; inasmuch as fluids and solids are frequently converted into one another. Thus water and watery fluids are by cold changed into ice; which by heat is again reduced to its fluid state. Metals of all kinds being melted become fluid, and upon cooling grow solid again. The most solid and ponderous woods, as also the hardest stones, may by the force of fire in a great measure be converted into water, as is well known to the chymists. And there are not wanting instances in nature of the grossest bodies being turned into the subtiler fluids, of air and light, and these again into gross bodies. Which changes can scarcely be accounted for, unless we suppose the minute particles of fluids to be of the same nature with those of solid bodies. But be this as it will, most certain it is, that fluids as well as solids consist of heavy particles, whose gravity is ever proportional to the quantity of matter which they contain. This having been found as far as experience reaches to be the universal property of matter, whatever be the form under which it appears. Most indeed of the ancient naturalists, not being sensible of any weight or pressure from the air about them, or from the incumbent water, when immersed therein, were of opinion, that the parts of one and the same element did

did not gravitate one upon another ; which opinion has been exploded by the moderns as erroneous ; and that it is so, will appear from the following experiment.

Let an empty phial close stopped and immersed in water, be suspended from one end of a balance, and poised ; then let the stopple be taken out, that the water may run in, the phial upon receiving the water will preponderate, and bear down the arm of the beam from which it hangs ; which evidently proves, that the parts of water retain their gravity in water, so as to press and bear down upon the parts beneath them ; otherwise the phial would not become heavier upon the admission of the water.

From the gravity of the parts it follows, that setting aside all external impediments, the surface of a liquid contained in a vessel must be smooth and level ; for should any part stand higher than the rest, it must descend by the force of its gravity, and in so doing, spread and diffuse itself till it comes to be on a level with the other parts. As the gravity of the parts reduces the upper surface to a level, so does it likewise occasion a pressure on the lower parts, greater or less in proportion to their depth below the surface, each part sustaining a pressure equal to the weight of all those which lie above it ; whence it follows, that the parts which are at equal depths below the surface, are equally pressed, and of consequence must be at rest, contrary to the opinion of those, who make the nature of fluidity to consist in the constant actual motion of the parts one among another. Should this equality of pressure at any time be destroyed, then indeed a motion will arise in the parts of the fluid, and continue till the pressure becomes equal again, as may appear from the following experiment ; whereby the truth of what has been said concerning the pressure of the superior parts of fluids on those beneath them, will likewise be confirmed.

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Exp. 2.

Take a glass tube open at both ends, and stopping one end with a finger, immerse the other in water to any depth whatever; upon the immersion, the water will rise in the tube, but the height to which it rises, whilst the upper orifice continues stopped, will be but small; but upon removing the finger, it will rise to the same height with the water without*.

When the tube is immersed, that portion of water which lies beneath the orifice ceases to be equally pressed with the other portions that are at the same depth; for that portion bears no other pressure than what arises from the spring of air included in the tube, (which pressure is equal, as shall be shewn hereafter, to the pressure arising from the weight of the external air) whereas, the other portions do not only bear the pressure of the air, but likewise the weight of the incumbent water; forasmuch therefore, as the portion of water which lies beneath the orifice, is pressed down less forcibly than the adjacent portions, it must give way and rise in the tube; but the height to which it rises, whilst the upper orifice of the tube continues stopped, can be but small; because, as the water rises it compresses the air in the tube, and thereby strengthens its spring, so as to make it press with greater force; and when the air is so far compressed by the rising water, as that the force of its spring, added to the weight of the elevated water, makes the same pressure on that portion of water which lies beneath the orifice, as the joint weight of the atmosphere and external water does on the other portions, which are at the same depth with the former, then the water ceases to rise. Upon opening the upper orifice of the tube, by the removal of the finger, the compressed air finding a passage thro' that orifice, expands and dilates itself till it becomes of an equal density with

* The water is tinged of a fine blue purple colour with a few grains of Sal Armoniack and Copper.

the external air; by which means, the pressure arising from the condensation of the air is taken off, and of consequence, the water which lies beneath the orifice is less pressed than the adjacent portions, and for that reason must rise, and continue so to do, till the elevated water in the tube gravitates as forcibly on the water beneath the orifice, as the external water does on the neighbouring portions; but this it cannot possibly do, till it comes to be of an equal height with the external water.

Should a lighter liquid be poured on the external water, the water within the tube will rise yet higher than before; and the height to which it rises above the surface of the external water, will be so much less than the height of the lighter liquor above the same surface, by how much the specific gravity of the water exceeds that of the lighter liquor; for instance, if the specific gravity of the water be to the specific gravity of the lighter liquor, as two to one, the height of the water in the tube above the level of the external water, will be to the height of the lighter liquid, as one to two; because in that case, one part of water makes an equal pressure with two parts of the lighter liquid. To illustrate this by an experiment.

Let oil of turpentine, whose specific gravity is to the specific gravity of water, as 83 to 100, be poured on the external water to the height of eight inches and an half, and the water will rise in the tube to the height of 7 inches and $\frac{3}{16}$ above the level of the external water, that is, the heights of the water and oil will be in the reciprocal proportion of their specific gravities; for 7 and $\frac{3}{16}$ is to 8 and $\frac{1}{8}$, or, which is the same thing, 73 is to 85, very nearly, as 83 to 100.

Exp. 3.

The same thing is in like manner confirmed by the following experiment.

Let one end of a small tube open at both ends, be immersed in mercury contained in a larger tube, and

Exp. 4.

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and let water be poured upon the mercury in the larger tube to the height of 34 inches; the mercury will rise in the smaller tube to the height of two inches and an half above the level of the mercury in the larger tube; so that the height of the mercury in the smaller tube above the level of the mercury in the larger, will be to the height of the water above the same level in the reciprocal proportion of their specifick gravities; for $2\frac{1}{2}$ is to 34, as 1 to $13\frac{3}{4}$, which numbers express the proportion of the specifick gravity of water to that of mercury.

The pressure which the lower parts of a liquid sustain from the weight of those which lie above them, exerts itself every way in all manner of directions, and that equally; or in other words, whatever be the force wherewith a drop of any liquid is pressed downward by the weight of the incumbent liquid, with the very same force is that drop pressed upward, as also laterally and obliquely, and in a word, in all kind of directions whatever; otherwise the drop, which from the nature of fluidity, readily yields and gives way to any impression, must by reason of the pressure from above move out of its place; but this it cannot possibly do, because the drops all around it being equally pressed from above, do on all sides resist the motion of that drop, with the same force that it endeavours to move; consequently, the drop must continue at rest, and be pressed on all sides with the same force that it is from above; and what has been thus proved of one drop, is in like manner demonstrable of all the rest; and therefore, the pressure on the lower parts of a liquid exerts itself equally every way, as will appear from the following experiment.

Exp. 5.

Let four tubes open at both ends be immersed in water to the same depth, their upper orifices being first stopped, and let the lower orifices be so situated, as that the water in entering may move directly upward in one, and directly downward in another, obliquely

obliquely in a third, and horizontally in the fourth; upon opening the upper orifices, the water will rise in all of them to the same height with the external water, as being pressed in the several directions with a force equal to the weight of the incumbent water.

From the pressure of liquids upwards it is, that solid bodies specifically lighter than liquids, are made to ascend when immersed therein. For when a solid body is immersed in a liquid, it presses that part of the liquid whereon it rests, with a force equal to the weight of a column composed of the body itself, and that portion of liquid which lies upon it; and the water presses upward against the body, with a force equal to the weight of a like column of the liquid alone; which force, inasmuch as the liquid is heavier than the solid, must overcome the force wherewith the body presses downward, and of consequence, the body must rise with the difference of those forces; as shall be shewn more fully in my next lecture. If by any contrivance the pressure of the liquid from beneath can be taken off, a body tho' specifically lighter will not rise in a liquid, but remain immersed, as in the following experiment.

A brass plate being joined to one end of a cylindrical piece of wood, and another plate of the same size and shape being fixed in water; let the cylinder be totally immersed, and let its plate be laid upon the other in such a manner, as that no water may get between; the cylinder tho' specifically lighter will remain beneath the water, being pressed down by its own weight and that of the incumbent water, whilst the contrary pressure of the water from beneath, is kept off by means of the plate whereon the cylinder rests. Exp. 6.

As bodies specifically lighter than liquid, are forced up, on account of the pressure from below being greater than the force wherewith the bodies

O. press

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press downward; so on the other hand, bodies specifically heavier must sink, because the force wherewith they press downward exceeds the pressure from beneath which opposes their descent; and the force wherewith they descend is equal to the difference of those forces; as shall likewise be shewn in my next lecture. If by any contrivance those two forces can be reduced to an equality; then the bodies will not descend, but remain suspended in the liquid; as in the following experiment.

Exp. 7. Let a brass plate, whose specifick gravity is to that of water, as 9 to 1, be adapted to the neck of a glass vessel in such a manner, as that being immersed in water no part of the water may get upon its upper surface; let it then be immersed to the depth of nine times its own thickness, (that is, to the depth of 2 inches and $\frac{7}{8}$, the thickness of the plate being $\frac{3}{8}$ of an inch) and it will remain suspended; but upon pouring ever so little water upon its upper surface, it will immediately descend and fall to the bottom.

The plate being immersed to the depth of nine times its own thickness is pressed upward by a force equal to the weight of a column of water, whose height is nine times as great as the thickness of the plate; which weight, inasmuch as the specifick gravity of the water is to that of the plate, as 1 to 9, is equal to the weight of the plate, that is, to the force wherewith the plate presses downward; for as none of the water lies on its upper surface, it can press downward with no other force than what arises from its own gravity; consequently, in this case, the force which resists the descent, is equal to the force which promotes it, and of course, the plate must remain in its place. When a little water is poured on the plate, the weight of that added to the weight of the plate, overcomes the resisting force

Exp. 8. of the water, and causes the plate to descend. Should the plate be immersed to twice the former depth, it

it will not descend tho' loaded with water to the height of nine times its own thickness; for as in this case, the depth to which it is immersed is double the former, so likewise is the force wherewith the water presses upward; consequently, that force is sufficient to support twice the weight of the plate, and therefore will sustain the plate when loaded with water, to the height of nine times its own thickness, such a quantity of water being just equal in weight to the plate.

If by pouring on more water, the force wherewith the plate presses downward be increased, or by raising the plate nearer to the surface of the water, the force wherewith the water presses upward be diminished, the plate will fall to the bottom; and on the other hand, if by immersing the plate to a greater depth, the pressure of the water upward be increased, the plate will be thrust upward against the glass, and would actually ascend were it not hindered by the glass.

From what has been said it follows, that if S be put to denote the number expressing the specific gravity of the plate, that of water being unity; D be the depth to which the plate is immersed expressed in the thickness of the plate, and H the height of the water upon the plate expressed likewise in the thickness of the plate; D must be equal to the sum of S and H in all cases where the plate remains suspended; and if there be no water upon the plate, then D and S must be equal; wherefore if in the former case, D be greater than S and H taken together, or in the latter, than S alone, the plate will ascend if not hindered by the glass; and on the other hand, if D be less, the plate will descend and fall to the bottom.

The pressure which the bottom of a vessel sustains from a liquid contained in it, whatever be the shape of the vessel, is equal to the weight of a pillar of the liquid, whose base is equal to the area of the

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the bottom, and whose height is the same with the perpendicular height of the liquor.

That this is the case in vessels that are equally wide from top to bottom is plain and obvious; inasmuch as the bottom of every such vessel does actually sustain such a pillar of liquor. But that the case should be the same in irregular vessels, is not so easy to conceive; for instance, that in a vessel which from a large bottom grows narrower as it rises, so as perhaps at length to be contracted into a tube, the bottom should bear the same pressure when the vessel is filled, as it would were the vessel equally wide throughout from bottom to top, seems strange and surprizing, and yet it is what necessarily follows from the nature of fluidity; for that part of the bottom which lies directly beneath the tube sustains the weight of a pillar of liquor which reaches to the top of the tube, the vessel being supposed to be full, and being pressed with the weight of that pillar, reacts with an equal pressure on that portion of the liquor which touches it; and that pressure inasmuch as it exerts itself equally in the liquor every way, is propagated laterally thro' the several portions of liquor which are contiguous to the bottom of the vessel; and forasmuch as this lateral pressure does in like manner exert itself equally every way, the bottom of the vessel must be equally pressed in every point; consequently, since that portion of it which lies beneath the tube, bears a pressure equal to the weight of a pillar of liquor, whose height reaches to the top of the vessel, every other equal portion must bear a pressure equal to the same weight; and of course, the whole bottom must be pressed as forcibly, as if the vessel continued of the same wideness to the top, and was filled with the liquor.

Exp. 9.
Pl. 6.
Fig. 10.
11.

To confirm this by an experiment. Let there be two glasses open at both ends, and of such shapes as are exhibited in the two figures, whose lower parts

M N

MN are cylindrical and equal, and of a capacity just sufficient to admit the brass plate made use of in the last experiment; which must be fitted to each of them successively, in order to constitute two vessels of equal bottoms, but of different capacities; and being so fitted, let it be immersed in water, as in the last experiment, to such a depth, as that it will be necessary to load it with water in order to make it sink; that is, let the depth be more than nine times the thickness of the plate, which depth must be the same in both cases; let then water be poured on the plate, and let it be observed what height of water is requisite to force down the plate when the wider vessel is made use of, and it will be found, that the same height will suffice in the narrower vessel; consequently, the small pillar of water in the narrower vessel, must press the plate with a force equal to the weight of a pillar of water of the same height, and of a base equal to the *area* of the plate; for such a pillar does actually press the plate in the larger vessel, as is evident from the bare inspection of the figures, and the pressure made on the plate in both vessels are equal, inasmuch as they overcome equal resistances.

From what has been said it appears, that where the base of a vessel is given, the pressures upon it are as the perpendicular heights of the liquid, whatever be the shape of the vessel. And universally, the pressure on any base is measured by the product of the *area* of that base into the perpendicular height of the liquor above it, without any regard to the quantity of liquor contained in the vessel; so that if we suppose a hoghead set on one end, and filled with a liquor, and a small pipe to issue perpendicularly upward from the other end to any height whatever, and to be filled with the same liquor, the bottom will be as strongly pressed, and be in as much danger of bursting out, as if the hoghead

was continued to the same height with the pipe, and filled with the liquor.

As the bottom of a vessel bears a pressure proportional to the height of the liquor, so likewise do those parts of the sides which are contiguous to the bottom; because the pressure of fluids is equal every way. And as the pressure which the lower parts of a fluid sustain from the weight of those above them exerts itself equally every way, and is likewise proportional to the height of the incumbent fluid, the sides of a vessel must every where sustain a pressure proportional to their distance from the upper surface of the liquor. Whence it follows, that in a vessel full of liquor, the sides bear the greatest stress in those parts next the bottom; and that the stress upon the sides decreases with the increase of the distance from the bottom, and in the same proportion; so that in vessels of a considerable height, the lower parts ought to be much stronger than the upper, that they may be able to withstand the greater pressure.

LECTURE XIII.

OF HYDROSTATICKS.

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IN this lecture I shall explain to you that part of HYDROSTATICKS which is of use in discovering the *densities* and *specifick gravities* of *bodies*.

The DENSITY of any body is measured by the proportion which its quantity of matter bears to its bulk. For the more numerous the particles of matter are in proportion to the space which they possess, the greater is the density of the body; and the fewer the particles, the less the density; wherefore, putting D to denote the density of a body,

Q its

Q its quantity of matter, and M its magnitude,
 $D = \frac{Q}{M}$; and forasmuch as the quantity of matter

in any body is ever proportional to, and measured by, the weight, as I shewed in my lecture upon gravity; if instead of the quantity of matter the weight of the body be substituted, and if that weight be denoted by W, then $D = \frac{W}{M}$; that is, the density is as the weight of the body directly, and the magnitude inversely.

By the specifick gravity of a body is meant the gravity peculiar to that species of matter, whereof the body is a part; and it is measured by the proportion of the absolute weight to the bulk; which proportion in one and the same kind of matter, remains unvaried; and in different kinds as this proportion is greater or less, so is the specifick gravity which is measured by it. Let S denote the specifick gravity of a body, its weight and magnitude being denoted by W and M as before; then, from what has been said, $S = \frac{W}{M}$; and by consequence,

since D is likewise $= \frac{W}{M}$, $S = D$; that is, the specifick gravity of a body is as its density. And therefore, by finding out the proportion which the specifick gravities of bodies bear to one another, the proportion of their densities is likewise discovered; for which reason I shall take no further notice of the densities of bodies, but confine myself to the consideration of their specifick gravities alone.

When a solid body is immersed in a liquid, it presses downward, and endeavours to descend by the force of its gravity; but forasmuch as it cannot descend without moving as much of the liquid out of its place, as is equal to it in bulk, it is manifest that it is resisted, and as I may say, pressed upward

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by a force equal to the weight of such a portion of the liquid as is equal to it in bulk; consequently, if the specifick gravity of the solid be greater than that of the liquid, that is, if the solid weighs more than an equal bulk of the liquid, the body will descend with a force equal to the excess of its gravity above the gravity of the liquid; on the other hand, if the gravity of the liquid exceeds that of the solid, the body being as it were pressed upward by a force greater than that whereby it endeavours to go down, will ascend with the difference of those forces, that is, with a force equal to the excess of the specifick gravity of the liquid above that of the solid. When the specifick gravities are equal, the body will neither rise nor fall, but remain suspended at any depth; being pressed as strongly upward by the resisting force of the liquid, as it is downward by its own weight. Hence it follows, that if by any contrivance the specifick gravity of a solid can be varied, so as to be one while greater, another less, and then equal to the specifick gravity of a liquid wherein it is immersed, the body will sink, or rise, or remain suspended according to the variation of its specifick gravity. And this is the case in that ludicrous experiment of the little glass images in water, which are made to descend, or rise, or remain suspended at pleasure; the reason of which I shall explain to you, after you have seen the experiment.

Exp. 1.

The images being set to float on the water, the top of the vessel must be covered with a bladder closely bound about the neck of the vessel, to the end that the air, which lies upon the surface of the water may not force its way out when it is condensed by the hand pressing on the bladder. The images themselves tho' lighter, are yet nearly of the same specifick gravity with the water, and being hollow, are full of air, which by means of small holes

holes in their heels communicates with the air without. When the air which lies beneath the bladder is pressed by the hand, it presses on the surface of the water; and forasmuch as the pressure is propagated thro' all the water, those portions which are contiguous to the heels of the images, are thereby forced into the holes, by which means the air within is condensed, and at the same time, the weight of the images is increased by the additional weight of the influent water. And when so much water is forced in, as to render the specifick gravity of the images greater than that of the water, the images descend and fall to the bottom, where they remain as long as the pressure above continues; but when that is taken off by the removal of the hand, the condensed air in the images dilates and expands itself, and in so doing drives out the water; upon which account the images become specifically lighter than the water, and of course ascend. As the pressure on the bladder is greater or less, so must the quantity of water which is forced into the images; and therefore, whenever it happens that during the ascent or descent of an image, such a pressure is made as suffices to force in just as much water as is requisite to reduce the image to the same specifick gravity with the water, the image stops and remains suspended, upon increasing the pressure it descends, and ascends if the same be lessened. Some of the images begin to descend sooner; as also to rise later, than others, for one or both of these reasons; first, because some are specifically heavier than others; and secondly, because the cavities in the legs are greater in some images in proportion to their magnitudes, than they are in others; upon both which accounts, a less pressure is requisite to make some descend, and to keep them down, than what is necessary to produce the same effects in others. For, first, let us suppose the specifick gravities of two images

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images to be different, but the cavities in their legs, when taken of a given height, to be proportional to their respective magnitudes; since the air is equally dense in both images, it is manifest, that it gives the same opposition in both to the influent water; consequently, the water when forced in by the pressure from above, must rise to equal heights in the cavities of both; since therefore the cavities whose heights are equal, are supposed to be proportional to the magnitudes of the images, it is manifest, that the quantities of water contained in those cavities must be so too; consequently, each image receives an addition of weight from the influent water proportional to its magnitude; or in other words, the specifick gravities of the two images are equally augmented; forasmuch therefore as one of the images is supposed to be specifically heavier than the other, it is evident, that when the specifick gravity of the former has received such an addition, from the influent water, as makes it a little exceed the specifick gravity of the water, the specifick gravity of the latter must fall short therefore; consequently, the former must sink and leave the other above.

Secondly, Let us suppose the specifick gravities of the two images to be equal; but let one image be less in proportion to the cavity in its legs, than the other is in proportion to its cavity, the height of the cavities being given; since the water does from the same pressure rise to an equal height in both, it is plain, from what I just now said, that the former must receive a greater quantity of water in proportion to its magnitude, and consequently, a greater addition to its specifick gravity than the latter, and of course must descend sooner.

From what has been said it follows, that if the proportion which the cavity in the legs bears to the magnitude of the image be given, the greater the

the specifick gravity of the image is, the more apt it will be to descend; consequently, in this case the aptitude or promptness of an image to descend is proportional to, and may be expressed by, the specifick gravity. In like manner, if the specifick gravity be given, the greater the proportion is which the cavity in the leg bears to the magnitude of the image, the more apt the image is to descend; and therefore in this case, the aptitude is proportional to, and may be expressed by, the cavity applied to the magnitude of the image. But if neither the specifick gravity of the image, nor the proportion of the cavity to the magnitude of the image, be given, the aptitude of an image to descend, is as the specifick gravity into the cavity applied to the magnitude of the image; that is, putting A to denote the aptitude, S the specifick gravity of the image, C the cavity in the leg, (the height whereof is always supposed to be given) and M the magnitude of the image; $A = \frac{SC}{M}$; or, substituting the abso-

lute weight of the image applied to its magnitude, in the room of the specifick gravity, $A = \frac{WC}{M^2}$; that is, the aptitude an image has to descend, is as the weight of the image into the cavity of the leg directly, and the square of the image's magnitude inversely.

A solid specifically heavier than a liquid, being immersed therein, loses as much of its weight as is the weight of a portion of the liquid equal to it in bulk; for it has been already shewn, that a solid is carried down in a liquid by the excess only of its gravity, above the gravity of a portion of the liquid equal to it in bulk; consequently, the other part of its gravity is lost, as to any effect it has on the body itself; as will appear from the following experiment.

Let

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Exp. 2.

Let a small cylinder of brass, suspended at one end of a balance and counterpoised, be immersed in water; upon the immersion it will become lighter, suppose by 200 grains, which is the weight of as much water as is equal in bulk to the cylinder; for a cylindrical vessel, just large enough to contain the cylinder, being hung at one end of a balance and poised, and then filled with water, preponderates with the weight of 200 grains.

Exp. 3.

Since a solid when immersed in a liquid, loses as much of its weight, as is equal to the weight of a portion of the liquid of the same dimensions with the solid, it follows, that all bodies whatever, whose magnitudes are equal, however different their specific gravities may be, do suffer an equal loss of weight in the same liquid. Thus a cylinder of block-tin, equal in dimensions to the brass cylinder, but specifically lighter, being immersed in water, loses 200 grains, as did that of brass.

Exp. 4.

Tho' a solid loses part of its weight when immersed in a liquid, yet it must not be imagined that the weight so lost by the solid, is actually destroyed, but that it is imparted to the liquid, the liquid constantly gaining in weight what the solid loses. For if the vessel with the water wherein the cylinders were immersed, be put into a scale and poised; upon the immersion of either cylinder, it will preponderate with the weight of 200 grains, which is what the cylinder loses.

Solids equal in weight, but of different specific gravities, being immersed in the same liquid, suffer losses of weight reciprocally proportional to their specific gravities; for as the loss of weight which any body suffers in a liquid, is equal to the weight of as much of the liquid as is equal in bulk to the solid, the loss sustained is ever proportional to the magnitude of the body; whatever proportion therefore the magnitudes of bodies have to one another, the same will the losses of weight have which they suffer;

suffer; but the magnitudes of bodies equal in weight, but of different specifick gravities, are to one another in the reciprocal proportion of their specifick gravities; consequently, so are the losses of weight which they suffer. Which is confirmed by the following experiment.

Let two cones, one of lead, the other of tin, whose specifick gravities are to one another, as 112 to 74, and the weight of each 400 grains, be immersed in water, after the manner of the cylinders; upon the immersion, the lead will lose $35\frac{1}{2}$ grains, and the tin 54; but $35\frac{1}{2}$ is to 54, as 74 to 112, that is, reciprocally as the specifick gravities of the metals.

Exp. 5.

From the losses of weight being reciprocally proportional to the specifick gravities, it follows, that if two bodies of different specifick gravities, which balance each other in air, be immersed in water or any other liquor, the *æquilibrium* will be destroyed, and that which has the greatest specifick gravity will descend; as will appear, by hanging the cones, one at each end of a balance, and then immersing them in water, for the lead will preponderate.

Exp. 6.

The specifick gravity of a solid specifically heavier than a liquid, is to the specifick gravity of the liquid, as the absolute weight of the solid, to the loss of weight which it suffers in the liquid; for the specifick gravities of bodies being as the absolute weights applied to the magnitudes, where the magnitudes are equal, the specifick gravities are directly as the absolute weights; if therefore we compare the solid with a quantity of the liquid equal to it in magnitude, their specifick gravities must be as their weights; but the absolute weight of such a quantity of the liquid, is equal to the loss of weight sustained by the solid; consequently, the specifick gravity of the solid, is to that of the liquid, as the whole weight of the solid, to the loss which it sustains in the liquid.

Hence

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Hence we have a method of discovering the specifick gravities of such solid bodies as are heavier than water; I mean, of discovering the proportions of their specifick gravities to the specifick gravity of water. For if we suppose the specifick gravity of water to be unity, and put L to denote the loss of weight which any body, whose specifick gravity we look for, sustains in water, and W its whole weight, then $L : W :: 1 : \frac{W}{L}$; consequently, $\frac{W}{L}$ expres-

ses the specifick gravity of the solid, that of water being unity; and therefore, in order to know the specifick gravity of any solid heavier than water, nothing more is requisite, but to discover the quantities denoted by W and L , and to divide the first by the last; the first is had, by taking the weight of the body in air, and the last, by taking the weight in water, and subducting it from the weight in air; for the remainder is the loss of weight, which dividing the weight in air, gives a quotient expressing the specifick gravity of the body. To apply this to a particular case, let it be proposed to discover the specifick gravity of a piece of tin, which being weighed in air, is found to be 300 grains, and in water, $259\frac{1}{2}$, which being subducted from the former, leaves $40\frac{1}{2}$ for the loss of weight; so that in this case, W denotes 300, and L $40\frac{1}{2}$; and therefore, dividing 300 by $40\frac{1}{2}$, we shall have $7\frac{4}{5}$ for the specifick gravity of tin, that of water being unity. Whence it appears, that tin, bulk for bulk, is more weighty than water, in the proportion of 74 to 10.

If the body, whose specifick gravity is required, be lighter than water; then, forasmuch as its gravity is not sufficient to cause a total immersion, the loss of weight which it suffers in water cannot be found out by weighing it alone in that liquid; let it therefore be joined to some other body so weighty, that the compound may sink; but first let the loss of

of weight which the heavier body alone sustains in water be found out, as before; and then let the loss of weight which the compound sustains be likewise discovered, whence deducting the loss of weight sustained by the heavier, the remainder will exhibit the loss sustained by the lighter; consequently, dividing the weight of the lighter by that remainder, the quotient will express the specifick gravity required; that is, putting W for the weight of the body whose specifick gravity is sought, L for the loss of weight sustained by the compound, and l for the loss sustained by the heavier body $\frac{W}{L-l}$

expresses the specifick gravity of the body. To apply this to a particular case; let the weight of a piece of wood specifically lighter than water be 220 grains, and let it be joined to a piece of tin of 160 grains, whose loss in water is found to be 17 grains; then the compound being weighed in water, will be found to lose 334 grains; so that in this case, W is equal to 220 grains, L to 334, and l to 17; and L less l , is equal to 317 grains. And therefore, dividing 220 by 317, we shall have $\frac{664}{1000}$ for the specifick gravity of the wood, that of water being unity. So that that kind of wood is bulk for bulk lighter than water, in the proportion of 664 to 1000.

Exp. 8.

If the body whose specifick gravity is sought be dissolvable in water, then instead of water, let some other liquor be made use of, which will not dissolve the body; and let the proportion of the specifick gravity of the body to the specifick gravity of that liquor, be discovered by the foregoing method; as also the proportion of the specifick gravity of that liquor to the specifick gravity of water, by the method which shall be shewn presently. Then in whatever proportion the specifick gravity of the liquor exceeds or falls short of the specifick gravity of water, in

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in the same proportion let the specifick gravity of the body with regard to that of the liquor be increased or diminished, and it will give the specifick gravity of the body with respect to that of water; that is, if we put A for unity or the specifick gravity of water, B for the specifick gravity of the other liquor, and C for the specifick gravity of the body with regard to that liquor; then by saying, as A is to B, so C to a fourth proportional, we shall have $\frac{BC}{A}$ for the specifick gravity of the body with

Exp. 9.

respect to that of water; or rejecting the divisor as being equal to unity, and putting S for the specifick gravity of the body with respect to water, we shall have $S = BC$. To apply this, let the specifick gravity of Roman-vitriol be required; let the weight of a piece in air be 67 grains, and in spirit of wine 41 grains; consequently, its loss of weight in the spirit is 26 grains, which dividing 67, gives 2.576 for the specifick gravity of the vitriol with regard to the specifick gravity of the spirit, which in this case is supposed to be unity; but the specifick gravity of the spirit with regard to that of water is less than unity, being only $\frac{87}{100}$, as shall be shewn presently; wherefore B is $= 0.87$, and C is $= 2.576$; consequently 2.24, which is the product arising from the multiplication of those two numbers, expresses the specifick gravity of Roman-vitriol with respect to that of water, which is as unity; and therefore in whole numbers, the specifick gravity of Roman-vitriol exceeds that of water, in the proportion of 224 to 100.

The specifick gravities of liquors are discovered by taking the losses of weight sustained by one and the same solid in the several liquors; for since the loss of weight in each liquor, is equal to the weight of as much of the liquor as is equal in bulk to the body; by taking the losses of weight sustained by the

the same body in the several liquors, we get the absolute weights of such portions of those liquors as are equal in bulk; and by consequence, the specifick gravities of the liquors, the specifick gravities of bodies equal in bulk, being to one another as their absolute weights; wherefore, putting L for the loss of weight which a body sustains in water, and little l for the loss of weight sustained by the same body in any other liquor; then, by saying, as L to l , so is unity to a fourth term, we shall have $\frac{l}{L}$ for the specifick gravity of the other liquor, that of water being unity; so that to discover the specifick gravity of any liquor, we have nothing more to do, but to weigh one and the same solid, both in the liquor whose quantity is sought, and in water, and to divide the loss of weight which the solid suffers in the liquor, by the loss which it sustains in water; for the quotient will express the specifick gravity of the liquor. Thus, a glass bubble, whose weight in air is 1727 grains, being weighed in water, is found to lose 641 grains, and 558 in spirit of wine; and therefore, dividing 558 by 641, we shall have a quotient of 0.87 for the specifick gravity of the spirit, that of water being unity.

Exp. 101

When a body specifically lighter than a liquid, is set to float upon it, the part immersed is equal in bulk to a portion of the liquid whose weight is equal to the weight of the whole body; for since the body sinks in part, by moving some of the liquor out of its place, and since the weight of the body is the power which moves the liquor, the body must continue to sink, till it has removed as much of the liquor as is equal to it in weight; consequently, the part immersed must be equal in magnitude to such a portion of the liquor, as is equal in weight to the whole body; which is abundantly confirmed by the following experiment.

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A ball

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Exp. 11.

A ball of pear-tree, a wood specifically lighter than water, being set to float on water contained in a glass vessel, let the vessel be placed in a scale and counterpoised; then, taking out the ball, let the vessel be filled up with water to the same height at which it stood when the ball was in it, and the same weight will counterpoise it as before.

From the vessel's being filled up to the same height at which the water stood when the ball was in, it is manifest, that the quantity poured in is equal in magnitude to that part of the ball which was immersed; and, from the same weight counterpoising, it is evident, that the water poured in, is equal in weight to the whole ball.

The part immersed is to the whole body, as the specific gravity of the body to the specific gravity of the liquid; for the specific gravities of two bodies, being to one another as their absolute weights applied to their magnitudes, if their weights be equal, their magnitudes are in the reciprocal *ratio* of their specific gravities; since therefore, such a portion of the liquid as is equal in magnitude to the immersed part of the solid, is likewise equal in weight to the whole solid; the magnitude of the immersed part is to the magnitude of the whole body, as the specific gravity of the solid to the specific gravity of the liquid.

When the same body is set to float successively in different liquors, the parts immersed are to one another in the reciprocal proportion of the specific gravities of the liquors. For the body descends in each liquor, till the part immersed takes up the room of as much liquor as is equal in weight to the whole body; and therefore, such portions of the several liquors as are equal in magnitude to the immersed parts of the body have all equal weights; but the magnitudes of bodies equal in weight, are to one another reciprocally, as their specific gravities; consequently, in one and the same body
floating

floating in different liquors, the parts immerſed are reciprocally as the ſpecifick gravities of the liquors. On this principle is founded the *HYDROMETER*; which is an hollow glaſs ball, with a ſmall hollow ſtem of about 5 or 6 inches in length, oppoſite to which, on the other ſide of the ball, adheres a ſmaller ball filled in part with mercury, or ſome other weighty body, to the intent, that when the ball is ſet to float in water, or any other liquor, the ſtem may be kept uppermoſt, and in a poſition perpendicular to the ſurface of the liquor; and at the ſame time, that the machine may be ſo far immerſed, as that the ſtem only, & ſome part thereof, may remain above the liquor: the ſtem being graduated from top to bottom, has numbers annexed to every degree, expreſſing the magnitudes of the parts which lie below the ſeveral degrees.

The uſe of this little machine is to diſcover the ſpecifick gravities of liquors, which is done in the following manner. The *hydrometer* being firſt ſet to float in water, the degree to which it ſinks muſt be obſerved, and the number thereto annexed; then being ſet to float in any other liquor, the degree to which it ſinks, with the number annexed, muſt likewise be noted; for as this number is to the former, ſo is the ſpecifick gravity of water, to that of the other liquor, as is evident from what was juſt now ſaid. To illuſtrate this in the caſe of water and ſpirit of wine. (The *hydrometer* being dropt into water, ſinks to the degree whoſe number annexed is 87; and being dropt into ſpirit of wine, ſinks to the degree whoſe number is 100; whence it appears, that the ſpecifick gravity of water is to that of ſpirit of wine, as 100 to 87.

Exp. 12i

Tho' *hydrometers* may be uſeful in diſcovering the ſpecifick gravities of liquors for looſe and inaccurate computations, yet they are not to be depended on in caſes where great exactneſs is required,

and that for two reasons; First, because it is extremely difficult to graduate the stems so exactly, as that the numbers annexed shall truly express the magnitudes of the parts below them. Secondly, because, partly from the motion of the *hydrometer* in the liquor, and partly from the rising of the liquor about the stem from the attractive force of the glass, it is hardly possible to determine with exactness the degree to which the *hydrometer* sinks. Upon both which accounts, as also because the method of determining the specific gravities of liquors by means of the glass bubble is much more easy and exact, this method by the *hydrometer* is intirely laid aside.

LECTURE XIV.

OF HYDROSTATICKS.

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IN this lecture I shall give you an account of the flux of water from RESERVOIRS thro' *orifices* and *pipes*.

If water, flowing out at an orifice in the bottom of a vessel, be kept constantly at the same height in the vessel, by being supplied as fast above as it runs out below, the velocity wherewith it flows out, is as the square root of the height of the water above the orifice.

For if we suppose the column of water which stands directly above the orifice, to be divided into an indefinite number of plates of an equal but exceedingly small thickness, it is manifest, that whatever be the force of gravity, wherewith the uppermost plate presses upon the second, the second presses upon the third with a double force, and the third upon the fourth with a triple force, and so on; so that the plate which is next the orifice is pressed downward

downward by the joint gravities of the several plates which lie above it, and likewise by the force of its own gravity, inasmuch as there is no other plate beneath it whereon to rest; consequently, from its own gravity, and that of the several plates above it, it does all at once receive as many equal impressions from gravity, as it would successively in falling down the height of the water; and of course, must pass thro' the orifice, with the same velocity that it would acquire in falling down that height; but I proved in my lecture upon gravity, that the velocity which a body acquires in falling thro' any space, is as the square root of the space; consequently, the velocity wherewith the water flows out, is as the square root of the height of the water above the orifice.

To confirm this by an experiment; let there be two vessels in all things alike, excepting that one is four times as tall as the other, the height of one being 20 inches, and of the other 5; let each of them have a circular orifice in the bottom, a fifth part of an inch in diameter; and being both filled with water; let them be set a running, and let the water be supplied as fast above as it runs out below; the taller vessel will discharge about twenty one ounces in the space of a quarter of a minute, and in the same time the shorter will discharge about 11 ounces. Now, forasmuch as the orifices thro' which the water flows are equal, and likewise the times of the flux, the quantities discharged are as the velocities; consequently, the velocity wherewith the water flows out of the taller vessel, is to the velocity wherewith it flows out of the shorter, as 21 to 11, that is, nearly as 2 to 1, which are the square roots of the heights of the water above the orifices.

Exp. 1.

As the pressure sustained by the lower parts of water from the weight of those above, exerts it-

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self with the same force laterally that it does downward, it matters not whether the orifice through which the water flows, be at the bottom or side of a vessel; for the water will flow out of both with the same velocity, provided they are at equal depths below the upper surface of the water; and therefore, the velocity of water flowing out of an orifice in the side of a vessel, is as the square root of the height of the water above the orifice; as will appear, by repeating the last experiment with vessels whose orifices are in their sides; for the quantities discharged will be the same as before.

Exp. 2.

Pl. 6.
Fig. 12.

From what has been said it follows, that if an orifice in the side of a vessel be situated as far above an horizontal plane, as it is below the upper surface of the water, the water will spout from that orifice, to the distance of twice the height of the orifice above the plane. For instance, if AOBC be a vessel full of water, O an orifice in the side, whose height OD above the horizontal plane DH, is equal to OA, the distance of the orifice from the top of the water; DH the horizontal distance to which the water spouts, will be double of OD, the height of the orifice above the plane. For the spouting water has two motions, one uniform from the pressure of the water in the vessel, in the direction OF perpendicular to the orifice, the other accelerated from the force of gravity in the direction OD perpendicular to DH; which two motions do by no means hinder one another, but by their combination cause the water to spout in the curve of a *parabola*. Now, the velocity wherewith the water moves in the direction OF, being equal to the velocity acquired by a body in falling from A to O, or from O to D; in the same time that it falls from O to D, and by so doing, reaches the horizontal plane, it will be carried in the direction OF, thro' a space equal to twice OD, (inasmuch as all bodies whatever that move

move uniformly, with a velocity equal to that which is acquired by a body in falling in thro' any height do in the same time with that of the fall, describe a space double of that of the fall); consequently, the horizontal distance to which the water spouts, will be equal to twice the height of the orifice above the plane. Thus, from an orifice in the side of a vessel, the depth whereof below the surface of the water is 20 inches, the water will spout to the distance of 38 inches on an horizontal plane, whose distance below the orifice is likewise 20 inches; and where the depth of the orifice below the top of the water is 5 inches, the water will spout to the distance of $9\frac{1}{2}$ inches on an horizontal plane situated at the distance of 5 inches below the orifice; so that in both cases the distances to which the water spouts, are nearly double the distances of the planes below the orifices; and they would be exactly double, were it not that the water is retarded a little by the opposition it meets with from the air.

Exp. 3.

The distances to which water spouts on an horizontal plane, from orifices in the sides of different vessels, the orifices being at equal heights above the plane, are to one another as the square roots of the heights of the water above the orifices.

For since the orifices are at equal heights above the plane, the times of the descent of the water from the several orifices to the plane must be equal; consequently, the horizontal distances to which the water spouts, must be as the velocities wherewith it spouts; but those velocities are as the square roots of the heights of the water above the orifice; consequently, so must the horizontal distances. Thus, if two vessels be so placed, as that the orifices in their sides shall be 20 inches above an horizontal plane, the height of the water in one vessel being 20 inches above the orifice, and in the other 5; the water will spout from the former, to the distance

Exp. 4.

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Pl. 6.
Fig. 13.

of 38 inches, and from the latter, to the distance of 19 inches; but 38 is to 19, as 2 to 1; that is, as the square roots of the heights of the water above the orifices, for the heights are as 4 and 1.

The distance to which water spouts from an orifice in the side of a vessel, whatever be the height of the orifice above the plane, as also of the water above the orifice, may be thus determined; let BR represent an horizontal plane, F an orifice in the side of a vessel at any height above the plane, and AB the height of the upper surface of the water above the plane. On AB as a diameter, describe the semicircle ADB, and at F set off FE perpendicular to AB, and meeting the circle in E. The distance to which the water spouts on the plane BR from the orifice F, is proportional to the line FE.

For, from the nature of motion, the space described, is as a rectangle under the time and velocity; but in this case, the time of the motion is as the square root of FB, and the velocity wherewith the water spouts, is as the square root of AF; consequently, the space thro' which the water runs in the horizontal direction, is as the square root of the rectangle AFB; but by the nature of the circle, the square root of the rectangle AFB is equal to FE; consequently, the horizontal distance to which the water spouts on the plane BR from the orifice F, is as FE.

Hence it follows, that the distance to which the water spouts, is as the sine of the arch AE, whose versed sine AF is equal to the height of the water above the orifice. And, forasmuch as any two sines, which are equally distant from the center, are equal, it follows that the water must spout to the same distance from two orifices as F and L, whose distances from the center are equal; as also, that it must spout to the greatest distance from an orifice in

the center, the sine CD being in that case equal to *radius*, and consequently the greatest.

To confirm what has been said; let a vessel whose height is 16 inches, and which is perforated in the middle, and likewise at the distance of $5\frac{1}{2}$ inches above and below the middle, be filled with water, and set upon an horizontal plane; the water will spout from the middle orifice to the distance of above 15 inches, and from each of the other two, to the distance of about 10 inches.

All things being supposed as before, the distances to which the water spouts, setting aside what little disturbance may arise from the resistance of the air, are equal to twice the sines of the arches, whose versed sines are equal to the heights of the water above the orifices. For, the distance to which the water spouts from the central orifice C, is to the distance to which it spouts from any other orifice as F, as the sine CD is to the sine FE; but forasmuch as the orifice C is as far distant above the plane as it is below the surface of the water, the distance to which the water spouts from that orifice is equal to twice CB, or twice CD; consequently, the distance to which it spouts from F must likewise be equal to twice FE, and so of any other orifice.

Water which spouts perpendicularly upward sets out with such a velocity, as is sufficient to carry it to the same height with the water in the vessel from which it spouts. For the velocity wherewith it sets out, is equal to the velocity acquired in falling down the height of the water; and, in my lecture upon gravity, I shewed, that a body thrown directly upward rises to such a height, whence if it be let fall, it will by the end of the fall acquire the same velocity wherewith it was thrown up; consequently, the water spouts with a velocity sufficient to carry it to an equal height with the water in the reservoir; however, it cannot possibly arrive at that height, by

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Exp. 5.

Exp. 6.

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by reason of the resistance it meets with from the air; which, as it cannot be taken off, must lessen the heights of all jets whatever, so as to make them fall short of the heights in the reservoirs; besides, the water in the uppermost part of the jet, when it has lost all its motion, rests for some time on the part next below it, and by its weight obstructs and retards the motion of the whole column, and thereby lessens its height; and so great is the resistance arising from this cause, as that the jet is frequently destroyed by it, the rising water being by fits and starts pressed down to the very orifice from which it spouts.

Exp. 7. By giving the jet a little inclination, the uppermost parts, when they have lost their motion upward, are made to fall off from the rest, whereby the resistance which arises from their weight is taken off. And this is the true reason why, *ceteris paribus*, such jets as are a little inclined, rise higher than those whose ascents are perpendicular.

The velocity wherewith water flows out of a cylindrical pipe inserted horizontally into the side of a vessel, is as the square root of the height of the water in the vessel above the place of the pipe's insertion directly, and the square root of the length of the pipe inversely; for since the pipe is cylindrical, the velocity wherewith the water flows out at one end, must be equal to the velocity wherewith it flows in at the other; but the velocity wherewith it flows in, is in the proportion laid down; for the pressure of the incumbent water in the vessel, cannot make the water which lies next the orifice flow into the pipe, unless at the same time it drives forward all the water contained in the pipe; for which reason, the water in the pipe may be looked upon as an obstacle which resists and impedes the moving cause. Now, where a cause
acts

acts under the disadvantage of a clog or impediment, the potency of such a cause is increased, either by diminishing the impediment, or augmenting the absolute strength and vigour of the cause itself; where the strength and vigour of the cause is given, the potency thereof increases in proportion as the impediment lessens, and lessens as that increases; and where the impediment is given, the potency of the cause increases, and lessens in proportion to the increase and diminution of the absolute strength and vigour of the cause; consequently, the potency is in a *ratio* compounded of the strength or magnitude of the cause, and of the weakness or smallness of the impediment; that is, it is as the magnitude of the cause directly, and as the magnitude of the impediment inversely; or as the magnitude of the cause applied to the magnitude of the impediment. Now, in the case before us, where the pressure of the water in the reservoir is the moving cause, and the water in the pipe is the impediment, the magnitude of the former is measured by a rectangle under the height of the water, and the orifice of the pipe, and the magnitude of the latter by a rectangle under the orifice of the pipe, and the length thereof; or rejecting the orifice as being ever the same in both, the magnitude of the moving cause, is as the height of the water, and that of the impediment, as the length of the pipe; and therefore, putting H for the height of the water in the reservoir above the place of the pipe's insertion, and L for the length

of the pipe; $\frac{H}{L}$ will denote the pressure of the water in the reservoir, as lessened by the resistance of the water in the pipe; and putting O for the orifice of the pipe, $\frac{HO}{L}$ will express the force which drives the water into the pipe; and forasmuch as the motion generated in any time by a force acting constantly

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constantly and uniformly, is as a rectangle under the force and time; putting T for the time that the water continues to flow into the pipe, $\frac{HOT}{L}$ will

be as the motion generated in the water flowing into the pipe; but the motion generated in the influent water, is as the quantity which flows in, multiplied into the velocity wherewith it flows; and therefore, putting Q and V for the quantity

and velocity, $\frac{HOT}{L}$ is as QV ; or, because the

quantity which flows in, is in a *ratio* compounded of the orifice, time, and velocity; by substituting O , T , V , which denote the orifice, time, and velo-

city, in the place of Q , we shall have $\frac{HOT}{L} = OTV^2$;

and striking out OT from both sides, we shall have

$\frac{H}{L} = V^2$; consequently, V is as $\sqrt{\frac{H}{L}}$; that is, the

velocity wherewith the water flows out of the reservoir into the pipe, and consequently, the velocity wherewith it flows out of the pipe, is as the square root of the height of the water in the reservoir, applied to the square root of the length of the pipe.

Hence it follows, that if the length of the pipe be varied whilst the height of the water in the reservoir continues the same, the quantities discharged in any given time, will be to one another inversely as the square roots of the lengths of the pipe; for since the diameter of the pipe, and the time of the flux are given, the quantities discharged must be as the velocities wherewith they run out, that is, in the inverse *ratio* of the square roots of the lengths of the pipe.

To confirm this by an experiment; let a pipe of 16 feet in length, and half an inch in diameter, be inserted horizontally into the side of a vessel; and let

let the water in the vessel be kept constantly at the height of 3 feet above the place of the pipe's insertion; the pipe when set a running will discharge above $161\frac{1}{2}$ ounces in half a minute; let it then be made shorter by 12 feet, and set a running again, and it will in the same space of time discharge 321 ounces, that is, near twice as much as before; so that the quantities discharged will be to one another reciprocally as the square roots of the lengths of the pipe which in this case are as 4 and 1.

TABLE I.

L	Q	T
1	$436\frac{1}{2}$	
4	321	$\frac{3}{4}$
9	$211\frac{3}{4}$	$1\frac{3}{4}$
16	$161\frac{1}{2}$	3
25	132	5
36	87	9
49	72	14
64	65	$20\frac{1}{2}$
81	$61\frac{1}{2}$	29
100	59	42

TABLE II.

D	Q
1	$87\frac{3}{4}$
4	$88\frac{1}{2}$
9	88
16	81
25	74
36	$67\frac{1}{4}$
49	65
64	$58\frac{3}{4}$
81	56
100	54

June

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XIV.

June the 21st, 1722, I made several experiments concerning the motion and discharge of water thro' pipes, in the following manner.

There was a reservoir of 3 feet in height, which was kept constantly full during the flux of the water; at the bottom was inserted horizontally a pipe of half an inch in diameter, whose length when greatest was 100 feet, but being composed of several pieces, was capable of being made of ten different lengths; which lengths were the squares of the natural numbers. Into this pipe were inserted horizontally (as occasion was) ten other pipes, each of them 6 inches long, and $\frac{1}{4}$ inch in diameter; the places of their insertion into the main pipe were distant from the reservoir the squares of the natural numbers in feet; the axes of the small pipes made an angle of 80 degrees, with that of the main pipe; the reason why they were inserted in such an angle was, that it had been observed that the water flowed out of orifices made in the main pipe nearly in that angle.

In TAB. I. *L* denotes the length of the main pipe (the small pipes not being inserted), *Q* the quantity in *troy* ounces discharged in half a minute of time, *T* the time in seconds which the water took to flow from the reservoir to the extremity of the pipe, the same having been first exhausted.

In TAB. II. *D* denotes the distance from the reservoir, at which the small pipe was inserted into the main pipe; *Q* the quantity in *troy* ounces discharged by the small pipe in half a minute of time, the main pipe being stopped.

TABLE

TABLE III.

1	2	3	4	5	6	7	8	9	10	P	Sum.
84 $\frac{3}{4}$	80	74	65 $\frac{1}{2}$	56 $\frac{1}{4}$	45 $\frac{3}{4}$	41 $\frac{1}{2}$	31 $\frac{1}{4}$	26	6 $\frac{1}{2}$	6 3 $\frac{1}{4}$	148
										59	139
										54 $\frac{1}{2}$	128 $\frac{1}{2}$
										49 $\frac{3}{4}$	115 $\frac{3}{4}$
										48	104 $\frac{1}{4}$
										45	90 $\frac{3}{4}$
										42	83 $\frac{1}{2}$
										43	74 $\frac{1}{4}$
										44 $\frac{1}{2}$	70 $\frac{1}{2}$
										59 $\frac{1}{2}$	66
										8	244 $\frac{1}{4}$
67 $\frac{1}{4}$	57	40	27	19 $\frac{1}{2}$	9 $\frac{1}{2}$	5	5 $\frac{1}{4}$	4	1 $\frac{3}{4}$		264 $\frac{1}{2}$
69 $\frac{1}{4}$	63 $\frac{1}{2}$	56 $\frac{1}{2}$	28	17 $\frac{1}{2}$	7 $\frac{1}{4}$	5 $\frac{1}{2}$	5 $\frac{3}{4}$	5	7		269 $\frac{1}{2}$
69 $\frac{3}{4}$	63 $\frac{1}{2}$	54 $\frac{1}{2}$	29 $\frac{1}{2}$	12 $\frac{1}{2}$	12 $\frac{3}{4}$	10 $\frac{1}{4}$	8 $\frac{3}{4}$	8			267 $\frac{1}{2}$
68	62	50	24 $\frac{1}{2}$	27	17 $\frac{1}{4}$	9 $\frac{3}{4}$	9				271
69 $\frac{1}{4}$	62 $\frac{1}{2}$	50 $\frac{3}{4}$	24 $\frac{3}{4}$	28 $\frac{1}{2}$	18 $\frac{3}{4}$	16 $\frac{1}{2}$					267 $\frac{1}{2}$
69 $\frac{1}{4}$	63 $\frac{1}{2}$	51 $\frac{1}{2}$	25	31	27						263 $\frac{1}{4}$
69 $\frac{1}{4}$	64	55	32	43							249 $\frac{3}{4}$
72 $\frac{3}{4}$	69	65	43								227 $\frac{1}{2}$
76 $\frac{1}{2}$	75 $\frac{1}{2}$	75 $\frac{1}{2}$									166 $\frac{3}{4}$
82 $\frac{3}{4}$	84										87 $\frac{1}{4}$
87											225 $\frac{3}{4}$
	76	50 $\frac{1}{2}$	36 $\frac{1}{2}$	27 $\frac{1}{2}$	13 $\frac{1}{4}$	6	5 $\frac{1}{4}$	6 $\frac{1}{2}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	166
		55	42	30 $\frac{1}{2}$	13 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{3}{4}$	7	2	4 $\frac{1}{2}$	135 $\frac{1}{4}$
			49	32	14 $\frac{3}{4}$	15 $\frac{3}{4}$	6 $\frac{3}{4}$	9	2 $\frac{3}{4}$	5 $\frac{3}{4}$	114 $\frac{3}{4}$
				40	30 $\frac{1}{4}$	17 $\frac{1}{4}$	8 $\frac{1}{2}$	9 $\frac{1}{4}$	3	6	92 $\frac{3}{4}$
					39	21	10 $\frac{1}{2}$	10 $\frac{1}{4}$	3 $\frac{1}{2}$	8 $\frac{1}{2}$	84
						37 $\frac{1}{2}$	16 $\frac{1}{4}$	11	6 $\frac{3}{4}$	12 $\frac{1}{4}$	69 $\frac{1}{2}$
							25	16 $\frac{1}{2}$	7	21	63 $\frac{1}{4}$
								24	8 $\frac{3}{4}$	30 $\frac{1}{2}$	62 $\frac{1}{4}$
									9 $\frac{1}{4}$	53	59
										59	59

In TAB. III. the numbers at the top denote the ten small pipes, P the main pipe, and the numbers below denote the quantities in *tro*y ounces discharged in half a minute of time, by the pipes denoted by the numbers directly above them. The blanks denote, that the pipes denoted by the numbers directly above them at the top, were stopped at the time that the others discharged.

LECTURE

LECTURE XV.

OF PNEUMATICKS.

LECT.
XV.

IN this lecture I shall give an account of the weight and pressure of the air, and of some remarkable effects arising from it.

Exp. 1.

Tho' the weight of the air which surrounds us is not perceived, by reason of the equal pressure which it makes on all parts of our bodies; yet that it is really heavy appears from hence, that vessels when exhausted are less ponderous than when filled with air. Thus a glass bottle, whose contents are nearly 40 cubick inches, being exhausted by means of the air pump, will be found to suffer a sensible loss of weight; when I formerly made the experiment, the loss of weight amounted to ten grains, and the magnitude of the exhausted air I found to be 34 cubick inches; for upon immersing the bottle in water, and opening the valve which covered the mouth, the quantity of water which flowed in and possessed the place of the exhausted air, amounted to 8628 grains, which being divided by $253\frac{1}{3}$, the number of grains in a cubick inch of water, give 34 in the quotient; so that from this experiment it is manifest, that 34 cubicks inches of that air, which more immediately surrounds us, are equal in weight to ten grains; and that the specifick gravity of the same air is to the specifick gravity of water, as ten to 8628, or, as one to $862\frac{4}{3}$; the specifick gravities of bodies equal in bulk, being to one another as the absolute weights of the bodies.

As the air rises above the surface of the earth, it grows rarer, and consequently lighter; a given bulk of air, being lighter at the distance of a mile than at the earth's surface, and lighter again at the distance

distance of two miles, and so on continually. And yet notwithstanding this diminution of gravity in the superior parts of air, so great is the height of the atmosphere, as to render the weight of the whole very considerable; as will appear from the following experiment.

Let a piece of common glass be placed as a cover on the top of the receiver; and upon exhausting the air, the glass will at first be pressed close to the receiver, and at length broken by the weight of the air, which rests upon it. Exp. 2.

While the air continues undiminished in the receiver, it does by virtue of its elasticity press as strongly against the lower surface of the glass, as does the incumbent air by means of its weight upon the upper surface; as shall be shewn hereafter; consequently, as long as the air remains undiminished in the receiver, the weight of the incumbent air can have no sensible effect on the glass; but upon lessening the quantity, and therewith the spring of the included air, the glass being no longer supported from below, is pressed down, and broken by the weight of the air above; and for the same reason, a square glass phial when exhausted cracks and flies to pieces.

From the weight and pressure of the air on the surface of liquors it is, that they are made to rise in exhausted tubes open at one end, as will appear from the following experiments.

Let a glass vessel with mercury be placed under a receiver, and let a tube open at one end be suspended above the vessel in such a manner, as that the open end may at pleasure be let down into the mercury; if then, the air being drawn out of the receiver, the tube be let down, the mercury will not rise therein as long as the receiver continues empty; but upon re-admitting the air, it will immediately ascend. The reason of which is, that upon exhausting Exp. 3.

Q

hausting

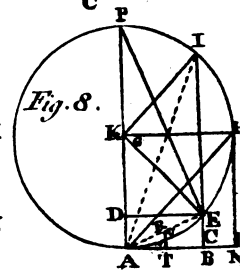
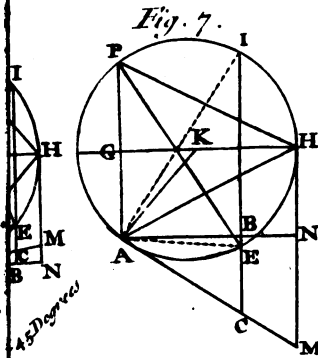
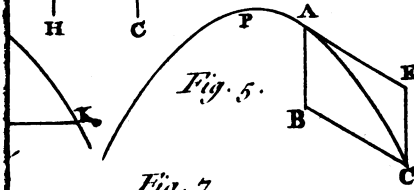
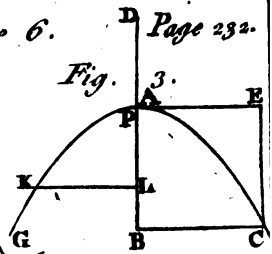
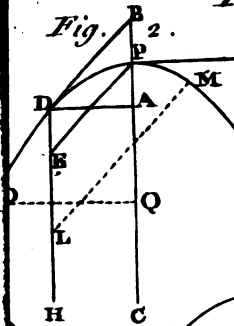
LECT.
XV.

hausting the receiver, the tube is likewise emptied of air; and therefore, when it is immersed in the mercury, and the air re-admitted into the receiver, all parts of the mercury are pressed upon by the air, except that portion which lies beneath the orifice of the tube; consequently, it must rise in the tube, and continue so to do, until the weight of the elevated mercury presses as forcibly on that portion which lies beneath the tube, as the weight of the air does on every other equal portion without the tube. But to proceed to a second experiment of the same kind.

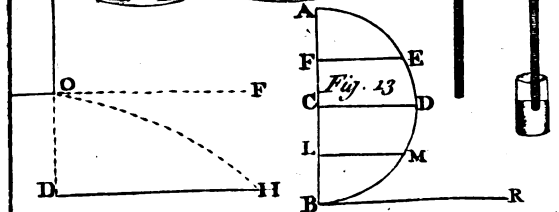
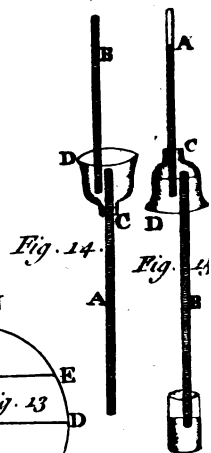
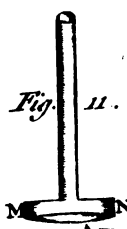
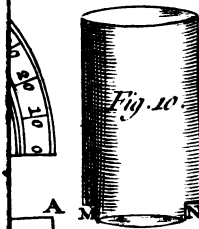
Pl. 6.
Fig. 14.
Exp. 4.

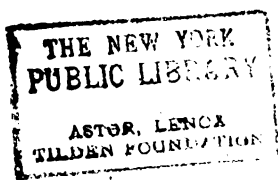
Let two glass tubes as A and B, each above 30 inches long, of which A is open at one end only, but B at both, be so contrived, as by means of screws to be let into the little glass vessel CD, in the manner represented in the figure. A being filled with mercury, and then screwed into the vessel, let mercury be poured into B, till both that and the vessel are full; let then the vessel be inverted; and let the extremity of B be immersed in a vessel of mercury, the mercury will descend thro' B, and continue so to do, till A is emptied, as also so much of the vessel CD as is above the level of the upper orifice of B. This being done, let A be so far unscrewed, as to permit the air to pass between the threads of the screw into the empty part of the vessel; upon the admission of the air, the mercury will rise in the tube A. For, from the circumstances of the experiment it is evident, that the part of A which stands above the level of the mercury remaining in the vessel, is perfectly void of air; consequently, while the mercury all around the tube is pressed by the newly admitted air, that portion which lies beneath the tube suffers no pressure from above; and of course must rise, and continue to rise, until the weight of the elevated mercury becomes a balance to the pressure of the air without.

By



45 Degrees





By the weight and pressure of the air, water is raised in common pumps, and fire engines, as will appear by considering their structures, and the manner in which they work. A B represents the body of a pump, which is commonly an hollow cylinder of wood or lead, C a plug fixed near the bottom of the pump, with an hole in the middle, covered by a leathern valve, so contrived as to open and give way to the water in passing upward, but to shut close and obstruct the passage downward; D a second plug of the same kind and perforated in like manner with the former. This plug is commonly called the *sucker* or *piston*, and being moveable, is drawn up and thrust down at pleasure, by means of the iron rod E to which it is fastened. The sides of the sucker are every where cased with leather, whereby it is made to fit the cavity of the pump so exactly, that neither air nor water can pass between. At some distance above the sucker is an orifice as O in the side of the pump, thro' which the water is discharged at the time of working, in the following manner. The sucker being drawn up, the space between that and the lower plug is left void of air; then forasmuch as the water, which stands about the pump, is every where pressed by the air, except in that part which answers to the hole of the plug, it must there give way, and pass up into the cavity of the pump; and upon depressing the sucker again, as it cannot return downward by reason of the valve, which shuts close upon the hole, and stops the passage, it rises up thro' the sucker, and lodges itself thereon; so that upon the next elevation of the sucker, it is carried towards the top of the pump, and thrown out at the orifice O.

If instead of an orifice above the sucker, we suppose one just above the lower plug, with a valve opening outwardly, so as to suffer the water to flow out, but not to return. And if we suppose the

Q 2

sucker

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fucker to be solid without a perforation, the figure will represent a forcing pump, or fire engine, in which the water rises above the lower plug in the same manner, and from the same cause, that it does in a common pump; and by the pressure made upon it by the sucker when thrust down, it is forced out at the orifice, and that so strongly, as by the help of leathern pipes to be conveyed to the tops of the highest houses.

The air in any particular place does not always continue of the same weight, but is sometimes heavier and sometimes lighter; which plainly argues a variation in the quantity, inasmuch as the gravity of any body is proportional to the quantity of matter which it contains. From what cause this variation arises, is not easy to determine. Dr. HALLEY is of opinion, that the diminution of the quantity of air in any place, is the effect of two contrary winds blowing from that place, whereby the air is carried both ways from it; and of consequence, the incumbent cylinder of air is diminished; as for instance, if in the *German* ocean it should blow a gale of *westerly* wind, and at the same time an *easterly* wind in the *Irish* sea; or if in *France* it should blow a *southerly* wind, and in *Scotland* a *northern*; that part of the atmosphere which is impendent over *England* would, he thinks, be thereby carried off and diminished. He likewise conceives, that the increase of the quantity of air in any place, is occasioned by the blowing of two contrary winds towards that place, whereby the air of other places is brought thither and accumulated. And upon this foot, he endeavours to account for what is commonly observed in this part of the world; namely, that the atmosphere, *ceteris paribus*, is always heaviest upon an *easterly* or *north-easterly* wind. This happens, says he, because, that in the great *Atlantick* ocean, on this side the thirty-fifth degree of *north* latitude, the *westerly* and *south-westerly* winds blow almost always; so that

that whenever the wind comes up here at *east* or *north-east*, it is sure to be checked by a contrary gale, as soon as it reaches the ocean; for which reason, the air over us must needs be heaped up in greater abundance, as often as those winds blow. To confirm this hypothesis of contrary winds being the cause of the variation in the weight of the air, he observes, that within the *Tropicks*, where there are no contrary currents of air, this variation does not obtain; but that the atmosphere continues much in the same state of gravity in all kinds of weather. Now, whether this, or whatever else be the cause of it, most certain it is, that the weight of the air does vary; and so considerable is the variation, that the weight of the air in its heaviest state, exceeds the weight thereof when it is lightest, in the proportion of almost ten to nine.

The changes which the air undergoes as to its gravity, are observed by means of the *Barometer* or weather-glass; which, as it was the invention of *TORRICELLIUS*, is known among the naturalists by the name of the *Torricellian tube or instrument*. It consists of a small glass-tube, about three feet long, closed at one end, which being filled with mercury well purged from air, is inverted into a cylindrical box of timber, wherein some mercury is lodged; upon the inversion some of the mercury falls out, whereby the upper part of the tube is left empty whilst the lower part continues full. Now, forasmuch as it has appeared from experiments, that the suspension of the mercury in the tube is owing to the pressure of the air on the stagnant mercury; the pillar of mercury which is kept up in the tube, must always be equal in weight to a pillar of the atmosphere of the same thickness; consequently, as the weight of the atmosphere varies, the height of the mercury in the barometer must do so too; the mercury constantly rising as the weight of the air increases, and sinking as that

Q 3

lessens.

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lessens. That the minute variations in the height of the mercury may be observed, that part of the tube which lies between the limits of the least and greatest height, to wit, from 28 to 31 inches, is graduated; each inch being divided into ten or twelve equal parts by means of a table, whereunto the tube is fixed; whereon likewise are inscribed in their proper places such constitutions of the air and weather, as have been observed to accompany different heights of the mercury. In contriving this instrument, care must be taken to make the box, which contains the stagnant mercury, so large, as that upon the rising or falling of the mercury in the tube, the height of that in the box may suffer little or no variation; for should the stagnant mercury sink upon the rising of the mercury in the tube, or rise as that sinks, which must be the case where the box is small; the rise or fall of the mercury in the tube will appear to be less than it really is; as for instance; if when the mercury rises half an inch in the tube, it does at the same time fall a quarter in the box, the rise in the tube, which appears to be only half an inch, is in truth three quarters; because the height of the mercury is always to be computed from the surface of that in the box. So, on the other hand, if the mercury by falling half an inch in the tube rises a quarter in the box, the true descent in the tube is three quarters of an inch; inasmuch as the height of the mercury in the tube above the surface of the stagnant mercury in the box, is less after the fall by three quarters of an inch. By making the circular *area* of the box thirty or forty times greater than that of the tube, (which is generally the case, the tubes of most barometers being but one fifth of an inch wide, and the boxes an inch and a quarter). the stagnant mercury in the box may be kept constantly at the same height very nearly, the greatest variation of the height not amounting to more than the tenth or twelfth

twelfth part of an inch, which is inconsiderable.

If the tube instead of being continued directly upward, be bent at the height of 28 inches, in the manner here represented, it is then called an *inflected* or *diagonal barometer*; in which the inclined part A B may constitute an obtuse angle of any magnitude with the perpendicular part B C; but the nearer the angle approaches to a right one, the longer must the inclined part be; for it must be continued until the perpendicular altitude thereof A H, above the horizontal line H B, becomes equal to three inches, which is the difference between the greatest and least height of the mercury in the barometer; otherwise, the mercury will not have room to rise to its utmost height, at such times as the constitution of the air requires it. This barometer shews the minute variations in the weight of the air much more accurately than the former; because the rise or fall of the mercury in the inclined part A B is very sensible, when an alteration in the perpendicular height is scarcely to be perceived. But then the box which contains the stagnant mercury, ought to be much larger in proportion in this than in the former; because in this, a much larger quantity of mercury rises into, and falls out of the tube, upon the changes of the weather.

If the lower part of the tube in the first barometer, instead of being inserted into a box, be turned up in the form of a crook, it is then called a *curved barometer*, in which the crooked part generally terminates in a large bubble open at top. The bubble contains the stagnant mercury, which, as it is pressed upon more or less by the incumbent air, is forced up to a greater or smaller height in the strait part of the tube. In this barometer the bubble ought to be so large in proportion to the tube, as that upon the greatest variation of the height of the mercury in the tube, the height thereof in the bubble may not vary above one tenth of an inch; the

Q 4

necessity

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Pl. 7.
Fig. 2.

Pl. 7.
Fig. 4.

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necessity there is for this, is evident from what was said concerning the magnitude of the box in the first kind of barometer.

Besides the barometers hitherto mentioned, there is the *wheel*, as also the *pendant* or *conical*, *barometer*, and others of various kinds; which, however they may differ as to their structures, do all agree in shewing the changes in the weight of the air, by the rising and falling of the mercury in their tubes; wherein it sometimes, tho' very rarely, descends as low as twenty eight inches; and at others rises to thirty one; the mean height thereof being twenty nine inches and an half. So that a pillar of the atmosphere, in the mean state of its gravity, is equal in a weight to a pillar of mercury of the same thickness, and whose altitude is twenty nine inches and an half. Whence it follows, that an inch square of the earth's surface, or of any other body contiguous thereto, sustains a pressure from the incumbent atmosphere, when in the mean state of its gravity, equal to seventeen pounds, eight ounces, and 374 grains; that being the weight of a square pillar of mercury one inch thick, and twenty nine and an half high.

From this great pressure of the air it is, that two brazen hemispheres, whose diameter is three inches and an half, being laid one upon another, and then exhausted, cling so fast together, as to require above 150 pounds to separate and draw them asunder. And it must be observed, that as the globe in this experiment cannot be perfectly exhausted, that small portion of air which remains within, by expanding itself, contributes to the separation of the hemispheres; for which reason, they are drawn asunder by a less weight than that wherewith the air presses them together; for the diameter of the sphere being three inches and an half, the *area* of its greatest circle is nine square inches and three fifths nearly; consequently, the weight of that pillar of
air

air which presses the hemispheres together, is not less than 162 pounds, even in its lightest state, when the mercury in the barometer stands at the height of 28 inches only. If the globe, after it has been exhausted, be hung within a receiver, upon drawing the air out of the receiver, the lower hemisphere will fall off from the other; which plainly shews, that their cohesion is owing to nothing else but the weight and pressure of the air upon them.

Since the atmosphere even in its lightest state is so ponderous, as that a square pillar of it one inch thick weighs sixteen pounds, nine ounces, and 461 grains; it follows, that a middle-sized man, the surface of whose body is generally allowed to contain about fifteen square feet, sustains a pressure from the atmosphere, when in its lightest state, equal to the weight of 31144 pounds; which pressure on larger bodies, and in heavier states of the air, is still greater; and therefore it may well be asked, how it comes to pass, that we are not sensible of this pressure great as it is. In answer to which it must be observed, that such pressures only are perceived by us, as do in some measure move our fibres, and put them out of their natural situation. Now, the pressure of the air being equal on all parts of the body, cannot possibly move the fibres of any one part, or force them from their situation; but on the contrary, must by reason of its uniformity keep all the fibres in their proper places, and as so doing, cannot be perceived. And that this is the case is evident from hence, that if the pressure of the air be taken off from one part of the body, the pressure on the neighbouring parts immediately becomes sensible. Thus, if a man covers the top of an open receiver with his hand, upon exhausting the receiver, and thereby taking off the pressure of the air from the palm of the hand, he will perceive
a weight

a weight on the back of his hand, and that so great as to put him to pain, and almost endanger the breaking of his hand.

LECTURE XVI.

OF PNEUMATICKS.

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XVI.

BY the elasticity of the air, whereof I intend to treat in this lecture, we are to understand that force wherewith the particles of air expand themselves, and recede from each other, whenever the pressure from without, which keeps them together, is taken off. The method which I shall observe in treating of this force is, First, to shew from experiments, that the air is really indued with such a force; and, Secondly, to enquire into its nature and laws.

Exp. 1.

As to the first, if a little warmed ale, or any other liquor somewhat glutinous, be put into a glass and included in a receiver, upon exhausting the receiver the liquor will rise in large frothy bubbles, and run over the glass.

As the liquor is glutinous, it retains a great number of airy particles, which upon the removal of the outward air, and therewith the pressure which it makes on the liquor, dilate and expand themselves; and forasmuch as they cannot readily extricate themselves from the liquor by reason of its clamminess, they raise it up, and carry it over in the form of froth. And for the same reason it seems to be, that meath, cyder, and most other domestick wines, after they have been bottled a while, upon drawing the cork, spurt out and fly. For as they are all in some measure glutinous, they retain a good quantity of air; which upon corking the bottle is condensed

densified by reason of the condensation of the air which is lodged in the neck of the bottle; besides, by the slight fermentation which such liquors commonly undergo in the bottle, a fresh supply of air is generated, equal in density to the former. When therefore upon drawing the cork, the extraordinary pressure arising from the condensed air in the neck of the bottle is taken off, the air which is dispersed thro' the liquor expands itself with great force, and not finding a ready passage between the parts of the liquor, which by reason of their clamminess do not easily separate, drives the liquor before it in the manner of a spout. But to proceed:

The expansive force of the air is likewise evident from the following experiment. Let a glass bottle of a globular form, and containing a small quantity of water, have a small glass tube open at both ends, inserted into it so far, as that the lower end may be below the surface of the water; and let the insertion be made by means of a screw and a collar of leathers, in such a manner as that no air may pass into or out of the bottle; let then the whole *apparatus* be placed under a tall receiver, and upon exhausting the air out of the receiver, the water will rise up thro' the tube in the form of a jet, which will be higher or lower in proportion as the receiver is more or less exhausted; the reason of which is, that the air included in the bottle, by endeavouring to expand itself, presses upon the surface of the water, which therefore must rise in the tube, as soon as the pressure of the outward air which keeps it down is lessened; and the greater the diminution of that external pressure is, the higher the water must be thrown.

Exp. 2.

If a bladder wherein a small quantity of air is included, be placed under a receiver, upon drawing the air out of the receiver, the bladder will swell, and the swelling will be greater or less in proportion as the receiver is more or less emptied; which plainly

Exp. 3.

LECT.
XVI.Exp. 4.
Exp. 5.
Exp. 6.

plainly argues an expansive force in the included air; as does likewise the bursting of a full blown bladder in an exhausted receiver; as also the cracking of a square glass phial when close stopped.

If a small siphon, having a weight fastened from the hand of the piston, and being closed at the end so as that upon drawing up the piston no air can get in, be suspended in an inverted position with the weight downward, and then covered with a receiver; upon drawing part of the air out of the receiver, the weight will descend, and draw down the piston; and upon the re-admission of the air it will rise again.

When part of the air is drawn out of the receiver, that portion which remains within expands itself, whereby its spring is so far weakened, as not to be able to stand against and support the weight, for which reason the weight descends; whereas, upon the return of the air which was carried off, the elastic force is so far increased, as to become an overbalance for the weight, and upon that account drives it up.

From this and the foregoing experiments it fully appears, that the air is endued with an expansive force. Whence that force arises, and what the law of its action is, comes now to be considered.

The naturalists were formerly of opinion, that the elasticity of the air was owing to the shape and figure of its parts; for they supposed each particle of air to consist of several branches, which being of a pliable nature, were capable of being compressed and squeezed together by any outward force, and of expanding and spreading themselves abroad upon the removal of the compressing force; and this has been thought by some to be a full and satisfactory account. But that great philosopher Sir ISAAC NEWTON was of opinion, that the expansive force of the air is altogether inexplicable on the foot of this,

this, or indeed any other hypothesis, except that of the air's being endued with a repelling power, whereby the particles recede and fly from each other. His words are these:

“ That there is a repulsive virtue, seems also to follow from the production of air and vapour
 “ The particles when shaken off from bodies by heat or fermentation, so soon as they are beyond the reach of the attraction of the body, receding from it, and also from one another with great strength, and keeping at a distance, so as sometimes to take up above a million of times more space than they did before in the form of a dense body; which vast contraction and expansion seems unintelligible, by feigning the particles of air to be springy and ramous, or, rolled up like hoops, or by any other means than a repulsive power.”

Now, supposing this to be the case, and that the repelling power of each particle exerts itself on the next adjacent particles only, as Sir ISAAC seemed to imagine, I shall shew you what the law of this repelling power is, or, in other words, how this power is varied, by varying the distance of the particles; and in order thereto, shall lay down the following PROPOSITION.

PROP. *If a fluid be composed of particles endued with a repulsive power, so as that each particle repels those, and those only, which are next it, and if the force wherewith two adjacent particles repel each other, be in a given reciprocal ratio of the interval of their centers; that is, putting I for the interval of the centers, and P for the index of the given power of that interval; I say, if two adjacent particles repel each other with a force that is as $\frac{1}{I^P}$, the force which compresses the fluid, is as the cubick root of that power of the*

LECT. *the density of the fluid, whose index is P increased by*
 XVI. *2, or $P + 2$; that is, putting F for the compressing*
 { *force, and D for the density of the fluid, F is as D*
raised up to the power whose index is $\frac{P+2}{3}$.

Exp. 7. For the proof of this, let a portion of the fluid
 be contained in a given cubick space, whose upper
 Pl. 7. surface is denoted by the square ABCG, the com-
 Fig. 5. pressing force being applied to that surface.

The elastick force of the fluid, which withstands
 the compressing force, and is exactly equal thereto,
 is the force of those parts only which compose the
 upper surface; because the repelling forces of the
 particles are supposed to exert themselves on those
 particles only which lie next them, and not to ex-
 tend to particles more remote. But the force of
 the superficial parts is as the number of particles in
 the surface, and the force wherewith any two adja-
 cent particles repel each other conjointly. Now, the
 number of particles in the given square surface, is
 reciprocally as the square of the distance of the
 centers of two adjacent parts; that is, as $\frac{1}{I^2}$; and
 by supposition, the force wherewith two parti-
 cles repel each other, is as $\frac{1}{I^P}$; and therefore, the
 elastick force of the fluid, and of consequence the
 compressive force, or F, is as $\frac{1}{I^{P+2}}$. The density
 of the fluid contained in the given cubical space, is
 inversly as the cube of the distance between the
 centers of the particles; that is, D is as $\frac{1}{I^3}$, and I
 is as $\frac{1}{D^{\frac{1}{3}}}$; and therefore, by substituting $\frac{1}{D^{\frac{1}{3}}}$ in the
 room of I, F is as $D^{\frac{P+2}{3}}$; that is, the compressive
 five

five force is as the cube root of that power of the density of the fluid, whose index is $P + 2$.

COROL. From this proposition it follows, that if the density of an elastick fluid be as the force which compresses it, the particles repel one another with forces that are inversly as the distances of their centers.

For since F is as $D, \frac{P + 2}{3}$ is equal to unity, and so likewise is P ; consequently, the P power of I , whose reciprocal expresses the repulsive force of the particles, is equal to I .

Hence the particles of air must repel one another with forces reciprocally proportional to the distances of their centers, because the density of the air is proportional to the force which compresses it; as will appear from the following experiment.

Let an inflexed tube as AB , open at both ends, be filled up with mercury to some small height, suppose DC ; then stopping the end B , so as that the air may not get out when it is compressed, and measuring the length of BC , that part of the shorter leg that is filled with air, which air, it is evident, is compressed by the weight of the atmosphere; let mercury be poured in at A , till the height thereof in the longer leg above the height of the same in the shorter, becomes equal to the height at which it stands in the barometer, by which means the air in the shorter leg will be compressed by twice the weight of the atmosphere; let then the length of that part of the leg which is possessed by the air under this double pressure be measured, and it will be found to be just one half of BC ; whence it appears, that the spaces which a given quantity of air possesses under different pressures, are reciprocally proportional to the pressures; and consequently, inasmuch as the densities of bodies where the quantity of matter is given are reciprocally as their magnitudes,

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Exp. 8.

Pl. 7
Fig. 6.

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Fig. 7.

tudes, the density of the air is directly as the compressing force. From this property of the air, COTES has deduced a method for determining the density thereof at any height; what he has delivered concerning this matter, is contained in the 5th chapter of his *Harmonia Mensurarum*, which I shall endeavour to explain to you; and in order thereto, shall lay before you such properties of the logarithmick curve, as I shall have occasion to make use of, referring you for their demonstrations to the forementioned author, and others who have wrote of that curve. Let then BDGI be a logarithmick curve, AH its asymptot, that is, a right line so situated with respect to the curve, as not to meet it till it is drawn to an infinite, or rather indefinite length, BA, DC, and GF, ordinates, that is, right lines perpendicular to the asymptot at the points A, C, and F, and terminating in the curve. BC a tangent to the curve at the point B. The properties of this curve, which I shall have occasion to mention, are these four.

First, Any portion of the asymptot intercepted between two ordinates, is the logarithm or measure of the *ratio* which those ordinates bear one to the other; thus AC measures the *ratio* of BA to DC; and CF measures the *ratio* of DC to GF; and so likewise AF measures the *ratio* of BA to GF. And if AC, AF, and AH be in arithmetick proportion, then DC, GF, and IH are in geometrick proportion; and if any portion of the asymptot be a given quantity, then is the *ratio* of the two ordinates, which intercept that portion, likewise given.

Secondly, That portion of the asymptot as AC, which is intercepted between a tangent and an ordinate, drawn to the same point of the curve as B, is a given quantity, or, in other words, to whatever point of the curve the tangent and ordinate are drawn,

drawn, the portion of the asymptot which they intercept is always of one and the same length. The portion so intercepted is called the *subtangent*; and it is the module, or that which regulates the magnitudes of all the logarithms in the same system; for they are greater or less in proportion to the magnitude of the subtangent; so that if in two logarithmick curves, the subtangent of one be double or triple the subtangent of the other, the measures of the same *ratios* are likewise twice or thrice as great in the former as they are in the latter.

Thirdly, The indefinite *areas* comprehended between the curve and the asymptot, drawn on to an indefinite length beyond HI, are to one another as the ordinates which bound them in their widest parts; thus, the indefinite *areas* B A H I, D C H I, and G F H I, are to one another as the ordinates B A, D C, and G F.

Fourthly, The indefinite *area* B A H I, is equal to the parallelogram B A C E, comprehended under the ordinate B A, and the subtangent A C.

These things being premised, let A B represent the earth's surface, and let A H be a line perpendicular thereto; then, forasmuch as the densities of the air at different heights, are as the pressures of the incumbent atmosphere, and the ordinates in the curve, as the indefinite *areas* which lie beyond them; if the indefinite *area* B A H I be made to denote the weight or pressure of all the air, and A B its density at the surface of the earth, then, by the third property of the curve, the indefinite *area* D C H I will denote the weight or pressure of all the air which lies above C, and the ordinate D C will denote the density of the air at that height; and thus it is with regard to any other height, so that at all heights, the densities of the air will be denoted by the respective ordinates; wherefore, by the first property of the curve, the difference be-

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tween any two heights, is the measure of the *ratio* which the densities of the air bear to one another at those heights; thus CF measures the proportion which the air's density at the height C bears to its density at the height F . Let us now suppose the force of gravity to cease, and that the air is so compressed by some external force, as to be every where from top to bottom of the same density, as it is at the surface of the earth; its weight or pressure, which before was denoted by the indefinite *area* $BAHI$, may now be denoted by the parallelogram $BACE$, inasmuch as by the fourth property of the curve, that *area* and this parallelogram are equal. Since then two fluids which balance each other must have their heights inversely as their specifick gravities, if we put unity to denote the specifick gravity of the air at the surface of the earth, and say, as unity to 11890, which is the specifick gravity of mercury with respect to that of air, so is $2\frac{1}{2}$ feet, which is the height of the mercury in the barometer, to a fourth number, we shall have 29725 feet for the height of the homogeneous atmosphere; and this height is equal to the subtangent AC . For since the pressure of this homogeneous atmosphere is as its density into its height, and likewise as the rectangle $BACE$; and since the density is denoted by BA , the height must be denoted by AC the module in this system of logarithms. Hence we have a method for determining the density of the air at any height; for putting H to denote the height at which the density of the air is required, by the second property of the curve, we have this analogy, as the integral number marked A , which is the module of this system, is to the fractional number marked B , which is the module of BRIGGS's system, so is H expressed in feet, to a fourth number, which in BRIGGS's tables is the logarithm of the *ratio* of the

the density of the air at the earth's surface, to its density at the height H , answerable to which in the tables is the natural number expressing that *ratio*.

$$\begin{array}{ccccccc}
 A & & B & & & & \\
 29725 & : & 0.43429448 & :: & H & : & \frac{0.43429448 \times H}{29725} \\
 & & B & & C & & D \\
 \frac{26400 \times 0.43429448}{29725} & = & 0.385661. & & & & 2.4303.
 \end{array}$$

Thus, for instance, if the density of the air at the height of five miles, or 26400 feet, be required, by multiplying that number by the fractional number marked B, and dividing the product by the integral number marked A, we shall have the logarithm marked C; answerable to which in the tables is the natural number marked D, expressing the *ratio* of the air's density at the surface of the earth, to its density at the height of five miles; whence it appears, that at the surface of the earth, the air is denser than it is at the height of five miles, in the proportion of almost $2\frac{1}{2}$ to one; but then, this is on supposition that the force of gravity continues the same at all heights, whereas in truth, that force decreases in the recess from the earth's center in the duplicate *ratio* of the distance, which causes the densities of the air at different heights to be somewhat different from what they would be in case the force of gravity did not vary.

In order therefore to determine the densities more accurately, let S be the earth's center, and AB , equal to AB in the last figure, the earth's surface, and let F be the height at which the density of the air is required; let SK be a third proportional to SF and SA , and at the point K , let the ordinate

R 2

K G

Pl. 7.
Fig. 8.

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KG be drawn, denoting the density of the air at F, then taking the point M at an indefinitely small distance above F, let SL be a third proportional to SM and SA; and at the point L, let the ordinate LN be drawn, denoting the density of the air at M; this being done, it will appear, that the curve BGN, which passeth thro' the points G and N, is the same logarithmick curve with the former, but in an inverted position. For since SL is to SA, as SA to SM, and since SK is to the same SA, as SA to SF, then by equality of *ratio*, SL is to SK, as SF to SM, and by division and permutation, KL is to FM, as SK to SM; or because FM is indefinitely small, as SK to SF; that is, as SA^3 to SF^3 ; whence reducing that analogy into an equation, and dividing by SF^3 , we shall have $KL = \frac{SA^3 \times FM}{SF^3}$;

and rejecting SA^3 , as being a given quantity, we shall have KL as FM directly, and SF^3 inverfly; but FM is as the quantity of air in the indefinitely little space FM, and SF^3 inverfly is as the gravitation of the same air, and KG is as its density; consequently, the rectangle under KL and KG, or the *area* KGNL, is as the gravitation, the quantity, and density of that air conjointly, that is, as its pressure on the air beneath it; and the sum of all the similar *areas* below KG, is as the sum of all the pressures above F, that is, as the density of the air at F, or as KG, which denotes that density; and KGNL, which is the difference of the two sums of all the similar *areas*, one of which sums begins from the point K, and the other from the point L, is as the difference of the air's densities at F and M, that is, as $KG - LN$. Let now KL be given; that is to say, let the small portion intercepted between KG and LN be always of one and the same length, in whatever parts of the line AS the points K and L are taken; then KG will be as the *area* KGNL,
and

and consequently, as $KG - LN$; whence by division, KG will be as LN , so that the *ratio* of KG to LN is given, and of course the given line KL will be the measure of that given *ratio*; whence by the first property of the logarithmick curve, the curve which passeth thro' the points G and N is a logarithmick curve; and it is also the same with the former; for taking AO the height above the earth's surface indefinitely small, it is evident, that the force of gravity is the same at O that it is at A , consequently, the density of the air at O will come out the same, whether the law of gravity be taken into the consideration or left out; let then the ordinate OP be drawn in the former curve, and at the same distance from A in the latter curve, let the ordinate PQ be drawn. Now, since one and the same density of the air at the earth's surface is denoted in both curves by the equal ordinates BA , it is evident, that the ordinates OP and PQ , which in the two curves denote one and the same density at O , must likewise be equal; whence it follows, that both curves have the same curvature, as also the same inclination of their tangents at the points B , and their subtangents equal; that is, the latter curve is the same with the former, but in an inverted position. Now, forasmuch as BA in the latter curve denotes the density of the air at the surface of the earth, and KG its density at F , it is evident by the first property of the curve, that in this system, AK is the measure of the *ratio* which the density at the surface has to the density at F ; the first thing therefore which must be done, in order to discover the density at F , is to find out the line AK , and this is done by diminishing AF in the same proportion that the earth's semidiameter SA is less than SF , the distance of F from the earth's center; for by the construction, SF is to SA , as SA to SK ; whence by division, $SF : SA :: AF : AK$. AK being thus obtained, let it be called H ; then, by the same

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process

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process as before, we may discover the density of the air at the height F.

$$\begin{array}{rcccl}
 & & E & \text{feet.} & \\
 4005 : 4000 :: 5 : \frac{4000 \times 5}{4005} = 4.99375 = 26367. & & & & \\
 & B & F & G & \\
 \frac{26367 \times 0.43429448}{A} = 0.385232. & & & 2.4279. &
 \end{array}$$

29725

For instance, if the density of the air at the height of five miles be required as before; then by saying, as 4005 miles, that is SF, is to 4000 miles, that is SA, so is five miles, that is AF, to a fourth, we shall have the number marked E, expressing miles, and parts of a mile, equal to 26367 feet, which being multiplied by the fractional number marked B, and the product divided by the integral number marked A, we shall have the fractional number of BRIGGS's tables marked F, answerable to which is the natural number marked G, expressing the *ratio* of the air's density at the surface of the earth, to its density at the height of five miles. After the same manner may the *ratio* of the air's density at the surface, to its density at any height be computed. The result of such computations I have set down in the annexed table; the first column of which contains the heights of the air in *English* miles, whereof 4000 make a semi-diameter of the earth. The numbers in the second column express the *ratio* of the air's density at the surface, to its density at the respective heights, and they likewise denote the rarity, or expansion of the air at those heights. The third column contains the densities and compressions at the several heights. The numbers at the bottom of the second column included in crotchets denote, that so many figures are to be annexed to the five preceding, and those included in the crotchets at the bottom of the third column

column denote, that so many decimal cyphers are to be prefixed to the five following figures.

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Heights of the air in Eng- lish miles.	Rarity and expan- sion.	Compreffion and density.
0	— — — — 1. —	1
$\frac{1}{4}$	— — — — 1.0454	0.95676
$\frac{1}{2}$	— — — — 1.0928	0.91509
$\frac{3}{4}$	— — — — 1.1424	0.87535
1	— — — — 1.1943	0.85405
$1\frac{1}{4}$	— — — — 1.2429	0.80456
$1\frac{1}{2}$	— — — — 1.3052	0.76616
$1\frac{3}{4}$	— — — — 1.3644	0.73290
2	— — — — 1.4263	0.70118
$2\frac{1}{4}$	— — — — 1.4871	0.67244
$2\frac{1}{2}$	— — — — 1.5586	0.64160
$2\frac{3}{4}$	— — — — 1.6292	0.61379
3	— — — — 1.7031	0.58716
$3\frac{1}{4}$	— — — — 1.7883	0.55919
$3\frac{1}{2}$	— — — — 1.8596	0.53775
$3\frac{3}{4}$	— — — — 1.9460	0.51387
4	— — — — 2.0336	0.49173
$4\frac{1}{4}$	— — — — 2.1257	0.47043
$4\frac{1}{2}$	— — — — 2.2221	0.45002
$4\frac{3}{4}$	— — — — 2.3226	0.43012
5	— — — — 2.4279	0.41187
10	— — — — 5.9182	0.16897
20	— — — — 34.288	0.029164
30	— — — — 198.34 —	0.0050418
40	— — — — 1136. —	0.00088028
50	— — — — 6449.2 —	0.00015505
100	33584[3] —	0. [7]26798
400	11271[24] —	0. [28]88723
4000	19316[150] —	0. [154]51770
40000	33097[276] —	0. [280]30214
400000	32859[301] —	0. [305]30433
1000000	12002[303] —	0. [307]45450
Infinite.	37311[304] —	0. [308]76802

R 4

COROL.

COROL. Since SF is by construction equal to SA, and since from the nature of musical proportion, the quotients arising from the division of one and the same quantity by quantities in arithmetick progression, constitute a series of musical proportionals, it follows, that if several distances from the earth's center as SF, be taken, in musical progression, their reciprocals as SK, must be in arithmetick progression; and by the first property of the logarithmick curve, the densities of the air as KG, must be in geometrick progression.

Since the density of the air is proportional to the compressing force, and since the compressing force is equal to the elastick force, it is manifest, that if the density of the air be increased, the elasticity will likewise increase in the same proportion; and on this principle are founded artificial fountains, which play by means of condensed air; they are of two kinds, single and double. The single fountain is made of brass, and is every where shut, excepting that thro' the middle of the basin BB, there passes down a pipe PP, whose lower end reaches nearly to the bottom of the fountain, and to the upper end is fitted a stopcock; by help of which the pipe may be shut or opened at pleasure.

Pl. 7.
Fig. 9.

Exp. 8. Some part of the fountain as ADC, being filled with water poured in thro' the pipe, a condensing or forcing syphon is screwed to the top of the pipe above the cock, by means whereof a great quantity of air is thrown into the pipe; which as it cannot return back, by reason of a valve which shuts close upon the hole of the syphon, forces its way thro' the water into the upper part of the fountain, and there remains in a state of condensation, greater than that of the outward air. When therefore the condenser is taken off, and the cock opened, the included air pressing strongly on the water which lies beneath

beneath it, throws it up thro' the pipe, and thereby makes a jet.

The force wherewith the water is thrown up, is proportional to, and may be expressed by the excess of the density of the included air above that of the external air. For if the included air be equally dense with that without, its elastick force must be equal to the compressive force of the atmosphere; consequently those two forces will balance one another, and the water will continue at rest, being pressed as strongly downward by the weight of the external air, as it is upward by the expansive force of the included air; but if the included air be more dense than the external, its elastick force will exceed the compressive force of the atmosphere, in the same proportion that its density exceeds the density of the outward air; consequently, that part of the expansive force of the included air which raises the water, is proportional to, and may be expressed by, the excess of the density of the included air above that of the external air. So that putting F for the force which raises the water, D for the density of the included air, and 1 for the density of the air without, F is as $D - 1$.

The height in feet to which the water rises, setting aside all impediments, is equal to the product arising from the multiplication of 33 into the excess of the density of the included air above that of the outward air; that is, putting H for the height of the jet, and x for 33, $H = x D - x$.

For as water which is driven out of a reservoir by the pressure of the incumbent water, if it spouts directly upward, rises to the same height with the water in the reservoir; so if it be driven by any other force, it must rise to an equal height with a pillar of water whose pressure is equal to that of the driving force; forasmuch therefore as the atmosphere makes an equal pressure with a height of water of 33 feet,

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33 feet, the water will be thrown to the height of 33 feet by the compressive force of the atmosphere; wherefore if we put 1 for the pressure of the atmosphere, and say, as one is to 33 or x , so $D - 1$, which expresses that part of the pressure of the included air which drives out the water, to a fourth proportional, we shall have $x D - x$, or $x \times D - 1$, for the height to which the water is thrown; whence it appears, that if $D - 1$ be equal to unity, which is the case when the air within is as dense again as that without, the water will rise to the height of 33 feet; and if $D - 1$ be equal to 2, which is the case when the included air is thrice as dense as the external, the height of the jet will be 66 feet, and so on.

Pl. 7.
Fig. 10.

Exp. 9.

The double fountain consists of two single fountains, whose bottoms are fastened to an hollow brass cylinder, one at each end, in the manner represented in the figure, wherein AA and BB denote the two fountains with their basons; CC the hollow cylinder, which plays upon the pins DD as upon an axle; each has a pipe as P , whose lower end reaches nearly to the bottom of the fountain. From the bason of the fountain AA , there issues another pipe as T , which passing thro' AA , and likewise the hollow cylinder CC , without communicating with either, opens at E into the fountain BB . And in like manner, such another pipe issuing from the bason of BB , and passing thro' that fountain and the cylinder, opens into the fountain AA . The hollow cylinder being placed in an upright posture by means of the carriage which supports it, and the pipes of the lower fountain being stopped, water is conveyed into it thro' the pipe T , which issues from the bason of the upper fountain; by the running in of the water, the air contained in the lower fountain is crowded into a smaller space, and thereby condensed; if then both the pipes of the upper fountain

fountain be stopped, and the lower fountain be brought into the place of the upper, by turning the cylinder on its pins, the water which it contains will fall to its bottom, and the lower end of the pipe P will be immersed therein, in the manner represented in the upper fountain; so that upon opening that pipe, the water will be driven thro' it by the expansive force of the condensed air; and as it falls into the basin, it will be conveyed thence by the pipe T into the lower fountain; and when the upper is exhausted and ceases to play, then stopping its pipes, and changing the places of the fountains as before, the other may be set a going in the same manner.

LECTURE XVII.

OF SOUNDS.

IN this lecture I shall first explain to you the NATURE OF SOUNDS, and then treat of the VIBRATIONS OF MUSICAL STRINGS.

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That SOUNDS have a necessary dependance on the air, will appear from the following experiment.

Let a bell be placed under a receiver in such a manner as that it may be rung at pleasure; and upon drawing the air out of the receiver, the sound of the bell will grow less and less audible in proportion to the degrees of exhaustion, so as at last almost to die away, and scarcely to be heard at all; and upon re-admitting the air, the sound will revive again, and increase in proportion to the quantity of air that is taken in. Exp. 1.

As this experiment proves the air to be necessary to the production of sounds, so the tremblings which great guns, bells, drums, and many other sounding bodies communicate, by means of the interme-

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intermediate air, to such bodies as are near them, plainly shew, that sounds depend on tremulous motions of the air; which therefore I shall endeavour to explain to you, together with the cause and manner of their production. When the parts of a bell, a musical string, or any other elastick body are set in motion by a stroke, they vibrate, that is, they go forward and return backward alternately thro' very short spaces; in going forward they propel, and thereby compress and condense the air which lies next them; and in returning backward, they suffer the compressed air to recede and expand itself, so that the parts of the air which are contiguous to the trembling body, go and return in the same manner with the parts of the body; and as they are endued with a repulsive power, they must by means thereof excite the same vibrations in those parts which lie next beyond them; and these again must in like manner agitate the parts beyond them, and so on continually; so that by one single vibration of an elastick body, a motion is excited in the air, and propagated directly forward, by which some parts go forward, whilst others return back, and that alternately, as far as the motion reaches.

Pl. 7.
Fig. 11.

That this motion may more readily be conceived, let ST represent an elastick string, stretched and made fast at both ends; and by a force applied to the middle point H, let it be drawn into the position SET; upon the removal of the force which inflects it, it will by virtue of its elasticity return to its former position SHT; and forasmuch as the restitutive force acts constantly upon it during the time of its motion from E to H, its motion thro' that space must be continually accelerated, and the velocity thereof must be greatest at H. When the string has recovered the position SHT, it will not remain therein; but by virtue of the velocity acquired in moving from E to H, it will be carried forward till it has moved thro' a space as HK, equal to

to EH, and then its motion forward will cease; for as it moves towards K, the elastick force acts continually upon it in drawing it back; and by so doing, retards the motion from H to K, in the very same manner that it accelerated the motion from E to H; consequently, by the time that the string has moved from H to K, it will have lost all that velocity which it acquired in moving from E to H; as soon as it ceases to go forward, it will be brought back again from K to H by the force of elasticity, with an accelerated motion, in the very same manner as it was at first from E to H; and when it has arrived at H, it will by virtue of the acquired velocity go on to E, with a retarded motion, in the same manner as it did from H to K. The motion of the string from E to K and back again, is called a *vibration*; and it is evident from what has been said, that setting aside all external impediments, a string which has made one vibration, must continue to vibrate for ever thro' the same space; but, whereas it meets with continual resistance from the air, the space thro' which it vibrates must on that account grow less and less continually, and at length vanish; and yet, notwithstanding this variation in the space, the times of the vibrations are all equal, as I shall demonstrate before the close of this lecture; but I take notice of it in this place, because one of the chief properties of the pulses of the air, whereof I shall have occasion to make mention presently, has a necessary dependance thereon.

When the string is drawn into the position SET, if we suppose A, B, C, and so forth, to be particles of air placed in a right line one beyond another, and that the distance of the first particle from the string at E, is equal to the interval of any two adjacent particles, as it must needs be, on supposition that the particles of the air fly from other bodies with the same force that they repel one another; upon letting

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letting the string go, as it cannot move forward without approaching to the particle A, it must in the very next instant after it begins its motion, propel that particle; which, for the same reason, must in the next instant after it begins to move propel the particle B, and that must in the same manner propel C, and C propel D, and so on; so that the string, and the several particles of air taken in their order, will begin to move forward successively one after another, at very small intervals of time. And whereas the string is accelerated in its motion from E to H, and retarded in its motion from H to K, the particle A must likewise be accelerated in one half of its progress, and retarded in the other; for since A is equally distant from the string, and from B, before the vibration commences, and since it begins to move forward a little later than the string; it is evident, that upon the first motion of the string, the distance between that and A, must become less than the distance between A and B; and forasmuch as the increments of velocity which are continually generated in the string by the action of its elasticity, are not communicated to the particle A, in the instant of time wherein they are generated, but a little later; it is manifest, that the string during its motion from E to H, must continually be nearer to A than A is to B; and consequently, must act more forcibly in driving A forward, than B does in driving it backward, and by so doing accelerate its motion. After the string has arrived at H the middle point of its progress, and ceased to be accelerated, in the very next moment A likewise reaches the middle point of its progress, and ceases to be accelerated, being driven as strongly backward by B, as it is forward by the string. But however, by virtue of the acquired motion it continues to go forward, but with a retarded motion; and is at length stopped by the

repulsive

repulsive power of B; in the same manner that the string in moving from H to K is retarded, and at last stopped by the action of its elastick force. After the string has reached K, the utmost limit of its progress, in the very next moment does A likewise reach the utmost limit of its progress, and then turning back, pursues the string, which had likewise turned back the moment before. And as the string is accelerated during its return from K to H, and retarded from H to E; so the particle A, during the first half of its return, being nearer to B than it is to the string, must be accelerated; and during the latter half, being nearer to the string, is thereby retarded, and at length stopped upon its arrival at the place from whence it set out, which happens immediately after the string has returned to E; and there it continues at rest, unless by a second vibration of the string it be again driven forward in the same manner as before. As this particle is made to go and return thro' a very short space, by the impulse of the string, so likewise are the several succeeding particles, by the impulses of the foregoing; and as the string and the several particles taken in their order, begin their motions forward, successively one after another at very small intervals of time, so likewise do they begin to return in their order at the same intervals of time; whence it follows, that some of them must go forward, at the same time that others return back. As the particles which go forward begin their motions successively one after another, they must necessarily come nearer together; that is, they must be condensed. And it must be observed, that the condensation goes forward continually; for in the very next instant after any particle as D, has made its nearest approach to E, E must make its nearest approach to F; and in the next instant F must make its nearest approach to G, and so on continually; so that the condensation

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fation must pass forward successively in a regular manner thro' the several particles of air.

But that I may explain this vibratory motion of the air more particularly, it must be observed, that as the string during the first half of its progress from E to H is continually accelerated, its distance from the particle A, must constantly grow less; and forasmuch as during the latter half of its progress from H to K, it is continually retarded, and that in the same uniform manner that it was accelerated from E to H, its distance from A must constantly be enlarged, and that in the same regular manner that it was diminished during the progress of the string from E to H; so that by the time it has arrived at K, the utmost limit of its progress, it is just as far distant from the particle A, as it was when it first set out. Upon the return of the string, inasmuch as it is continually accelerated from K to H; its distance from the particle A must still be enlarged; and forasmuch as it is retarded in its motion from H to E, in the very same manner as it was accelerated from K to H, its distance from A must constantly grow less in the same regular manner that it was enlarged during its motion from K to H; so that upon its return to E, it is again just as far distant from the particle A, as it was at its first setting out. From what has been said, it is evident, that the string during the time of its progress is always nearer to the particle A, than it was before its motion began, and that its least distance from the particle is at H, the middle point of its progress; it is likewise manifest, that during the time of its return, it is always more distant from the particle than it was before its motion began; and that its greatest distance from the particle is at H, the middle point of its return. And what has been thus shewn of the string with respect to the particle A, is in like manner true of that particle with respect

spect to the particle B, and of B with respect to C, and so on of every particle, with respect to that which lies immediately beyond it, as far as the motion reaches; so that each particle with regard to that which lies immediately beyond it, is in a state of condensation during its progress, and of rarefaction during its return, its greatest condensation being at the midst of its progress, and its greatest rarefaction at the midst of its return. What proportion these rarefactions and condensations bear to the density of the air in its natural state, in every point of that small space thro' which a particle of air vibrates, shall be shewn in my next lecture, as also the law of this vibratory motion.

As the parts which go forward, do in their progressive motion strike such obstacles as they meet in their way, they are for that reason called *pulses*; and the sensations which are excited in the mind by the strokes of these pulses on the drum of the ear are called *sounds*; so that sounds as considered in their physical causes, are nothing else but the pulses of the air. In order therefore to explain the nature of sounds, I shall lay before you the chief properties of these pulses.

The first of which is, that they are propagated from the trembling body all around in a sphaerical manner. For tho' the parts of the body, by whose vibrations the pulses are generated, do go and return according to certain directions, yet forasmuch as every impression which is made on a fluid is propagated every way throughout the fluid, whatever be the direction wherein it is made, the pulses must spread and dilate, so as to form themselves into concentrick sphaerical surfaces, or rather thin shells, whose common center is the place of the sounding body. And hence appears the reason why one and the same sound may be heard by several persons, tho' differently situated with respect to the sounding body.

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A second

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A second property of the pulses is, that they grow less and less dense as they recede from the sounding body, and that in the same proportion with the squares of their distances from the body. For whatever be the force wherewith the sounding body acts on the first sphaerical shell of air, with the very same force does that shell act upon the second, and that again upon the third, and so on continually; so that the force which condenses the air in the several shells is given; consequently, the condensations which it produces in those shells must be inversely as the resistances it meets with; but the resistances are as the shells; and therefore, since those increase continually in the same proportion with the squares of their distances from the center, their densities must decrease in the same manner.

By reason of this diminution in the densities of the pulses, those which are farther removed from the sounding body, make slighter impressions on the drum of the ear, than those which are less distant; and hence it is, that sounds grow less and less audible, the farther they go from the sounding body; and at certain distances become so weak as not to be heard at all.

A third property of the pulses is, that all of them whether denser or rarer, move equally swift, so as to be carried thro' equal spaces in equal times, as I shall demonstrate in my next lecture.

From this property it follows, that all sounds, whether they be loud or low, grave or acute, move equally swift, the softest whisper making equal speed with the noise of a cannon, or the loudest thunder-clap; and it has been found by experiment, and I shall likewise demonstrate in my next lecture, that sounds move at the rate of 1142 feet in a second of time or thereabouts; for the velocity is not precisely the same in all seasons of the year, but is somewhat greater in *summer* than in *winter*, on account of the heat which renders the air more elastic

tick in proportion to its density, than it is in the cold *winter* season.

A fourth property of the pulses is, that all those which are excited by the vibrations of one and the same body, are at equal distances from one another. For since each pulse is excited by one single vibration of the sounding body, and since all the pulses move with equal and uniform velocities, it is manifest that they must succeed one another at distances proportional to the times of the vibrations; but the times of the vibrations of one and the same body are all equal; consequently, the intervals of the pulses are so too. And it must be observed, that the interval between two pulses, which is by some called the *length*, and by others the *breadth* of a pulse, is that space thro' which the motion of the air is carried, during the time wherein any one particle performs its vibratory motion, in going forward and returning back.

On the intervals of the pulses depend the tones of sounds; and here I must observe to you, that all the variety there is in sounds, respects either their *strength* or their *tone*; with regard to their strength, they are distinguished into *loud* and *low*; and with respect to their tone, into *grave* and *acute*, otherwise called *flat* and *sharp*. The strength of any sound depends on the magnitude of the stroke, which is made by a pulse on the drum of the ear; the greater the stroke is, the louder is the sound which it excites, and the weaker the stroke, the lower the sound; and whereas all the pulses move with equal velocities, the magnitude of the stroke, and consequently the strength of the sound, must be as the quantity of matter in the pulse; that is, as a rectangle under the density and breadth of the pulse; and supposing the breadth of the pulse to be given, it must be as the density.

The tone of a sound depends on the duration of a stroke; the longer a stroke is which a pulse makes

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on the drum of the ear, the more grave is the sound which it produces ; and the shorter the stroke, the more acute is the sound ; but since all the pulses move equally swift, the duration of a stroke must be proportional to the interval between two successive pulses ; and of consequence, a sound is more or less grave or acute, in proportion to the length of that interval. Hence it follows, that all the sounds from the loudest to the lowest, which are excited by the vibrations of one and the same body, are of one tone. It likewise follows, that all those sounding bodies, whose parts perform their vibrations in equal times, have the same tone ; as also, that those bodies which vibrate slowest, have the gravest or deepest tone ; and on the contrary, those which vibrate quickest have the sharpest or shrillest tone.

As there may be an infinite variety in the times wherein sounding bodies perform their vibrations so may there likewise be in the tones of the sounds which depend thereon ; and yet amidst this great variety, musicians acknowledge but seven principal notes in an *octave* ; for tho' the eighth be requisite to compleat the seven intervals in an octave, yet are there in truth but seven notes ; for that which is called the *eighth*, becomes the base or ground note in the next octave ascending ; and as it stands in the limits of the two octaves, it is called the *eighth* with respect to the base note below it, and the ground or base note with respect to the 15th which is above it ; which 15th is likewise the base in the next ascending octave ; for by a repetition of notes, wherein the proportions of the times of the notes in the first octave are preserved, the octaves may be continued on both ways, ascending and descending, and that in *infinitum* ; and yet, notwithstanding this infinite progression in the octaves, the number of harmonick sounds is limited. Mr. SAUVEUR is of opinion, that all the harmonick sounds, that is, such sounds as can be heard distinctly and with pleasure,

and

and in whose tones a difference can be clearly perceived by the ear, lie within the compass of ten octaves; as also, that all sounds whatever, from the lowest harmonick sound, to the highest that the human ear can well bear, are contained within the limits of two octaves more. And if this be the case, it follows, that that body which gives the shrillest sound that the ear can bear, makes 4096 vibrations in the same time that one vibration is performed by that body which gives the gravest harmonick sound; for since in every octave, the time of the eighth is $\frac{1}{2}$ of the time of the base note, if $\frac{1}{2}$ be raised up to the 12th power, it will exhibit the time of the shrillest sound, that of the gravest being unity; but the 12th power of $\frac{1}{2}$ is the 4069 part of an unit; consequently, the time of the shrillest sound that the ear can well bear, and likewise of the vibration which produces it, is to the time of the gravest harmonick sound, and of the vibration whereby it is produced, as 1 to 4096; but the times of the vibrations of two bodies are inversely, as the numbers of vibrations which they perform in a given time; consequently, the body which gives the shrillest sound performs 4096 vibrations in the same time that the body which gives the gravest harmonick sound performs one; and forasmuch as Mr. SAUVEUR has found by some experiments which he made on organ pipes, of which I shall give you an account in my next lecture, that a body which gives the gravest harmonick sound, vibrates twelve times and a half in a second, the shrillest sounding body must perform 51100 vibrations in the same time; which argues great swiftness in the vibrating parts; and yet, great as it is, it has nothing extraordinary or surprising in it, if compared with the velocity of some other motions; for if we suppose the parts in each vibration to run thro' a space equal to the 10th part of an inch, tho' it is highly probable, that the lengths they run are much shorter;

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and

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and if we suppose them to move with the same velocity during the whole time of their motion; it follows, that they are carried at the rate of 425 feet and 10 inches in a second; consequently, they do not move with much more than two third parts of the velocity wherewith a ball flies from the mouth of a cannon.

The fifth and last property of the pulses is, that they may be propagated together in great numbers from different bodies, without disturbance or confusion; as is evident from consorts, wherein the sounds of the several instruments are conveyed distinctly to the ears of the audience; as they move along, some of them coincide and strike the drum of the ear at one and the same time, and thereby excite a smooth regular motion, that is pleasing and agreeable; whilst others which do not mix and unite, at least not frequently, strike the ear at different instants of time, and thereby disturb each other's motion, so as to render them harsh, grating and offensive. And hereon depends almost the whole of concords and discords in musick; such sounds, generally speaking, being deemed concords, as are excited by pulses which have frequent coincidences; and on the other hand, such sounds being called *discords*, as arise from pulses which coincide but rarely.

The frequency or infrequency of the coincidences, depends on the proportions which the intervals of the pulses bear one to another; as I shall shew you in relation to the several notes in an octave; in doing of which, instead of the pulses and their intervals, I shall consider the vibrations of the bodies which excite the pulses, and the times of those vibrations; because the number of pulses is always equal to the number of vibrations in the sounding bodies, and the intervals of the pulses proportional to the times of the vibrations.

If two vibrating bodies begin their motions together, and vibrate in equal times, it is manifest, that

that their vibrations must keep pace together, and constantly coincide. But if the vibrations be performed in unequal times, it is plain, that they cannot constantly keep pace together; for which reason some of them only will coincide; and which those are may be determined from the times of the vibrations; for since the numbers of vibrations, which are performed in a given time, are inversely as the times of the vibrations, if the numbers which express the times of the vibrations of two bodies be taken reciprocally, they will exhibit the coincident vibrations of the respective bodies. For instance, if the time of the vibrations of one body, be to the time of the vibrations of another, as 8 to 9, which is the case of two bodies, whereof one sounds a second or tone major to the other, every ninth vibration of the former coincides with every eighth of the latter. So again, if the times of the vibrations be to one another, as 5 to 6, which is the case, where one body sounds a lesser third to the other, every sixth vibration of the former falls in with every fifth of the latter.

In this scheme, I have set down those fractional numbers which express the proportions that the times of the vibrations of those bodies, which sound the several notes in an octave, bear to the time of the vibration of that body which sounds the base note; by the help of which numbers the coincident vibrations may be readily discovered. For in each fraction, the denominator exhibits the coinciding vibration of that body which sounds the note, and the numerator the coinciding vibration of the body which sounds the base note.

S 4

Having

$\frac{1}{2}$	<i>Eight.</i>
$\frac{8}{13}$	<i>Greater seventh.</i>
$\frac{5}{9}$	<i>Lesser seventh.</i>
$\frac{3}{5}$	<i>Greater sixth.</i>
$\frac{5}{8}$	<i>Lesser sixth.</i>
$\frac{2}{3}$	<i>Fifth.</i>
$\frac{3}{4}$	<i>Fourth.</i>
$\frac{4}{5}$	<i>Greater third.</i>
$\frac{5}{6}$	<i>Lesser third.</i>
$\frac{8}{9}$	<i>Second or tone major.</i>
1	<i>Base note.</i>

Having thus explained the nature and properties of sound, I come now to give you an account of the VIBRATIONS of MUSICAL STRINGS, and to shew you in what proportions the times of the vibrations are varied, by varying the length, thickness, or, tension of the strings; and in order thereto, I shall lay down the following PROPOSITION.

Pl. 7.
Fig. 13.

Let an elastick string as AB, fastened at A, and passing over a small pin or pulley at B, be stretched by an appending weight as P, (which I shall call the tending force); and by a force applied at the middle point C, (which I shall call the inflecting force), let it be drawn into the position ADB; if the distance between C and D be exceedingly small in proportion to the length of the string, or, to speak in the mathematical phrase, if CD be a nascent quantity, the inflecting force will be measured by a rectangle under the space CD, and the tending force applied to the length of the string. For since the tending force acts upon the string in the direction DB, it may be denoted by that line, and being so denoted, it may be resolved into two forces, whereof one acts in pulling the string horizontally in the direction CB, and is therefore to be expressed by CB; whilst the other acts in drawing the string perpendicularly upward from D towards C, and is therefore to be expressed by the line DC, so that that portion of the tending force which acts in moving the string upward, is to the whole force, as DC to DB; or, because D and C are supposed to be indefinitely near, as DC to CB; but the force which acts in drawing the string upward, is equal to the inflecting force, because they balance each other; consequently, the inflecting force is to the tending force, as CD to CB; and turning this analogy into an equation, by multiplying the extremes and means, and then dividing by CB, we shall have the inflecting force equal to a rectangle under the tending force, and the line CD, applied

to

to half the length of the string; and therefore, forasmuch as whole quantities are in proportion as their halves, the inflecting force will be as a rectangle under the tending force and the line CD, applied to the length of the string; so that putting F for the inflecting force, P for the tending force, S for the line CD, and L for the length of the string, F is as $\frac{SP}{L}$.

Hence it follows, that if P and L , that is, if the tending force and length of the string be given, the inflecting force is as the line CD, as will appear from the following experiment.

Let a small brass wire three feet long, fastened at one end, and passing over a pin so as that when stretched it may be in an horizontal position, be tended by a weight of three pounds; and let half an ounce, and an ounce, be appended successively to the middle of the wire; in the former case, the point of suspension will be drawn down $\frac{4}{10}$ th parts of an inch, and in the latter $\frac{8}{10}$.

Exp. 2.

Since the force which inflects the string of a given length, and tended by a given force, is as the space CD, thro' which the string is bent; the force wherewith the string restores itself, must likewise be as CD, because the restitutive force is in all cases equal to the inflecting force; consequently, the point D is carried towards C, by a force that varies with the distance; and therefore, whatever be the distance at which it begins its motion, the time wherein it arrives at C will still be the same; as I proved in my lecture on the pendulum. Whence it follows, that the vibrations of one and the same string, whether they be through larger or smaller spaces, are all performed in equal times.

If L and S be given, F is as P ; that is, if the length of the string, and the space thro' which it is bent be given, the inflecting force is as the tending force; or, in other words, one and the same string
being

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being tended by different forces will upon the inflexion be drawn down equal spaces by inflecting forces, which are to one another in the same proportion with the tending forces.

Exp. 3.

Let the same wire as before be tended by a weight of six pounds, and it will require one ounce to draw it down $\frac{4}{16}$ ths of an inch, and two ounces to draw it down $\frac{8}{16}$ ths; whereas, when it was tended by a weight of three pounds only, it was drawn down the same spaces by half an ounce, and an ounce.

If P and S be given, F is as $\frac{1}{L}$; that is if the force which tends the string, and the space thro' which it is bent be given, the inflecting force is inversely as the length of the string; or, in other words, if strings of different lengths be tended by equal forces, they will be drawn thro' equal spaces by inflecting forces, which are to one another inversely as the lengths of the strings.

Exp. 4.

Let a small brass wire a foot and a half long, be tended by a weight of three pounds, and it will require an ounce to bend it down $\frac{4}{16}$ ths of an inch, whereas half an ounce was sufficient to give the same bent to the wire which was of a double length, and under the same tension.

The time of a vibration of an elastick string is measured by a rectangle, under the length and diameter of the string, applied to the square root of the tending force. For if, as in the case of gravity, we suppose the force wherewith the inflected string restores itself to act uniformly, as we safely may because the space thro' which it acts is exceedingly small; then the motion generated will be as a rectangle under the force and the time of its acting; so that putting M for the motion, F for the restitutive force, and T for the time of its acting, M is as FT;



F T; but the motion is as the quantity of matter moved into the velocity wherewith it moves; and in this case, the quantity of matter is as a product under the length of the string, and the square of its diameter; wherefore, putting D , L , and V , to denote the diameter, length, and velocity, $F T$ is as $D^2 L V$; and dividing both sides by F , T is as $\frac{D^2 L V}{F}$; but the restitutive force of the string be-

ing equal to the force which inflects it, and that having been proved to be as $\frac{SP}{L}$, wherein S denotes

the space thro' which the string is bent, P the tending force, and L the length of the string; if in-

stead of F we substitute $\frac{SP}{L}$, T will be as $\frac{D L^2 V}{P S}$;

but the velocity applied to the space is inversely as the time, that is, $\frac{V}{S}$ is as $\frac{1}{T}$; and therefore, in-

stead of that, substituting this, and multiplying both sides by T , we shall have T^2 , as $\frac{D L^2}{P}$;

and therefore extracting the root, T is as $\frac{DL}{P^{\frac{1}{2}}}$;

that is, the time of a vibration, is as a rectangle under the diameter and length of the string, applied to the square root of the tending force.

Hence it follows, that if D and P be given, T is as L ; that is, if the diameter of the string and the tending force be given, the time of the vibrations varies with the length of the string; as is manifest from the division of the *monochord*, wherein the parts of the chord which sound the several notes in an octave, have the same proportions to the whole chord, that the times of the respective notes have to the time of the base note; as for instance, one half of the chord sounds an octave to the whole, whose time is one half of the time of the base note;

Exp. 5.

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and $\frac{2}{3}$ of the chord sound a fifth, the time whereof is $\frac{2}{3}$ of the base time of the note, and so of all the rest.

Exp. 6. If P and L be given, then T is as D; that is, if the tending force and length of the string be given, the time of the vibration is as the diameter of the string; as will appear, if two wires of equal lengths be tended by equal weights, the diameter of one being the 90th part of an inch, and that of the other the 45th part; for the former will sound an octave to the latter.

Exp. 7. If D and L be given, then T is inversly as the square root of P; that is if the diameter and length of the string be given, the time of the vibration is inversly as the square root of the tending force; as will appear, if eleven wires equal as to length and thickness, be tended by weights, whose square roots are to one another inversly as the times of the notes in an octave; for the wires so tended will sound the respective notes.

<i>Eighth</i>	— — — —	$\frac{1}{8}$	—	240
<i>Greater seventh</i>	— — — —	$\frac{2}{7}$	—	$210\frac{1}{6}$
<i>Lesser seventh</i>	— — — —	$\frac{5}{9}$	—	$194\frac{2}{3}$
<i>Greater sixth</i>	— — — —	$\frac{3}{5}$	—	$166\frac{2}{3}$
<i>Lesser sixth</i>	— — — —	$\frac{5}{8}$	—	$153\frac{2}{3}$
<i>Fifth</i>	— — — —	$\frac{2}{3}$	—	135
<i>Fourth</i>	— — — —	$\frac{3}{4}$	—	$106\frac{2}{3}$
<i>Greater third</i>	— — — —	$\frac{4}{5}$	—	$93\frac{3}{4}$
<i>Lesser third</i>	— — — —	$\frac{5}{6}$	—	$86\frac{2}{3}$
<i>Tone major, or second</i>	— — — —	$\frac{8}{9}$	—	$75\frac{1}{6}$
<i>Base note</i>	— — — —	1	—	60

In the left hand column of this table, the numbers express the times of the several notes; and the numbers in the right hand column, express the weights in ounces, whereby the wires which sound the respective notes are tended; the square roots of which

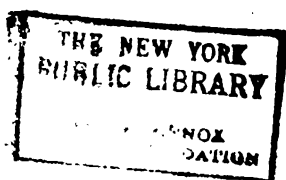


D
A

A
P

K
L

S



which weights, are to one another inverſly as the times of the reſpective notes; as for inſtance the weight which tends the ſtring that ſounds the octave, is to the weight whereby the ſtring that ſounds the baſe note is tended, as 4 to 1, whoſe ſquare roots are as 2 to 1, that is, inverſly as the time of the octave, to the time of the baſe note; and ſo of all the reſt.

LECTURE XVIII.

OF THE MOTION OF SOUND.

IN my laſt lecture, wherein I treated of that motion of the air, which is productive of ſounds, I ſhewed you, that each particle of air in going forward and returning back, is twice accelerated, and as often retarded; but I did not then enquire into the law of that acceleration and retardation. I likewiſe told you, that all the pulſes of the air move equally ſwift, the demonſtration of which, I promiſed to give you in this lecture.

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Now Sir ISAAC NEWTON, having in a moſt elegant manner, in the 47th *propoſition* of the *Second Book* of his *Principles*, demonſtrated that each particle of air, during its vibratory motion, is accelerated and retarded, in the very ſame manner as a pendulum vibrating in a cycloid; and having likewiſe, in the 49th and 50th *Propoſitions* of the ſame book, determined the velocity of ſound, I ſhall in this lecture lay before you what he has ſaid, in relation both to the one and the other, in the cleareſt light that I am able.

As to the firſt, let the line AB denote the length of a pulſe; or that ſpace thro' which the motion of the air is propagated, during the time that a particle performs its vibration, by going forward and
returning

Pl. 8.
Fig 1.

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XVIII.Pl. 8.
Fig. 2.
Fig. 1.Pl. 8.
Fig. 2.

returning back ; and let E, F and G, be three particles, or physical points of air situated in the right line at equal distances, and at rest ; and let EQ, FR, and GT, be three equal, but exceedingly short spaces, thro' which these particles go and return in their vibrations ; which spaces tho' they be here taken of some length, to avoid confusion in the scheme, are in reality so exceedingly small, as to bear no proportion to AB, the length of a pulse. Let x, y, and z denote any intermediate points, in which the particles are found during their motion forward or backward. Let EF, and FG be small physical lines, or little portions of air, situated in strait lines between those physical points ; which lines are successively moved into the places xy, yz, and QR, RT. Let the right line PS, be drawn equal to EQ, and on that line as a diameter, let the circle SIPi be described ; and let the circumference of that circle denote the time of the vibration of a particle, and the parts of the circumference, the proportional parts of the time ; so as that after any time as PH, or PHSh, if right lines as HL and hl be drawn from the points H and h perpendicular to SP, and Ex be taken equal to PL, or Pl, the particle E may be found at x. By this means the particle or physical point E, in moving forward thro' x to Q, and thence back again thro' x to E, will be accelerated and retarded, in the same manner with a pendulum vibrating in a cycloid ; inasmuch as in my lecture on the pendulum, I shewed you, that the spaces described by such a pendulum, and the times of describing those spaces, are (as we have now supposed them to be in the case of the air's motion) as the versed sines and arches of a circle, whose diameter is equal in length to the whole cycloid.

Now, in order to prove that the several little portions of air are agitated in the forementioned manner by

by their elasticity, which in this case is the true moving cause, let us suppose them to be so moved by some cause or other, be that cause what it will; and their elasticity will be found to be such in every point of their progress and return, as must of necessity produce in them the same degrees of acceleration and retardation, that gravity does in a pendulum vibrating in a cycloid.

In the circumference of the circle, let the equal arches HI and IK, or hi and ik, be taken, bearing the same proportion to the whole circumference, that the little right lines EF and FG do to AB the length of a pulse; and drawing the lines IM and KN, or im and kn perpendicular to PS, inasmuch as the points or particles E, F and G, are moved in the same manner successively one after another, the motion beginning with E, and each of them performs its intire vibration, in going forward and returning back, in the same time that the motion is propagated thro' a space equal to AB, the length of a pulse; if PH or PHSh denotes the time from the beginning of E's motion, PI, or PHSi, will denote the time from the beginning of F's motion; and in like manner PK, or PHSk, will denote the time from the beginning of G's motion. And if the points E, F and G be found at x, y and z; the lines Ex, Fy, and Gz, in the first figure, will be respectively equal in the second, to PL, PM, and PN, in the progress of the points; and in their return, equal to Pl, Pm, Pn, those being the versed sines of the arches which denote the times. Whence it follows, that xz, which is equal to the difference between Ex, and the sum of EG and Gz, is in the progress of the points, equal to EG — LN, and to EG + ln in the return; but xz is as the expansion of the little portion of air EG, when it is in the place xz; consequently, that expansion is to the mean ordinary expansion, or that expansion

Pl. 8.
Fig. 1.
Fig. 2.

Fig. 1.

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Pl. 8.
Fig. 2.

sion which it has when at rest before it is put into its vibratory motion, as $EG - LN$, to EG , when that portion of air in going forward is found in the place xz ; and it is as $EG + ln$; or, because LN and ln are equal, as $EG + LN$, to EG , when the portion of air in returning back, is found in the same place. Let now ID be drawn from the point I , perpendicular to HL , and the nascent triangle HID , will be similar to the triangle OIM , because the angles at D and M are right ones, and the angles at I are equal, as being each of them the complement of one and the same angle DIO , to a right one; consequently, DI , or its equal LM , is to HI , as IM to the *radius* OI , equal to OP ; and double LM equal to LN , is to double HI equal to HK , as IM to OP ; and by the construction, HK is to EG , as the circumference of the circle to AB ; or putting R for the *radius* of a circle, whose circumference is equal to AB , as OP to R ; whence reducing these two analogies into equations, we shall have $\frac{LN}{HK} = \frac{IM}{OP}$, and $\frac{HK}{EG} = \frac{OP}{R}$; wherefore, multiplying these equations together, we shall have $\frac{LN}{EG} = \frac{IM}{R}$; and resolving this into an analogy, we shall have $LN : EG :: IM : R$; and of course, by substituting IM and R , in the places of LN and EG , the expansion of the small portion of air EG , or of the physical point F , when in the place xz or y , is to its mean ordinary expansion, as $R - IM$ to R , in its going forward, and as $R + im$ to R , in its returning; and forasmuch as its elasticity is inversely as its expansion, its elasticity, when at the point y , is to its ordinary elasticity, as $\frac{1}{R - IM}$ to $\frac{1}{R}$ in its progress, and in its regress in the same point, as $\frac{1}{R + im}$ to $\frac{1}{R}$; and

and by the same way of arguing, the elastick forces of the physical points E and G, when in going forward they are found at x and z, will be to their or-

dinary elasticity as $\frac{I}{R-HL}$, and $\frac{I}{R-KN}$ to $\frac{I}{R}$;

and by subducting the latter of these quantities from the former, the difference of those forces will be as

$\frac{HL-KN}{R^2-R+HL-R+KN+HL \times KN}$ to $\frac{I}{R}$; or

rejecting all the terms of the divisor except the first, as being indefinitely small with respect to that, as

$\frac{HL-KN}{R^2}$ to $\frac{I}{R}$; or, multiplying both sides by

R^2 , as $HL-KN$ to R ; but forasmuch as R is a given quantity, $HL-KN$ is as unity; consequently, the difference of the forces is as $HL-KN$. But from the similarity of triangles, $HL-KN$ is to HK , as OM to OI or OP ; consequently, since HK and OP are given, $HL-KN$ is as OM ; or because SP and EQ are equal, if EQ be bisected in C , as cy . And by the same way of reasoning, the difference of the elastick forces of the same points, when in their return they are found at x and z, is as the same cy ; but that difference, or the excess of the elastick force of the point x above the elastick force of the point z, is the force by which the little line or portion of air xz , which lies between those points is accelerated in its progress; and on the other hand, the excess of the elastick force of the point z above that of the point x, is the force by which the same little line or portion of air is accelerated in its return; so that the force by which that little portion is accelerated, is every where as its distance from C , the middle point of its vibration; consequently, during its vibratory motion, it must be accelerated and retarded in the same manner with a pendulum vibrating in a cycloid; inasmuch as I proved in my lecture on the pendulum, that the

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force which agitates the pendulum in the, forementioned manner, is every where as its distance from the middle or lowest point of the vibration. And what has been thus proved of the little portion EG, is in like manner demonstrable of every other little portion of air, thro' which the motion is propagated.

As to the velocity of sound, or what amounts to the same thing, of the pulses of the air, if a pendulum be made equal in length to the height of an homogeneous atmosphere, whose weight is equal to that of our atmosphere, and its density the same with that of the air at the surface of the earth; which height is, as I shewed you in a former lecture, equal to 29725 feet, and which I shall now denote by the letter H; in the same time that such a pendulum performs an intire vibration by going forward and returning back, a pulse of the air will move thro' a space equal to the circumference of a circle described with the *radius* H. For if the little portion of air EG, vibrating thro' a small space as PS, be acted upon at P and S, the extremities of the space thro' which it vibrates by an elastick force equal to its gravity, it will perform its vibrations in the same time that it would in a cycloid whose length is equal to PS; because equal forces must of necessity move equal bodies thro' equal spaces in equal times. Since then, the times of vibrations are in the subduplicate *ratio* of the lengths of the pendulums, and the length of any pendulum is equal to half of the cycloid, wherein it vibrates; the time in which the small portion of air would vibrate by the force of its gravity in a cycloid equal in length to PS, must be to the time of the vibration of a pendulum whose length is H, in the subduplicate *ratio* of PO to H. But the elastick force which acts upon the little portion of air in the extream points P and S, was proved to be to its whole or ordinary elastick force,

Pl. 8.
Fig. 2.

force, as $HL - KN$ to R ; that is, in the case before us, where the point K coincides with P , as HK to R ; for upon the coincidence of K and P , KN vanishes, and HL , which in this case is their difference, becomes the sine of HK , and equal to it, inasmuch as HK is a nascent arch. And the whole elastick force of that little portion of air, or, which is the same thing, the weight which compresses it, is to its own weight, as the height of the homogeneous atmosphere or H , to the small line EG ; whence putting e to denote the elastick force, which agitates the small portion of air in the extrem points of its vibration P and S , and w for its weight, W for the whole elastick force, or the weight of the compressing atmosphere, and reducing the two last analogies into equations, we shall have $\frac{HK}{R} = \frac{e}{W}$, and $\frac{W}{w} = \frac{H}{EG}$; whence multiplying the two middle terms together, and likewise the extremes, we shall have $\frac{e}{w} = \frac{HK \times H}{R \times EG}$; and by substituting PO and R for HK and EG , to which they are proportional, $\frac{e}{w}$ is equal to $\frac{PO \times H}{R^2}$; that is, by resolving this equation into an analogy, the elastick force which agitates the little portion of air in the extrem points of the space thro' which it vibrates, is to its weight, as $PO \times H$ to R^2 ; since then, from the nature of motion, the times wherein equal bodies are moved thro' equal spaces, are reciprocally in the subduplicate *ratio* of the moving forces, it follows, that the time wherein the little portion of air performs its vibration by virtue of the elastick force denoted by e , is to the time wherein it can vibrate thro' an equal space by the force of its gravity, in the subduplicate *ratio* of R^2 to $PO \times H$, and of course, to the time of the vibration of a pendulum whose length is H , in a *ratio*

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compounded of the last-mentioned *ratio*, and of the subduplicate *ratio* of PO to H; that is, as $R^2 \times PO$ to $H^2 \times PO$; that is, by dividing by PO, and extracting the square roots in the simple *ratio* of R to H. But in the time that the little portion of air performs one vibration by going forward and returning back, the pulse is carried thro' a space equal to AB; consequently, the time in which a pulse moves from A to B, is to the time in which a pendulum whose length is H, swings forward and backward, as R to H, or as BC, the circumference of a circle whose *radius* is R, to the circumference of a circle whose *radius* is H; but the time of the pulse's motion from A to B, is to the time in which it moves thro' a space equal to the circumference of a circle whose *radius* is H, in the same proportion; wherefore, in the same time that a pendulum whose length is H, swings forward and backward, a pulse will move thro' a space equal to the circumference of a circle whose *radius* is H, which was the thing to be proved.

As a *Corollary* it follows, that the pulses move with such a velocity as a heavy body acquires in falling down half the height denoted by H; for in the same time with the fall they will, with a velocity equal to that acquired by the fall, describe a space double that of the fall, that is, a space equal to H; and of consequence, in the time that the pendulum vibrates forward and backward, they will run thro' a space equal to the circumference of a circle whose *radius* is H. For, in my lecture on the pendulum, I shewed you, that the time of the fall thro' half the length of the pendulum, is to the time of one vibration, as the diameter of a circle to its circumference; and of course, to the time of a double vibration, as the *radius* to the circumference. Since then it has been proved, that the pulses move with such a velocity as carries them thro' a space equal to the circumference of a circle whose

whose *radius* is H , in the same time that a pendulum whose length is H , performs a double swing; and since it appears that the velocity acquired by a heavy body in falling down half the height H , will carry the pulses thro' the same space in the same time, it is manifest, that they move with that velocity.

As a second *Corollary* it follows, that the velocity of the pulses is in a *ratio* compounded of the subduplicate *ratio* of the air's elasticity directly, and of the subduplicate *ratio* of its density inversely; for since the velocity wherewith they move, is such as a body acquires in falling down half the height H , and since the velocities acquired by falling bodies, are in the subduplicate *ratios* of the heights from which they fall, it is manifest, that the velocity of the pulses is as the square root of H , but the height H , is directly as the air's elasticity, and inversely as its density; consequently, the velocity of the pulses is in the subduplicate *ratio* of the air's elasticity directly, and the subduplicate *ratio* of its density inversely. Whence it appears, that the velocity of the pulses is given, forasmuch as, *ceteris paribus*, the elasticity is as the density. In the *winter* time indeed, the motion of the pulses is somewhat slower than in *summer*, because the coldness of that season does in some measure weaken the elasticity, and at the same time increase the density. From what has been said, the space thro' which sound moves in any given time, may readily be determined; for since it is known by experience, that a pendulum $39\frac{1}{2}$ inches long, performs a double vibration by going forward and returning back in two seconds of time, a pendulum whose length is H , that is 29725 feet long, will perform a like double vibration in $190\frac{3}{4}$ seconds; consequently, in that time sound will move thro' a space equal to the circumference of a circle whose *radius* is 29725 feet; that is, it will move thro' 186768 feet, which being

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divided by $190\frac{1}{4}$, gives a quotient of 979 feet, for the space thro' which sound moves in one second of time. But it must be observed, that in this computation no regard has been had to the thickness of the solid particles of air, thro' which sound is propagated in an instant; if that therefore be allowed for, the velocity of sound will come out greater in the proportion of about ten to nine; for since the specifick gravity of air is to that of water, as 1 to 870, if we suppose the particles of air to be equally dense with those of water, and that the greater rarity of air is owing to the greater interval between its particles, it follows, that that interval is about nine times as great as the diameter of a particle; consequently, a tenth part of the space thro' which sound is propagated is possessed by the particles of air; if therefore to 979 feet, which is the space thro' which sound would move in a second, in case the particles of air had no magnitude, we add a ninth part, or 109 feet more on account of the thickness of the particles, we shall have 1088 feet for the space thro' which sound is carried in a second of time. Besides, as there are vapours dispersed thro' the air, which being of a different tone and elasticity, do not partake of that motion of the true air by virtue whereof sound is propagated, the moving cause having on that account fewer particles of matter to agitate, must of necessity give them a greater velocity; and from the nature of motion it is evident, that the velocity will be greater in the inverse subduplicate *ratio* of the quantity of matter to be moved; that is to say, if we suppose the atmosphere to consist of ten parts of true air, and one part of vapours, the motion of sound will be quicker in such an atmosphere, than in an atmosphere consisting intirely of true air, in the subduplicate *ratio* of 11 to 10, or in the simple *ratio* of about 21 to 20. If therefore the velocity last found be augmented in that proportion, we shall have

1142

1142 feet for the space thro' which sound moves in one second of time ; and this agrees with the most accurate experiments that have been made, for discovering the velocity of sound.

The space thro' which sound moves in a second of time being thus discovered, the length of the pulses excited by the vibrations of a sounding body may likewise be found, provided the number of vibrations performed by the sounding body in a given time, can by any method be determined ; for since each vibration excites a new pulse, all that is requisite to be done, is to divide 1142 by the number of vibrations which the sounding body performs in a second, and the quotient will express the length of a pulse in feet. Now, the number of vibrations which a sounding body performs in a given time, has been determined by Mr. SAUVEUR, in the following manner ; “ Musicians having frequently observed, that if two organ pipes which are nearly unisons, be made to sound together, there are certain instants of time, and those, as well as they can be judged of by the ear, at equal intervals, wherein their joint sound is stronger, than in the intermediate times.” This, Mr. SAUVEUR with great appearance of reason, thinks is owing to the coincidence of their vibrations at those instants ; for when by the coincidence of their vibrations, they strike the ear at one and the same instant, they must needs make a stronger impression upon it, than when they strike it separately one after another. Taking this for granted, he by the help of a pendulum, took the time between two successive coincidences in the vibrations of two pipes of considerable lengths, and nearly of the same tone ; he made choice of long pipes, because the coincidences of their vibrations are rarer, and consequently, the intervals between the coincidences are more easily measured, in long pipes than in short ones. Having thus found the time which passed between two

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successive coincidences, he readily found the number of vibrations performed by each pipe in the same time, they being inversely as the numbers expressing the proportion of the tones of the pipes; as for instance, if the time between two successive coincidences was found to be the sixth part of a second, and the numbers which expressed the proportion of the tones of the pipes were 45 and 46, the longer pipe performed 45 vibrations, and the shorter 46, in the sixth part of a second. From these experiments he found, that a pipe, whose length was about five *Parisian* feet, had the same tone with a string that vibrates an hundred times in a second; consequently, of the pulses excited by the sounding of such a pipe, there are about one hundred in the space of 1142 *English*, or 1070 *Parisian* feet; and of course, the length of one pulse is about 10 *Parisian* feet and $\frac{7}{10}$ ths, that is about twice the length of the pipe; whence it is probable, that the lengths of the pulses excited by the soundings of open pipes, are in all cases equal to twice the length of the pipes.

In a former lecture, speaking of the increase which motion received by being communicated from a smaller elastick body to a larger, I took occasion to give a reason for the augmentation of sound in speaking trumpets; I shall close this lecture, by accounting for it from the nature of the pulses of the air. From what has been said in relation to the properties of those pulses, it is manifest, that the greater their condensation is, the stronger is the sound which they excite; now, when the voice acts upon a portion of air confined within a trumpet, it must necessarily make a stronger impression upon it, and of course condense it more, than when it acts upon it in an unconfined state; inasmuch as in the former case, the force of the voice is wholly employed in giving motion to that small portion of air which lies within the trumpet, whereas

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in the latter case, not only that portion of air is put in motion by the force of the voice, but likewise all that body of air which immediately surrounds it; the air then in the trumpet being by reason of its confinement, more strongly agitated and more closely condensed, than it would otherwise be, must at the *exit* of the trumpet, communicate to the air without greater degrees of condensation; and of consequence, produce a louder sound, than could possibly be excited by the same force of the voice, were it immediately impressed on the unconfined air.

LECTURE XIX.

OF LIGHT.

LIGHT, whereof I intend to treat in this lecture, is a most subtile fluid, consisting of particles exceedingly small, but of different magnitudes, as shall be shewn hereafter, which are thrown off from luminous bodies by the vibrating motions of their parts, with a velocity surprizingly great; for they do not spend above seven or eight minutes of an hour in passing from the sun to the earth, as was observed first by Mr. ROMER, Professor of Astronomy to the late King of *France*; and after him by others, by means of the *eclipses* of the *satellites* of JUPITER; for these eclipses, when the earth is between the sun and JUPITER, are observed to happen about seven or eight minutes sooner than they ought to do by the astronomical tables; and on the contrary, when the earth is beyond the sun with respect to JUPITER, they happen about seven or eight minutes later than they ought to do; so that in the latter situation of the earth, they are observed to happen fourteen or sixteen minutes later than in the former; forasmuch therefore as the satellites

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lites cannot disappear, but must continue visible to the eye of an observer, till all that light which they reflect before their immersions has passed by the place of observation, it follows, that the reflected light of the satellites spends fourteen or sixteen minutes in passing from one end of the diameter of the earth's orbit to the other; and consequently, half that time in moving from the sun to the earth. Hence, if the distance of the sun from the earth be 70 millions of miles, as it must be on supposition that its horizontal parallax is twelve seconds of a degree, and such the most accurate observations of the latest astronomers make it; then light moves at the rate of about 150 thousand miles in a second of time, and its velocity exceeds the velocity of sound, in the proportion of above seven hundred thousand to one.

The motion of light is in its own nature rectilinear, as is evident from the shadows which all opaque bodies cast when placed in the light of the sun, or of any other luminous body; and yet the beams or rays of light in passing out of one transparent body or medium into another of a different density, are bent and turned out of their way; or to speak more properly, they are made to change the direction of their motion; and this bending or change of direction is commonly called *refraction*; and it has been found by experience, that the rays in passing out of a rarer *medium* into a denser, are bent in such a manner as to be brought nearer to a line drawn perpendicular to the refracting surface at the point of incidence; and on the contrary, in their passage out of a denser medium into a rarer, they decline from the perpendicular.

Pl. 8.
Fig. 3.

For the illustration of which, let AB represent a ray of light moving in air from A to B, and passing into water at B, and let HK be perpendicular to the surface of the water at the point B; when the ray goes into the water, it does not continue its motion

motion strait forward in the line BC, but in some other line as BD, which is more inclined to the perpendicular BK. And on the other hand, if the line DB be supposed to be a ray of light moving in water from D to B, and there passing into air, instead of continuing its motion in the direction BE, it goes on in some other direction as BA, which being less inclined to, is more distant from, the perpendicular BH; as will appear from the following experiment. Let an empty vessel as BCDE, have a small object as A, placed at its bottom; and let it be so situated as that the sight of the object may be intercepted by the side of the vessel from an eye placed at Q; let then the vessel be filled with water, and the ray AB, which before the pouring in of the water, moved in a right line from A to K, and by so doing passed above the eye, will upon its emersion out of the water be bent downward, so as to strike upon the eye, and thereby render the object visible.

Exp. 1.
Pl. 8.
Fig. 4.

This bending of the rays in their passage out of one medium into another, seems to be owing to the attractive force of the denser medium acting upon the rays at right angles to the surface, as may appear by considering the consequences of such an attraction.

Let then AC be a ray of light moving from A to C, and there entering into a denser medium, the surface which separates the two mediums being denoted by the line HK. The motion of the ray in the direction AC, being resolved according to the known method into two, one in the direction AD, and the other in the direction AB or DC, whereof the former is parallel, and the latter perpendicular to HK; it is manifest, that as the ray enters into the denser medium at C, its perpendicular motion must be accelerated by the attraction, whilst its parallel motion continues the same; let then the line CG be taken in the same proportion to CD, that the

Pl. 8.
Fig. 5.

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the velocity of the perpendicular motion after refraction has to the velocity thereof before the refraction; and forasmuch as the parallel motion is the same before and after refraction, let CE be taken equal to AD or BC, and letting fall EF equal and parallel to CG, and drawing the diagonal CF, the ray after refraction will describe the line CF in the same time that it moved from A to C before the refraction; and forasmuch as GF is equal to AD, LM, that is, the sine of the angle MCL, must be less than AD, the sine of ACD; consequently, by the attraction of the denser medium, the ray in passing into that medium is brought nearer to the perpendicular.

Again, let FC denote the motion of a ray in the denser medium from F to C, and let this motion be resolved into two others, one in the direction FG or EC, and the other in the direction FE or GC, the former being parallel, and the latter perpendicular to HK; when the ray passes into the rarer *medium* at C, the parallel motion does not suffer any change from the attraction; but the perpendicular motion is retarded by the attractive force, which in this case acts in direct opposition to it; let then CD be to GC, as the perpendicular velocity of the ray in the rarer *medium*, to the perpendicular velocity thereof in the denser; and let DA be drawn equal and parallel to FG, in order to denote the parallel motion of the ray after refraction; and the diagonal CA will be the line described by the ray after refraction, in a space of time equal to that wherein it described the line FC before refraction; and forasmuch as AD is equal to GF, it must be greater than LM; consequently, the angle ACD is greater than FCG; and therefore, the ray in passing out of a denser *medium* into a rarer, is by the attraction of the denser *medium* bent from the perpendicular; so that in both cases, the refraction seems to be owing to the attractive force of the denser

denser medium, acting upon the rays at right angles to its surface; and what farther confirms this opinion is, that the denser any medium is, and consequently the stronger its attraction, the greater, *ceteris paribus* is its refractive power; thus oil of vitriol, whose density exceeds the density of water in the proportion nearly of three to two, acts more forcibly than water on the rays of light, in bending and turning them out of their way; as will appear from the following experiment; let the sixth figure represent a quadrant, whose *radius* A B is parallel to the horizon; and let A be a small coloured object, placed on the limb of the quadrant at the extremity of the horizontal *radius*; this being viewed thro' an empty glass vessel as C, of a prismatick form, placed at the center of the quadrant, with its refracting angle downwards, will appear in its real place at A. Let then the vessel be filled with water, and let the object be raised on the limb of the quadrant as high as D, that is to say, to the height of fifteen degrees and twenty minutes, and the rays as DB, which go from it towards the prism, will be so bent in passing thro' the water as to enter the eye in a direction parallel to the horizon, and represent the object as if placed at A. And the same thing will happen when the vessel is filled with oil of vitriol, excepting only that the object must be raised to a greater height, suppose to E, so as to have an elevation of twenty degrees and eight minutes; which plainly shews, that the rays are more bent, and suffer a greater refraction under the same circumstances from oil of vitriol, than they do from water.

Exp. 2.
Pl. 8.
Fig. 6.

Refractions

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Refractions taken by the Quadrant and prismatick vessel.

	Density.	Degrees and min.	Sines.
Water	1	15.20	2644342
Oil of vitriol	1.497	20.8	3442060
Salt water	1.2	17.52	3068029
Spirit of hartshorn	1.011	16.	2756374
Spirit of wine	0.835	17.	2923717
Oil of turpentine	0.869	22.34	3837582
Oil of linseed	0.939	22.57	3889277

The denser medium begins to attract the rays at some distance from its surface, and it acts upon them more and more forcibly in proportion as their distance from its surface lessens; but however, in what follows I shall suppose the attractive force to act with the same vigour in all parts of the space thro' which it extends itself; because, as that space is indefinitely small, no sensible error will arise from such a supposition. If then CD be the surface of the denser medium, and AB the space thro' which the attractive force extends itself from A to B; a ray of light in passing from B to A will be accelerated in such a manner, as that the perpendicular velocity thereof at the point A will be equal to the square root of the sum of the square of the perpendicular velocity of the ray at its incidence on the point B, and of the square of the perpendicular velocity

Pl. 8.
Fig. 7.

velocity which it would have at A, supposing it began its motion at B, from a state of rest. For since the attractive force is supposed to act uniformly thro' the space B A, the motion which it generates will as to its properties correspond with the motion arising from gravity; if therefore the triangle E G H be taken to denote the space B A, G H will express the velocity of a ray at A, on supposition that from a state of rest it begins its motion at B; but if at B it has a velocity expressed by any right line as I K, parallel to G H, let the triangle be continued on till the portion I F L K becomes equal to E G H, and F L will express the velocity of the ray at the point A; and forasmuch as the triangle E F L, is equal to the sum of the two triangles E G H and E I K, F L is equal to the square root of the sum of the squares of G H and I K; that is, the perpendicular velocity of the ray at A, is equal to the square root of the sum of the square of the perpendicular velocity of the ray at its incidence on the point B, and of the square of the perpendicular velocity which it would have at A, on supposition that it began its motion at B from a state of rest. And this being so, the course and velocity of a ray of light after refraction, in passing out of a rarer *medium* into a denser, may be determined in the following manner. Let Z be a rarer *medium*, and X a denser, separated by the common surface E F, on which let a ray of light as A C, fall obliquely, and let A C measure the velocity of the ray in the rarer *medium*; which velocity is the same, whatever be the inclination of the ray. From the center C with the *radius* C A, let a circle be described, in which let N M be drawn thro' the center perpendicular to E F, and from A let fall A Q perpendicular to E F, as also A O perpendicular to N C. The motion of the ray in the direction A C being resolved into two others, one in the direction A O or Q C, and the other in the direction A Q or O C; the line O C will

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will measure the velocity of the perpendicular motion; and therefore, if CP be taken to denote the perpendicular velocity generated by the attraction of the denser medium, the line PO will measure the perpendicular velocity of the ray in the denser medium; and forasmuch as the velocity of the parallel motion is no way altered by the attraction, if CV be taken equal to QC , and VB be drawn parallel to CM , and equal to PO , it is evident, that the ray after refraction, will describe the line CB , and that the velocity of its motion will be measured by that line.

As a *Corollary*, from what has been proved it follows, that the velocity of the refracted ray in the denser medium is no way varied by varying the inclination of the incident ray; for the square of BC being equal to the sum of the squares of BV and CV , or of PO and AO , and the square of PO being equal to the sum of the squares of CO and PC , the square of CB is equal to the sum of the squares of AO , CO , and PC ; but the squares of AO and CO are equal to the square of CA or CN : consequently, the square of CB is equal to the sum of the squares of PC and CN , which quantities continue unvaried, whatever be the inclination of the incident ray; and therefore PN or CB is a given quantity; that is, the measure of the velocity, and of consequence, the velocity wherewith the rays move after refraction in the denser medium, is always the same, however differently inclined the rays may be to the surface of the denser medium at their incidence thereon.

The angle ACN , which the line described by the incident ray contains, with the perpendicular to the refracting surface at the point of incidence, is called the *angle of incidence*; and the angle BCM , which the line described by the refracted contains, with the perpendicular to the refracting surface at the point of incidence, is called the *angle of refraction*.

As

As a second *Corollary*, from what has been proved it follows, that the sines of these angles are to one another in a given *ratio*; or, in other words, that whatever proportion the sine of any one angle of incidence bears to the sine of the corresponding angle of refraction, the same does the sine of any other angle of incidence bear to the sine of the respective angle of refraction. For since C B is cut by the circle in the point T, if from B and T, B S and T R be drawn perpendicular to the *radius*, B S will be equal to A O, which is the sine of the angle of incidence, and T R will be the sine of the angle of refraction; and from the nature of similar triangles, B S is to T R, as C B to C T; that is, the sine of incidence is to the sine of refraction in the same proportion with two standing quantities; consequently, that proportion is given, whatever be the inclination of the incident ray. And what has been thus proved, with respect to the sines of incidence and refraction, when rays pass out of a rarer medium into a denser, is in like manner demonstrable of those lines, when the rays move out of a denser medium into a rarer, with this difference only, that whereas in the former case the angle of incidence exceeds the angle of refraction, in the latter it is exceeded by it; for as the attraction of the denser medium by accelerating the perpendicular velocity of the rays in their passage from a rarer medium turns them out of their way, so as to bring them nearer the perpendicular, so on the other hand, by retarding their perpendicular velocity in their passage into the rarer medium, it turns them out of their way so as to remove them farther from the perpendicular, as has been already shewn; and forasmuch as the rays are turned out of their way in both cases by one and the same cause acting in the same uniform manner, it is manifest, that in both cases, they must be equally bent; consequently, as

U

much

Pl. 8.
Fig. 9.

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much as the angle of incidence exceeds the angle of refraction when a ray passes out of the rarer medium into the denser, so much must it be exceeded by it, when the passage of the ray is made the contrary way.

Pl. 8.
Fig. 10.

Now that the sine of the angle of incidence is to the sine of the angle of refraction in a given *ratio*, whatever be the inclination of the incident ray, may be proved experimentally in the following manner. Let a brass quadrant graduated on both sides, and fixed at its center to a perpendicular pillar in the manner represented, have two indices as A and B, one on each side, moveable on the center C; and let the index A, whereof the stem D is a continuation, be made to point to the 15th degree, and the index B to the 15th minute of the 20th degree; let then the pillar be immersed in water, so far as that CE the horizontal edge of the quadrant may touch the surface of the water, and upon viewing the stem D which lies within the water, it will by reason of the refraction seem to have changed its situation, and appear to lie in the same plane with the index B. And the same thing will likewise obtain, if the index A be set at the 30th degree, and B at the 30th minute of the 42d degree. Now in both these cases, the angle of incidence is equal to the angle contained between FC, the perpendicular edge of the quadrant, and the index A; and the angle of refraction is the angle made by the perpendicular edge of the quadrant, and the index B; so that one of the angles of incidence is 15 degrees, and the other 30, and the corresponding angles of refraction are nineteen degrees fifteen minutes, and 41 degrees 30 minutes; and 25, which is the sine of the lesser angle of incidence, is to 33, the sine of the corresponding angle of refraction, as 50, the sine of the greater angle of incidence, to 66, the sine of the angle of

of refraction, which corresponds thereto ; as in the following TABLE.

	<i>Angles of incidence.</i>	<i>Sines.</i>	<i>Angles of refraction.</i>	<i>Sines.</i>
Out of water into air.	d.		d. m.	
	15.	2588	19. 15	3296
	d.		d. m.	
	30.	5000	41. 30	6626
Out of oil of turpentine into air.	d.		d.	
	15.	2588	22.	3746
	d.		d.	
	30.	5000	47.	7313

LECTURE XX.

OF COLOURS.

NATURALISTS were formerly of opinion, that LIGHT was in its own nature simple and uniform, without any difference or variety in its parts. And that COLOURS, which are to be the subject of this lecture, were nothing else than certain changes or modifications of light caused by *refractions*, *reflexions*, and *shadows*. But Sir ISAAC NEWTON, to whom we are indebted for almost every thing that we know with certainty concerning the nature of light, has shewn from experiments, that notwithstanding the uniform appearance of light, the particles whereof it is composed are of different colours ; and that the colour of each particle is lasting and permanent, so as not to be changed either by refraction or reflexion. He has likewise shewn, that those particles which differ

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Exp. 1.
Pl. 8.
Fig. 11.

as to colour, differ also in degrees of refrangibility; by means whereof, the rays of different colours may be separated from each other, and exhibited apart.

Let a beam of the sun's light pass into a darkened chamber thro' a round hole as H, about the sixteenth or twentieth part of an inch wide, so as to fall directly on the middle of a double *convex lens* as L, ground to a *radius* of five or six feet, and placed at the distance of ten or twelve feet from the hole; by which means the image of the hole will be projected to I, on the other side of the *lens*, at the distance of ten or twelve feet more, and there appear white and round. Let then a prism of solid greenish glass as P, be placed close behind the *lens*, and in such a posture as that the beam of light may fall upon it perpendicular to its axis, which is an imaginary strait line, running thro' the middle from one end to the other parallel to its edges; this being done, the image of the hole, instead of being round and white, and projected to I, will be long and coloured, and cast sidewise from I; and the colours of the image taken in their order from that which lies nearest to I, will be *red, orange, yellow, green, blue, purple, and violet*; as in the image MN, where the several colours are denoted by their initial letters.

From the lengthening of the round image by the refraction of the prism, it is evident, that of the particles of light which form the image, some are more refrangible than others; for were they all alike refrangible, the distances to which they are thrown sidewise from their first situation at I, would be all equal, and of consequence, the second image would be round as the first.

As in the coloured *spectrum* the *red* lies nearest to, and the *violet* farthest from I, it is manifest, that the red particles in their passage thro' the prism, are pushed out of their way less, and the violet more,

more than any other; and consequently, that the red particles have the smallest degree of refrangibility, and the violet the greatest; and that the particles of intermediate colours have intermediate degrees of refrangibility, greater or less in proportion as they lie nearer to the one or the other of the two extremes.

This difference of refrangibility in the particles of light, argues a difference likewise in their magnitudes; for since one and the same cause, to wit, the attraction of the glass, acting upon them all with equal force, and under like circumstances, produces unequal changes in the directions of their motions, it must needs be that they move with unequal forces, and consequently, that their quantities of motion are unequal, which inequality of motion can arise from nothing else but the different size of the particles, in case they all move equally swift, as they are generally supposed to do; and that they are all perfectly solid, as their power of penetrating and dissolving the densest bodies, without suffering any change themselves, seems to require; consequently, the particles of light which differ as to colour, differ also in magnitude; those of violet being smallest, and the particles of other colours increasing continually one above another, as they are more and more removed from the violet, and approach nearer to the red, whose particles are largest of all; and here it will not be improper to observe, that as the red particles are, of all others, the largest, they must, on that account, act with the greatest force, and excite the strongest vibrations in the nervous coat of the eye; which may be one reason why reds are found to be more offensive to the eyes than any other colour whatever.

The seven colours whereof the long image is composed are permanent and lasting, and cannot possibly be changed, either by refraction or reflexion, as will appear from the following experiments.

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Let

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Exp. 2.

Let a small hole be made in the paper whereon the coloured image is formed, thro' which let each of the seven colours pass successively, and falling upon a prism, be again refracted, and they will be found to continue the same, without the least change or alteration; thus, the *red*, when refracted, will continue totally of the same red colour as before; neither *orange*, *yellow*, *green*, *blue*, nor any other new colour, will arise from the refraction; and the like constancy and immutability will be found in the other six colours, when refracted singly and apart from the rest. And as these colours are not changeable by refraction, so neither are they by reflexion; for if bodies of different colours be placed in the red light, they will all appear red, and in the blue light, they will appear blue, in the green light, green, and so of the other colours; in the light of any one colour, they will all appear totally of that same colour, with this difference only, that in some the colour will be more strong and full, in others more faint and dilute, every body appearing most splendid and luminous in the light of its own colour.

Exp. 3. Thus, for instance, if a deep red, as *carmine*, and a full blue, as *ultramarine*, be held together in the red light, they will both appear red; but the *carmine* will appear of a strongly luminous and resplendent red, and the *ultramarine* of a faint obscure and dark

Exp. 4. red; and on the other hand, if they be held together in the blue light, they will both appear blue; but the *ultramarine* will appear of a strongly luminous and resplendent blue, and the *carmine* of a faint dark blue.

Since the colours of the rays are not capable of being changed, either by refraction or reflexion, it is manifest, that if the sun's light consisted of but one sort of rays, there would be but one colour in the world; and by consequence, that the variety of colours depends upon the composition of light. It is likewise manifest, that the permanent colours of
natural

natural bodies arise from hence, that some bodies reflect some sort of rays, and others other sorts more copiously than the rest, and upon that account appear of this or that colour. Thus *minium*, and other red bodies, reflect the red rays most copiously, and thence appear red; *violets*, and all other bodies of the like colour, reflect the violet rays in greater abundance than the rest, and thence have their colour; and so of other bodies, every body reflecting the rays of its own colour more copiously than the rest, and deriving its colour from the excess and predominancy of those rays in the reflected light; for tho' all bodies appear of the same colour, when placed together in the light of any one colour, yet every body looks more splendid and luminous in the light of its own colour than in that of any other, which puts it past dispute, that every body reflects the rays of its own colour in greater abundance, than it does the rest, and thence has its colour.

As natural bodies appear of divers colours, accordingly as they are disposed to reflect most copiously the rays originally indued with those colours, so from the different proportions which the predominant rays bear to the rest of the reflected light, arise different shades or degrees in those colours. Where the predominant rays are very numerous in proportion to the rest, the colour appears strong and full; but as the excess of the predominant rays lessens, the colour, from the mixture of the other rays, abates of its liveliness, and becomes more faint and dilute; and when all the rays are equally reflected, so as that no one kind predominates, the colour becomes white; for whiteness is a mixture of all the colours, and it is more or less intense in proportion as the reflected rays are more or fewer in number; all *grays*, *duns*, *russets*, *browns*, and other dark and dirty colours, down to the deepest *black*,
 U 4 being

being but so many lesser degrees of *white*, and differing from perfect whiteness on no other account but that they consist of a lesser quantity of light, and consequently appear less glaring and luminous.

The reason why bodies reflect this or that kind of ray more copiously than the rest, and consequently appear of this or that colour, depends altogether on the size and density of the particles whereof the bodies are composed. Particles of coloured bodies reflecting rays of different colours according to their different magnitudes and densities, as has been fully proved by Sir ISAAC NEWTON, from experiments and observations made on the colours of thined bodies of *air*, *water*, and *glass*; by the help of which, he has in the second book of his *Opticks*, given us a table, containing *seven orders*, or *series* of colours, together with the thicknesses of the particles of air, water, and glass, which exhibit the several colours in each order; which thicknesses are expressed in parts, whereof ten hundred thousand make an inch. The first part of that table is here laid before you; and by inspection thereof, it will be found, that in each order of colours, the *red* is reflected by particles of the greatest thickness, and that the thicknesses of the particles which reflect the other colours, grow less and less, as the colours which they reflect are more and more removed from the *red*. It is likewise manifest from the same table, that among the particles which reflect one and the same colour, those which have the greatest density, have the least thickness; thus, for instance, the thickness of a particle of glass which reflects the *scarlet* of the *second order*, is but $12\frac{2}{3}$; whereas, the thickness of water which reflects the same colour, is $14\frac{3}{4}$, and that of air still greater, to wit, $19\frac{2}{3}$; so that the thicknesses of the particles which reflect any colour, increase as their densities lessen; for which reason,

particles

particles of the same thickness may reflect different colours, provided their densities be unequal; thus the particles of air which reflect the *violet* of the *second order*, have very nearly the same thickness with particles of water which reflect the *green*, as also with the particles of glass which reflect the *orange* of the same order.

		Thicknesses of		
		Air.	Water.	Glass.
The colours of the first order.	Very Black	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$
	Black	1	$\frac{3}{4}$	$\frac{20}{31}$
	Beginning of Black	2	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue	$2\frac{2}{3}$	$1\frac{1}{3}$	$1\frac{11}{20}$
	White	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{3}$
	Yellow	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	8	6	$5\frac{1}{8}$
	Red	9	$6\frac{3}{4}$	$5\frac{4}{5}$
Of the second order.	Violet	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{3}$
	Indico	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue	14	$10\frac{1}{2}$	9
	Green	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow	$16\frac{2}{7}$	$12\frac{1}{3}$	$10\frac{2}{3}$
	Orange	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright red	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$

From what has been said concerning the colours of natural bodies, it follows, that if any change be made in the size or density of the particles whereof a body is composed, the colour of the body will likewise be changed; for which reason, if two colourless liquors be mixed together, they may in the mixing suffer such changes in the size and density of their parts from their mutual actions one upon another, as to become opaque and coloured; and such

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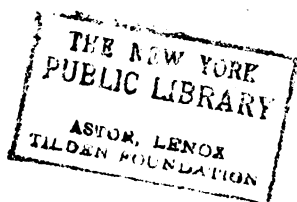
such liquors as are coloured, may for the same reason, when mixed together, either become transparent and colourless, or of such a colour as is different from the colour of either, before the mixture; as will appear from the experiment now to be made.

Colours produced by the mixture of liquors void of colour.

1. Rosated spirit of wine, and spirit of vitriol, a *Red*.
2. Solution of mercury, and oil of tartar, *Orange*.
3. Solution of sublimate, and lime water, *Yellow*.
4. Tincture of roses, and oil of tartar, *Green*.
5. Tincture of roses, and spirit of urine, *Blue*.
6. Solution of copper, and spirit of sal armoniack, *Purple*.
7. Solution of sublimate, and spirit of sal armoniack, *White*.
8. Solution of sugar of lead, and the solution of vitriol. *Black*.

Colours arising from the mixture of such liquors as are coloured.

- | | | | | | |
|----|---|----------------|-------------------------------|---|-----------------|
| 1. | { | <i>Yellow.</i> | Tincture of saffron | } | <i>Green.</i> |
| | { | <i>Red.</i> | Tincture of red roses | } | |
| | { | <i>Blue.</i> | Tincture of violets | } | |
| 2. | { | <i>Brown.</i> | Spirit of sulphur | } | <i>Crimson.</i> |
| | { | <i>Red.</i> | Tincture of red roses | } | |
| 3. | { | <i>Brown.</i> | Spirit of hartshorn | } | <i>Blue.</i> |
| | { | <i>Blue.</i> | Tincture of violets | } | |
| 4. | { | <i>Blue.</i> | Solution of Copper | } | <i>Violet.</i> |
| | { | <i>Blue.</i> | Tincture of violets | } | |
| 5. | { | <i>Blue.</i> | Solution of Hungarian vitriol | } | <i>Purple.</i> |
| | { | <i>Blue.</i> | Tincture of cyanus | } | |
| 6. | { | <i>Blue.</i> | Spirit of sal armon. coloured | } | <i>Green.</i> |
| | { | <i>Blue.</i> | | } | |
| | | | | | 7. <i>Blue.</i> |



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7.	{ <i>Blue.</i>	Solution of Hungarian vitriol	}	<i>Yellow.</i>	LECT. XX.
	{ <i>Brown.</i>				
8.	{ <i>Blue.</i>	Solution of Hungarian vitriol	}	<i>Black.</i>	
	{ <i>Red.</i>				
9.	{ <i>Blue.</i>	Tincture of red roses	}		
	{ <i>Green.</i>				
	{ <i>Blue.</i>	Tincture of cyanus	}	<i>Red.</i>	
	{ <i>Green.</i>				
	{ <i>Blue.</i>	Solution of copper	}		
	{ <i>Green.</i>				

Colours changed and restored.

1. A solution of copper, which is *Green*, by spirit of nitre is made *colourless*, and is again restored by oil of tartar.

2. A limpid infusion of galls, is made *black* by a solution of vitriol, and *transparent* again by oil of vitriol, and then *black* again by oil of tartar.

3. Tincture of red roses, is made *black* by a solution of vitriol, and becomes *red* again by oil of tartar.

4. A slight tincture of roses, by spirit of vitriol becomes a fine *red*, then by spirit of sal armoniack turns *green*, and then by oil of vitriol becomes *red* again.

5. Solution of verdegrease, from a *green* by spirit of vitriol becomes *colourless*, then by spirit of sal armoniack turns a *purple*, and then by oil of vitriol becomes *transparent* again.

Among the various *Phænomena* of colours, there is none more remarkable than that of the *rainbow*, which is an appearance observable in those places only where it rains in the sunshine, and where the spectator is placed in a due position between the sun and the rain, with his back to the former; for which reason it is generally allowed, that the bow is made by the refraction of the sun's light in drops of falling rain; the manner wherein it is formed, has in some measure been explained by ANTONIUS DE DOMINIS, archbishop of *Spalato*, and after him by DES CARTES; but as neither of them understood

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the true origin of colours, it was impossible for them not to be defective in their accounts; and therefore Sir ISAAC NEWTON, after he had discovered the true nature and rise of colours, set himself to the consideration of this subject, and towards the latter end of the first book of his *Opticks*, has given a full and satisfactory account of the whole matter; the substance of what he has there delivered concerning the rainbow is as follows.

Pl. 9.
Fig. 1.

Let a drop of rain, or any other spherical transparent body, be represented by the sphere BNFG, and let AN be one of the sun's rays, incident upon it at N and thence refracted to F, where let it either go out of the sphere by refraction towards V, or be reflected to G; and there let it either go out by refraction to R, or be reflected to H, where let it go out by refraction towards S, cutting the incident ray in Y; let AN and RG be produced till they meet in X. Parallel to the incident ray AN, let the diameter BQ be drawn, and let BL be a quadrant, on every point of which let us suppose a ray to fall parallel to BQ; as the point of incidence removes from B towards L, the angle AXR which the rays AN and RG contain, will first increase, and then decrease; and on the other hand, the angle AYS, contained between the rays AN and YS, will first decrease and then increase. This being so, if we suppose N to be that point of the quadrant BL, whereon if the incident ray AN falls, it makes the greatest angle with the ray GR, which emerges after one reflexion; then all the rays which fall on each side at a very little distance from N, and go out after one reflexion, will emerge parallel or very nearly parallel to GR; whereas those which fall on the quadrant at greater distances from N, will notwithstanding their parallelism before their incidence be scattered, and diverge from one another after their emergence. If therefore an eye be situated in the direction of the former rays
which

which go out parallel, they will enter it so copiously as to exhibit the image of the sun in the drop of rain which reflects them; but if the eye be so placed as to receive the latter rays which go out diverging, those which enter the eye will be too few to excite any sensation; and, of consequence, the image of the sun will not appear in the drop to an eye so situated.

If N be the point, whereon if the incident ray AN falls, it makes the smallest angle with the ray HS, which emerges after two reflexions; then, as before, all the rays which are incident near N, and which emerge after two reflexions, will go out parallel, and for that reason, will exhibit the sun's image to an eye situated in their direction; but those rays which are incident at any sensible distance from N, and which emerge after two reflexions, will be scattered as they go out, and upon that account will be too few, and consequently too feeble to excite any sensation in the eye of the spectator.

Now, forasmuch as the rays which are of different colours have likewise different degrees of refrangibility, the greatest angle AXR which can be made by the incident rays, and those which go out after one reflexion, will be of different magnitudes in rays of different colours; so likewise will the smallest angle AYS, that can be made by the incident rays, and those which go out after two reflexions; and it has been found, by computation, that in the least refrangible or red rays, the greatest angle AXR, is 42 degrees and two minutes; and the least angle AYS, 50 degrees and 57 minutes; and in the most refrangible or violet rays, the greatest angle AXR, has been found to be 40 degrees and 17 minutes; and the least angle AYS, 54 degrees and 7 minutes.

Suppose now, that O is the spectator's eye, and OP a line drawn parallel to the sun's rays; and let
POE

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POE be an angle of 40 degrees and 17 minutes; POF of 42 degrees 2 minutes, POG of 50 degrees 57 minutes, and POH an angle of 54 degrees 7 minutes; and these angles turned about their common side, shall with their other sides OE, OF, OG, and OH, describe verges of two rain-bows AFBE and CHDG. For if E, F, G, and H, be drops of rain, placed any-where in the conical surfaces described by OE, OF, OG, and OH, and be illuminated by the sun's rays SE, SF, SG, and SH, the angle SEO being equal to the angle POE, or 40 degrees and 17 minutes, shall be the greatest angle in which the most refrangible rays can after one reflexion be refracted to the eye; and therefore, all the drops in the line OE, shall send the most refrangible rays most copiously to the eye, and thereby strike the senses with the deepest *violet* colour in that region. And in like manner, the angle SFO being equal to the angle POF, or 42 degrees 2 minutes, shall be the greatest in which the least refrangible rays after one reflexion can emerge out of the drops; and therefore, those rays shall come most copiously to the eye from the drops in the line OF, and strike the senses with the deepest *red* colour in that region. And by the same argument, the rays which have intermediate degrees of refrangibility, shall come most copiously from drops between E and F, and exhibit the intermediate colours in the order which their degrees of refrangibility require, that is, in the progress from E to F, or from the inside of the bow to the outside, in this order, *violet, indigo, blue, green, yellow, orange, and red.*

Again, the angle SGO being equal to the angle POG, or 50 degrees and 57 minutes, shall be the least angle in which the least refrangible rays can after two reflexions emerge out of the drops; and therefore the least refrangible rays shall come most copiously to the eye from the drops in the line OG, and

And strike the sense with the deepest *red* in that region. And the angle SHO, being equal to the angle POH, or 54 degrees and 7 minutes, shall be the least angle, in which the most refrangible rays, after two reflexions, can emerge out of the drops; and therefore, those rays shall come most copiously to the eye from the drops in the line OH, and strike the senses with the deepest *violet* in that region. And by the same argument, the drops in the regions between G and H, shall strike the senses with the intermediate colours, in the order which their degrees of refrangibility require, that is, in the progress from G to H, or from the inside of the bow to the outside, in this order, *red, orange, yellow, green, blue, indigo, and violet*. And since these four lines OE, OF, OG, and OH, may be situated any where in the abovementioned conical surfaces, what is said of the drops and colours in these lines, is to be understood of the drops and colours every where in those surfaces. Thus then shall there be made two bows of colours, an interior and stronger by one reflexion in the drops, and an exterior and fainter by two, (for the light becomes fainter by every reflexion), and their colours shall be in a contrary order to one another, the *red* of both bows bordering upon the space GF, which is between the bows. The breadth of the interior bow measured cross the colours, shall be one degree and 45 minutes, and the breadth of the exterior, shall be three degrees 10 minutes, and the distance between them, shall be 8 degrees 55 minutes; the greatest semidiameter of the innermost, or the angle POF, being 42 degrees and 2 minutes, and the least semidiameter of the outermost, or the angle POG, being 50 degrees and 57 minutes. And these are the measures of the bows as they would be, were the sun but a point; for by the breadth of his body, the breadth of the bows will be increased, and their distance lessened by half a degree; and so the breadth

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Pl. 9.
Fig. 3.

breadth of the interior will be 2 degrees 15 minutes, and that of the exterior 3 degrees 40 minutes, and their distance 8 degrees 25 minutes; the greatest semidiameter of the interior bow 42 degrees 17 minutes, and the least of the exterior 50 degrees 42 minutes; and such Sir ISAAC NEWTON says he has found the dimensions of the bows in the heavens, when he measured the same. This explication of the rainbow, is confirmed by the following experiment; let a glass globe filled with water, as AB, be hung up in the sun-shine, with a black cloth placed behind it, and let IS be one of the sun's rays incident thereon; let the eye of a spectator, whose back is to the sun, be placed at O, and let it be directed to such a point in the lower part of the globe, suppose C, as that a strait line drawn from the eye thro' that point, and continued on till it meets the incident ray likewise produced, may therewith make an angle OXI, of 42 degrees 2 minutes; and the spectator shall then see a full *red* colour in that side of the globe opposed to the sun, as at F; let then the eye be raised up gradually to P, till the angle PZI becomes equal to 40 degrees and 17 minutes, and as the eye rises, it will perceive other colours, to wit, *yellow*, *green*, and *blue*, successively in the same side of the globe.

Again, let the eye be placed at Q, and let it be directed to such a point in the upper part of the globe, suppose D, as that a strait line, drawn from the eye thro' that point and meeting the incident ray protracted, may therewith make an angle QSI of 50 degrees and 57 minutes, and there will appear a faint red colour in that side of the globe towards the sun; let then the eye be gradually depressed to R, till the angle RTI is 54 degrees 7 minutes, as the eye sinks, the *red* will turn successively to the other colours, *yellow*, *green*, and *blue*, as in the former case, upon the raising of the eye.

LECTURE

LECTURE XXI.

OF DIOPTRICKS.

INTENDING in my next lecture to enquire LECT. XXI.
 into the NATURE of VISION, where I shall
 have occasion to take notice of *Defective Eyes*, I
 shall in this lecture, by way of preparation, lay be-
 fore you some of the chief properties of such *lenses*
 or glasses as are most commonly in use for assisting
 defective eyes; and they are of two sorts, First,
 such as are equally convex on both sides, and se-
 condly, such as are on both sides equally concave.
 The former sort is represented in the fourth figure,
 and the latter in the fifth.

Let ABC be an object placed before the double Pl. 9.
convex lens HK at any distance greater than the Fig. 6.
radius of the sphere, whereof the *lens* is a segment;
 the rays, which issue from the several points of the
 object, and fall upon the *lens*, will in their passage
 thro' it be so bent by the refractive power of the
 glass, as to be made to convene at so many other
 points behind the *lens*, and at the place of their
 concourse they will form an image or representa-
 tion of the object; and this image will be invert-
 ed, because the rays which flow from A, the up-
 permost point of the object, are united at F, the
 lowermost point of the image, whilst those which
 flow from C, the lowerest point of the object, are
 brought together again at D, the highest point of
 the image. So likewise those rays which issue
 from the right side of the object, are united in the
 left side of the image, whilst those which proceed
 from the left side of the object, concur in the right
 side of the image; as will appear by placing a light-
 ed candle before a double *convex lens*, at such a dis-

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tance as that the image thereof may be formed on a piece of white paper placed at a due distance behind the *lens*; for the flame will appear inverted with its point downward; and if either side of the flame be intercepted by the interposition of a dark body, the contrary side of the image will be obscured.

With regard to this experiment, I must observe to you, that tho' there is one certain distance, at which the paper must be placed, in order to exhibit the image with the greatest distinctness, yet may the distance be a little varied without rendering the image confused; and it is remarkable, that when the image is projected on the paper at the nearest distance that it can with any degree of distinctness, it appears bordered all around with red; which redness continually decreases, as the paper is more and more removed from the *lens*; and when it is removed to such a distance as is requisite to give the image the greatest advantage in point of distinctness, the redness intirely vanishes, and leaves the image equally white all over; but upon a farther removal of the paper, the edges of the image which at the nearest distance were tinged with red, do now appear tinged with blue. If a candle, which is placed at A before the *convex lens* CD, has its image projected on a paper at EF, supposing that to be the least distance at which it can be projected distinctly, its edges will appear red, but upon the removal of the paper to GH, they will become white; and when the paper is removed to IK, they will appear blue; the reason of these different appearances is this, the rays of light as AC and AD, which flow from the candle, being compounded of particles of different colours, whereof the red are least refrangible, and the blue most so, upon passing thro' the *lens*, the blue rays are made to convene soonest, and the red latest; as in the figure where the blue are denoted by the pricked lines, and the red

Pl. 9.
Fig. 7.

red by the continued ; so that an image is formed at E F, by the concurrence of some of the more refrangible rays, and it is tinged around its edges by the red rays, which converging more slowly than the rest lie outermost.

After the blue rays have concurred, they cross one another, and go on diverging towards G H, where meeting with the red rays which have not yet concurred, and there mixing with them and the rays of other colours, they produce a white image, whiteness resulting from a due mixture of all the colours ; as they proceed forward toward I K, they, by reason of their greater divergence, spread themselves on all sides beyond the other rays, and by so doing, tinge the outlines of the image blue.

On the formation of pictures by means of a double *convex lens*, depend the appearances of the *camera obscura*, which is a small square box with a tube issuing horizontally from one side, at the extremity whereof is fixed a double *convex lens* ; within the box is placed a looking-glass in a slanting position, so as to be at half right angles with the bottom of the box, which is parallel to the horizon. On the top of the box is placed horizontally a plate of glass rough on one side, whereon the pictures of objects are represented in the following manner.

Let A B be an object placed before C D, the *lens* fixed in the tube which issues from the box ; G H the looking-glass inclined to the bottom of the box, in an angle of 45 degrees, L M the plate of rough glass covering the top of the box horizontally. The rays which flow from A, the uppermost point of the object, after they have passed the *lens*, converge towards F, and would actually meet at that point, but that they are intercepted by the looking-glass G H, which reflects them, and throws them upward ; and so far as the inclination of the rays towards one another is no way altered by the reflexi-

Pl. 9.
Fig. 8.
Exp. 2.

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on, they must meet at some point as K, as far distant above the *speculum*, as the point F is behind it. In like manner, the rays which flow from B, the lowest point of the object, and which after they have passed the glass are tending towards E, being reflected upward by the *speculum*, are made to convene at I, whose distance above the *speculum* is equal to the distance of E behind the *speculum*; and as the rays from the extreme points A and B, are made to convene at K and I, so those which flow from the intermediate points of the object, are brought together at corresponding points between K and I, whereby the image is projected horizontally, but with its right and left sides corresponding to the contrary sides of the object; as may appear by placing a man before the *lens*, and causing him to stir one of his hands; for in the image the other hand will appear to move.

Pl. 9.
Fig. 9.

The distance of the image behind the glass is always varied by varying the distance of the object before the glass; the image approaching as the object recedes, and receding as that approaches. For if we suppose A and C to be two radiating points, from which the rays AH, AK, and CH, CK fall upon the *lens* HK, it is manifest, that the rays from the more distant point diverge less than those from the nearer point, the angle at A being less than that at C; consequently, when they pass thro' the glass they must be brought together sooner, and must convene at some point as B, less distant from the *lens*, than is the point D, whereat the more diverging rays from the point C are made to convene.

Where the distance of the object and the *radius* of the *lens*'s convexity are given, and where the thickness of the *lens* is but small, as is commonly the case; the distance of the image from the *lens* is determined very nearly, by saying, as the distance of the object from the *lens*, lessened by the *radius* of

of the *lens's* convexity, is to the *radius*, so is the distance of the object from the *lens*, to the distance of the image from the *lens*; that is, putting D for the distance of the object, R for the *radius* of the convexity, and F for the distance of the image,

$$D - R : R :: D : F ; \text{ consequently, } F = \frac{RD}{D - R}.$$

The truth of this rule is demonstrated by the writers of DIOPTRICKS; but as all the demonstrations which I have hitherto met with are tedious and intricate, I shall not at present trouble you with them, but shall proceed to confirm the rule by experiments.

Let then the flame of a candle be placed at the distance of twelve feet and an half from a double *convex lens*, the *radius* of whose convexity is four feet two inches; that is, let the distance of the flame from the glass be equal to thrice the *radius*, and the image will be projected behind the *lens* at the distance of six feet three inches, that is, at the distance of a *radius* and an half; for in this case, R being put equal to unity, RD is three, which being divided by D — R, that is, by two, gives one and an half in the quotient.

Exp. 3.

If the flame be brought nearer to the *lens*, the image will move farther from it, and when the distance of the flame becomes equal to twice the *radius* of the *lens's* convexity, the distance of the image will be equal to that of the flame, the *lens* standing in the midway between them; for in this case D — R is equal to R, and of consequence, F is equal to D.

Exp. 4.

The flame being placed at the distance of the *radius*, the distance of the image becomes infinite. For in this case D — R is nothing, and F is equal to $\frac{DR}{0}$, which expression denotes an infinite quantity;

Exp. 5.

so that in this case, there will not be any image of the

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the flame; but the rays of light which flow from the candle, after they have passed thro' the *lens*, will go on parallel to one another; and by so doing, form a bright circular image, equal in size to the *lens*, and the magnitude thereof will remain the same at all distances from the glass.

Where the distance of the flame is less than the *radius* of the convexity, $D - R$ becomes a negative quantity, and so of consequence does the quotient arising from the division of DR by $D - R$; which shews, that the place at which the rays meet, lies on the same side of the *lens* with the flame; or to speak more properly, that the rays after they have passed the *lens*, proceed diverging from one another in such a manner, as if they had flowed from a point before the *lens*, more distant than the place of the flame. For the easier understanding of which, let the rays AB and AC flow from the point A , whose distance from the *lens* BC is less than the *radius* of the *lens*'s convexity; after they have passed the glass, they will not continue to go on in the directions BD and CE , but in the directions BF and CG , as if they had proceeded from some point as H , more distant from the *lens* than is the point A , from which they really flow; so that in this case, the rays after they pass the glass, go on diverging from one another, but however they do not diverge as much as they did before they passed the glass.

Fig. 13.
Pl. 9.

When the distance of the flame from the glass is so great as that neither the breadth of the *lens*, nor the *radius* of its convexity bears any sensible proportion to it, then $D - R$ is equal to D ; and of consequence, F is equal to R ; that is, the distance of the image is equal to the *radius* of the glass's convexity, and this is the least distance at which an image can be projected by such a *lens*; and forasmuch as the rays of the sun, which by reason of the immense distance of his body are always united
at

at the smallest distance, are apt to burn at the place of their union ; that place is usually called the *focus* or *burning point*, and sometimes the *absolute focus*, in contradistinction to those places whereat the images of less remote objects are formed, and which are frequently called the *respective foci*.

The length or breadth of an object, is to the length or breadth of its image, as the distance of the object from the *lens*, to the distance of the image from the *lens*. For if AC be the length or breadth of an object, and DF the length or breadth of its image; AB, which is one half of AC, is to FE, which is one half of FD, as BL to EL, the triangles ABL and FEL being similar. Hence it follows, that the nearer an object approaches the *lens*, the larger is its image, the image receding, and consequently enlarging, as the object approaches ; and thus it appears to be from experiments ; for the flame of a candle being placed at a distance greater than the diameter of the *lens's* convexity, in which case the distance of the image is less, appears larger than the image, but being brought within the distance of the diameter, the image, which in that case is at the same distance, becomes equal to it ; and upon bringing the flame still nigher, the image becomes larger in proportion to the square of its greater distance.

Pl. 9.
Fig. 6.

Exp. 6.

Exp. 7.

The same thing is likewise evident from the magick lantern ; which is a lantern out of which issues an horizontal arm, capable of being lengthened or shortened at pleasure, by means of one part sliding in and out of the other ; to the extremity of the moveable part is fitted a double *convex lens* ; and to that part of the arm which joins the lantern is adapted a glass, plane on one side, and *convex* on the other, the plane side looking towards the lantern ; in the body of the lantern there is placed a candle, whose distance from the *plano-convex* glass

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is somewhat less than the focal distance; so that the light which passes thro' that glass, is thrown very strongly upon little images painted in dilute colours on pieces of plane thin glass; which being fixed in a slider that moves to and fro across the arm, are placed at a small distance behind the *plano-convex* glass in an inverted position, and by means of the *lens* in the moveable part of the arm, are projected in an erect position, on a paper or white cloth placed at a proper distance; if by drawing out the moveable part of the arm, the pictures be removed to a greater distance from the *lens*, the lantern must be brought nearer to the cloth, in order to a distinct representation; because, as the object recedes from the *lens*, the image approaches, and at the same time the images will be diminished. But on the other hand, if by thrusting in the arm the pictures be brought near the *lens*, the lantern must be removed farther from the cloth, and in this case the images will appear larger.

As convex glasses cause the rays of light to converge and unite, so those which are concave make them separate and diverge; for which reason, if diverging rays fall upon a concave *lens*, they will diverge more after they have passed thro' it, than they did before; and such rays as converge before their incidence, will after their passage converge less; for instance, if the rays *AB* and *AC*, which diverge from *A*, pass thro' the concave *lens BC*, they will not go on in the directions *BD* and *CE*, but in some other directions as *BH* and *CG*, so as to widen faster than before. On the other hand, if *HB* and *GC* be two rays converging towards *K*, after they have passed thro' the glass, they will not go on towards *K*, but towards a more distant point as *A*, so as to converge more slowly than before. All which is fully confirmed by experiments. For a candle being placed before a *convex lens*, so as to have

Pl. 9.
Fig. 11.

Exp. 8.

have its image projected on a white paper, placed at a due distance behind the *lens*, if a concave glass be placed between the *convex* and the image, so as that the rays which are converging towards the image may pass thro' it, the image will thereby be thrown to a greater distance behind, the rays being made to converge more slowly, and of consequence, to meet at a greater distance than they did before the concave was interposed; and it must be observed, that as the image is thrown to a greater distance, it must for that very reason be enlarged; and forasmuch as the larger image is composed of the same number of rays, or rather fewer, some of the rays being reflected by the concave *lens*, it must on that account appear less bright and luminous than the smaller. If by the removal of the *convex-lens*, the rays which flow from the candle be suffered to fall diverging on the concave, and a white paper be placed close behind the glass, there will appear thereon a dark circle of some breadth, occasioned by the shadow of the hoop which contains the glass; and the circular *area* contained within the shadow will be enlightened by the rays which pass thro' the glass; and because all the rays which fall upon the glass do not pass thro' it, some of them being reflected, the circular *area* will appear somewhat darker than the other parts of the paper, which are exposed to the light of the candle, without the interposition of the glass; upon removing the paper gradually from the glass, the circular *area* will gradually enlarge, and as that enlarges, the shadow which environs it will grow narrower, and at length vanish; and upon the vanishing of the shadow, if the paper be removed a little farther, there will arise a bright circle all around the circular *area*, which will grow broader, but less bright, as the paper is more and more removed from the glass; and at the same time, the circular *area* will continue to widen, and grow darker. All which appearances

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appearances are the natural and necessary consequences of the divergency or spreading of the rays, occasioned by their passage thro' the glass; for the farther they go from the glass, the more they must diverge, and by so doing, must on all sides spread themselves into the place of the shadow, and render it equally luminous with the rest of the *area*; and when they have spread themselves a little beyond the limits of the shadow, they fall upon such parts of the paper as were before inlightened, and there, by their additional light, exhibit that bright circle which surrounds the darker *area*; and the bright circle, by the farther spreading of the rays, as the paper is more and more removed from the glass, grows broader and less luminous; as does likewise the circular *area*, from the spreading of the rays wherewith it is inlightened.

Tho' concave glasses do not collect the rays of light, and consequently, have not a real *focus*; yet inasmuch as the rays after they have passed thro' such glasses, do flow in such a manner as that they either tend to some point behind the glass, or appear to flow from some point before it, those points are usually called the *foci*; and in double concaves of equal concavities, the *foci* for converging rays are found, by saying, as the *radius* of the glass's concavity lessened by the distance of the point of convergence from the glass, is to the *radius*, so is the distance of the point of convergence to the *focus*. And the *foci* for diverging rays are found, by saying, as the sum of the *radius* and the distance of the point of divergence from the glass, is to the *radius*, so is the distance of the point of divergence to the *focus*. So that putting F for the *focus*, R for the *radius*, and D for the distance of the point of convergence, or divergence, $F = \frac{RD}{R \mp D}$; the negative sign being to be prefixed to D when the rays converge, and the affirmative when they diverge.

The

The demonstration of this *Theorem* I shall for the present omit, on account of its tediousness and intricacy, and shall close the lecture with this observation; that if rays which are converging towards a *focus* be intercepted by a concave *lens*, whose distance from the *focus* is equal to the *radius* of its concavity, after they have passed thro' the glass, they will cease to converge and become parallel, for R and D being equal, $R - D$ is 0; consequently, F is infinite; that is, the point to which the rays converge, is at an infinite distance, and the rays of course must be parallel.

LECTURE XXII.

OF VISION.

MY design in this lecture, is to explain the manner of VISION with the naked eye; and likewise to shew you, what assistances the sight receives from glasses; and in order thereto, I shall give you a short description of the eye.

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If a small portion be cut off of a globe, and in the room thereof a portion of a smaller globe, but of an equal circular base, be substituted, the compound will exhibit the true figure of the eye; for it is of a globular form, but more *convex* before than in any other part. It consists of several membranes which lie contiguous one to another, of which the outermost is called the *tunica adnata* or *conjunctiva*; it has its rise from that membrane which invests the skull, and it covers the whole ball of the eye, except the foremost transparent part, that portion of it which is visible, is called the *white of the eye*. Besides, this membrane, which is not reckoned among the proper coats of the eye, there are three others, which constitute the proper coats; the first of which is called the *sclerotica*, it is a tough membrane

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XXII.

membrane derived from the *dura mater*, which passes to the eye from the brain along with the *optick nerve*, and is thence propagated over the whole globe of the eye; on the fore part it becomes transparent like thin polished horn, which has given anatomists occasion to make two membranes of it, and to call the transparent part *cornea*; this part is represented by A B F.

Pl. 9.
Fig. 12.

The second membrane, called *tunica choroides*, is derived from the *pia mater*, and transmitted likewise from the brain along with the *optick nerve*; this is much thinner and tenderer than the former, and tinged on the hinder part with a black liquor. The fore part is called the *uvea*, and sometimes the *iris*, from its variety of colours. In its middle is a small hole called the sight or pupil; the *iris* consists of several circular concentrick muscular fibres, which are cut across at right angles by other strait fibres in the manner of so many *radii*; by the contraction of the former the pupil is lessened, and is enlarged by the contraction of the latter.

The third coat is usually called the *retina*, and sometimes the *nervous coat*, being nothing else but the *optick nerve*, which spreads itself in the form of a membrane over the bottom of the eye, over-against the sight. These coats lying contiguous, form a *capsula* or bag, wherein are contained the three humors of the eye, called the *aqueous*, the *chrystalline*, and the *vitreous*.

At a little distance behind the pupil is placed the chrystalline humor, which is *convex* on both sides, but somewhat flatter before than behind; it is supported by small muscular fibres, called the *ciliary ligaments*, which are inserted into the edges of the chrystalline humor at one end, and at the other, into the *tunica choroides*, and being closely united, form a kind of membrane, whereby the cavity of the eye is divided into two parts; in the foremost of which is lodged the *aqueous humor*, so called, because

because in consistence and colour it somewhat resembles water, being almost equally limpid and transparent. In the hindmost is lodged the *vitreous humor*, which has its name from the resemblance it is supposed to bear to melted glass.

It has been generally thought by anatomists, that the humors of the eye are of different densities, and that the chrySTALLINE is much more dense than either of the other two; but Doctor ROBINSON has informed us in his lecture upon the eye, that upon weighing these humors in an hydrostatical balance, he found the *aqueous* and *vitreous* to be very nearly of the same specifick gravity; and that the specifick gravity of the chrySTALLINE, did not exceed the specifick gravity of the others, in a greater proportion than that of eleven to ten; whence it follows, that the chrySTALLINE is not of such great use in bringing the rays together, and thereby forming on the *retina* the pictures of outward objects, as it has been commonly thought to be by *optical* writers; for tho' in shape it resembles a double *convex lens*, and on that account is fitted to make the rays converge, yet forasmuch as it is situated between two humors, which are nearly of the same density with itself, it can have but little force on the particles of light; for they are found by experience, to be refracted very little in passing out of one *medium* into another, when the difference in the densities of the *mediums* is but small.

Behind all the coats and humors is situated the *optick nerve*, which passes out of the skull thro' a small hole in the bottom of the *orbit* which contains the eye. O represents the *optick nerve*, SS the *sclerotica* or outermost coat, whose foremost transparent part ABF, is the *cornea*, CC is the *choroides*, the fore part whereof AP, and FP constitutes the *uvea* or *iris*, with the pupil PP in the middle; RR is the *retina*, AD and FE the *ciliary ligaments*, DE

Pl. 9.
Fig. 12.

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the *chryſtalline humor*, VV the *vitreous humor*, and WW the *watry humor*.

Underneath the white of the eye are inserted into the *sclerotica* fix muscles, which take their rise from different parts of the orbit, and are distinguished by different names, taken from the different motions which they give the eye; their tendons spread themselves over the *sclerotica*, so as to terminate in the confines of the *cornea*; by which means, when the six muscles act together, they press the sides of the eye towards each other, whereby the eye is lengthened, and at the same time the convexity of the *cornea* is increased; both which effects are in some cases absolutely necessary in order to distinct vision, as will appear presently.

Having given this short account of the constituent parts of the eye, I now proceed to lay before you, the *manner of vision*. If an object as AB, be placed at a convenient distance before the eye, the rays which flow from the several points of the object, and falling on the *cornea* pass thro' the pupil, will be brought together by the refractive power of the eye on so many corresponding points of the *retina*, and there paint the image or representation of the object, in the same manner as the images of objects placed before a *convex lens* are exhibited on white paper, placed at a proper distance behind. Thus the rays which flow from the point A, are united on the *retina* at C, and those which issue from B, are collected at D; and in like manner, the rays which proceed from the intermediate points of the object, are again united at so many intermediate points on the *retina*. On this union of the rays at the bottom of the eye, depends distinct vision, for should they be united before they arrive at the *retina*, or should the point of their union lie beyond the *retina*, it is evident, that the rays from each point must take up some space on the *retina*, and

Pl. 9.
Fig. 13.

Pl. 9.
Fig. 14.
Pl. 10.
Fig. 1.

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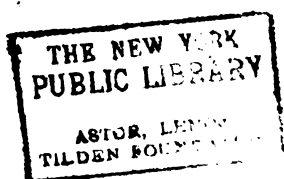
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rd of consequence, those which flow from continuous points of the object will be mixed and blended together on the fund of the eye, so as to exhibit a confused representation of the object.

Now forasmuch as the rays which fall upon the eye from radiating points, whose distances from the eye are different, have different degrees of divergence, the divergency of the rays increasing as the distance of the radiating point lessens, and lessening as that increases; and whereas those rays which have greater degrees of divergence, require a stronger refractive power to bring them together at a given distance, than what is requisite to make those meet which diverge less, it is manifest, that in order to see objects distinctly at different distances, the eye must have a power of increasing and lessening its refractive force, and thereby of adapting itself to the different distances of objects; and this it does by means of the six muscles which are inserted into the *sclerotica*; for when a radiating point is placed so near, as that the rays which issue from it fall upon the eye with a considerable degree of divergence, the muscles act strongly on the eye, whereby the *cornea* is rendered more convex, and of consequence refracts the rays with greater force; besides by the lengthening of the eye from the joint action of the muscles, the *retina* is removed to a greater distance from the *cornea*, by which contrivance, the rays are made to convene at the *retina*, notwithstanding the great degree of divergence wherewith they enter the eye. As the radiating point recedes from the eye, and the divergency of the rays of course grow less, the muscles relax themselves in order to lessen the convexity of the *cornea*, and to shorten the eye, a less convexity of the *cornea*, as also a less distance between the *cornea* and *retina*, being requisite to distinct vision in greater distances of the object than in smaller.

Tho'

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Tho' most mens eyes are so framed as to be able to see distinctly at different distances, yet some there are which are defective in this point, as being unable to see any thing distinctly but when placed very near; and this is the case of their eyes who are called *myopes*, purblind, or short-sighted; in such the *cornea* is too convex in proportion to the length of the eye; for which reason, all those rays which issue from distant points, and of consequence diverge but little when they enter the eye, are made to converge before they reach the *retina*. As these men advance in years, their eyes like those of other old men, for want of a due supply of humors, abate of their convexity and grow flatter; upon which account they begin to see objects distinctly at a distance, without the help of spectacles, and are for that reason deemed to have the most lasting eyes.

By the help of concave glasses, purblind persons, may see distant objects distinctly; for as it is the property of such glasses to make the rays diverge, if the rays which flow from a distant point, and fall upon the eye with a small degree of divergencie, be made to pass thro' a concave *lens* of a proper concavity, they will thereby be made to diverge so much, as that the eye, notwithstanding the great convexity of the *cornea*, shall not be able to bring them together till they arrive at the *retina*.

Pl. 10.
Fig. 2.

If CD be a concave *lens*, and if B be the *focus* of the rays which flow from the point A; that is, if the rays which diverge from A, pass thro' the glass, and by the refraction which they suffer in their passage, proceed in such a manner as if they had diverged from B; and if the distance at which a purblind person sees distinctly with his naked eye, be equal to the distance of B from the glass, such a person will by the help of the glass CD, be able to see the point A distinctly; because the rays which flow from A, after they pass thro' the glass, fall
upon

upon his eye with the same degree of divergence, as if they had issued from B, the point of distinct vision. Hence it follows, that if in the *Theorem* laid down in my last lecture, for finding the *focus* of double concaves exposed to diverging rays, namely

$$F = \frac{RD}{R+D},$$

wherein F denotes the *focus*, D the distance of the point of divergence, and R the *radius* of the concavity, we suppose F to denote the distance at which the purblind person sees distinctly without a glass, and D the distance at which he sees distinctly by the help of the glass, by clearing R

$$\text{we shall have } R = \frac{FD}{D-F};$$

that is to say, the *radius* of the concavity of a double concave of equal concavities, which enables a purblind person to see an object distinctly, when placed beyond the reach of his naked eye, must be equal to a rectangle, under the distance at which he sees distinctly with his naked eye, and the distance at which it is required he should see distinctly by the help of the glass, divided by the difference of those distances. For instance, if a person with his naked eye can read at the distance of three inches only, and it be required to find the *radius* of such a glass as shall enable him to read at the usual distance of eighteen inches; in this case, F being equal to three inches, and D to eighteen, their product is 54; which being divided by their difference, which is 15, gives three and $\frac{2}{3}$ in the quotient, which shews, that the *radius* of the glass must be three inches and $\frac{2}{3}$ ths nearly.

Where the distance at which it is required the purblind person shall see distinctly is infinite, or in other words, where it is so great, as that the distance to which the power of his naked eye reaches, bears no sensible proportion to it, there D—F becomes equal to D, and of course, R becomes equal to F; so
Y
that

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that in order to see such objects as are very remote, purblind persons must make use of concave glasses, whose *radii* are equal to the distances at which they see distinctly with their unarm'd eyes.

As purblind persons cannot see remote objects distinctly, so on the other hand, those who are old cannot, generally speaking, see such as are nigh; the reason of which is, that in old men the *cornea*, for want of a due supply of humor to plump out the eye, has not a degree of convexity sufficient to bring the rays together on the *retina*, when they fall upon the eye with a considerable degree of divergence; as is the case of all those rays which flow from points situated near the eye. The proper remedy for this defect is a *convex lens*, because it lessens the divergency of the rays, and brings them nearer to a parallelism. If with respect to the *convex lens* CD, A be the *focus* of the rays which diverge from B; that is to say, if the rays which flow from B and pass thro' the *lens*, do afterwards proceed in such a manner as if they had diverged from A, and if the distance at which an old man can see distinctly with his naked eye, be equal to the distance of A from the glass, he will be able by the assistance of the glass, to see the nearer point B distinctly; because the rays which issue from that point in passing thro' the glass acquire the same degree of divergence, with those which flow from A, the point of distinct vision, and of consequence, may as easily be brought together on the *retina*, by the refractive power of the eye; hence, if we take the *Theorem* laid down in my last lecture for finding the *foci* of double *convexes* of equal convexities, and fit it to the case before us, where the *focus* is imaginary, by making $F = \frac{RD}{R-D}$, if then we suppose F to denote the distance at which an old eye sees distinctly, and D the nearer distance at which it is

Pl. 10.
Fig. 3.

is required to make it see distinctly with the assistance of a glass, by clearing R, we shall find it equal to $\frac{FD}{F-D}$. So that the *radius* of such a dou-

ble convex of equal convexities as enables an old man to see a nigh object distinctly, must be equal to a rectangle under the distance at which he sees distinctly with his naked eye, and the distance at which he is to see by the help of the glass, divided by the difference of those distances. To illustrate this by an example; suppose an old man cannot with his naked eye read at a less distance than of four feet, and it is required to assign the *radius* of spectacles which shall enable him to read at the distance of a foot and a half; in this case, F is four feet, and D is one and an half, and their product is six, which when divided by their difference, to wit, two and an half, gives $2\frac{4}{5}$ in the quotient; which shews, that the spectacles must be ground to a *radius* of two feet and four tenths.

If F be infinite, which is the case where the eye can see nothing but what is extremely remote, then $F-D$ is equal to F, and of consequence, R is equal to D; so that where an old man can see no objects distinctly but such as are very far off, in order to see distinctly at nearer distances, he must for each distance use such spectacle glasses as have their *radii* equal to the distance.

If D be given, then R becomes equal to $\frac{F}{F-1}$; and forasmuch as the proportion of F to $F-1$ increases as F lessens, R must do so too, which shews, that where the distances at which two old eyes when unarmed can see distinctly are different, in order to make them see distinctly at any lesser given distance, the eye which can see at the smaller distance must be furnished with a glass of a greater *radius* than the other. And herein lies the whole secret of younger and older spectacles, those being deemed

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the youngest, which are ground to the largest radius.

Having shewn you of what use both convex and concave glasses are in assisting defective eyes, I shall now lay before you the alterations which they produce in the appearances of objects; and First, as to *convexes*; if an object be viewed through a *convex lens*, at a less distance than the *focus*, it appears more remote and bigger than it does to the naked eye. That it must appear more remote, will be evident, if we consider what has been already proved in a former lecture, namely, that where rays fall upon a *convex lens*, from a point less distant than the *focus*, after they have passed the glass, they proceed in such a manner as if they had issued from a more distant point; and since this is the case of the rays which flow from each point in the object, the object must of consequence seem to be more distant than it is, and it must likewise appear greater; for if *A B* be an object exposed to a naked eye at *O*, its extremities *A* and *B* will be perceived by the eye by means of the rays *AO* and *BO*, which flow directly from those points to the eye, but if a *convex lens* as *C*, be interposed, the eye will no longer perceive the extremities by means of the rays *AO* and *BO*, because as they are refracted by the *lens*, they are made to concur before they can reach the eye; the eye therefore must now perceive those points by means of some other rays as *A E* and *B D*, which falling upon the glass at a greater distance from each other, are by the refractive power of the glass thrown into the directions *EO* and *DO*, and made to concur at *O*; so that continuing those lines directly backward as far as the object, to wit, to *I* and *H*, the eye at *O* will perceive the extremities of the object as situated at *I* and *H*; that is, it will perceive the object magnified. And if the eye be farther removed from the glass suppose to *P*, the object will appear still greater, its extremities in that

Pl. 10.
Fig. 4.

that case appearing at L and K in the lines PG and PF produced. And on the other hand, if the eye continuing in its place, the object be farther removed from the *lens*, it will appear larger; for whereas at the nearer distance the eye perceives the extream points of the object by means of the rays AE and BD, which fall upon the *lens* at E and D, and are thence refracted to O; when the object is at the greater distance, its extremities cannot be seen by means of the rays incident on the glass at E and D; for since the interval between the extremities continues the same, the rays which flow from them and fall upon the *lens* at E and D, will diverge less at a greater distance of the object than at a smaller; consequently, they will concur before they reach the eye; and therefore in this case the extream points of the object must be conveyed to the eye by some rays as aG and bF, which diverging more than the former, fall without them at G, and F, whence they are refracted to the eye at O, in the lines GO and FO, which being continued backward as far as the nearer distance of the object, to wit, to L and K, shew that the object which at the nearer distance appeared to extend itself only from I to H, does at the greater distance seem to reach from L to K, and of consequence, appears more magnified.

If the object be removed beyond the *focus*, it will appear still greater; but whereas before it passes the *focus* it appears distinct, as also more and more distant the farther it is removed from the glass, when it gets beyond the *focus* it appears confused, and the farther it is removed from the glass, the more confused it appears, and the nearer it seems to approach the eye, provided its distance from the glass be not so great as to make it project its image between the eye and the glass.

This seeming approach of the object at a time when it really recedes, and in a case where, accord-

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ing to the received principles of *Dioptricks*, it ought to appear at a distance, if possible more than infinite, has very much puzzled the writers of *opticks*, and was looked upon as an insuperable difficulty, till Doctor BERKELEY took it into consideration in his *Essay upon Vision*, wherein, among other difficulties which he has cleared up relating to vision, he has given us a natural and satisfactory account of this. The substance of what he has there delivered concerning this matter is, that by custom and experience we are taught to judge those objects near which appear confused, because, according to the ordinary course of nature, those objects, and those only, appear confused which are brought very near the eye, and therefore if an object shall at any time appear confused, tho' from another cause, the mind will immediately connect nearness of distance in the object, with that confusion in the appearance, as having always experienced them to go together; and the greater the confusion is, the nearer it will judge the object to be, because it has always observed the nearest distances to be attended with the greatest confusions: now if in the case before us, we suppose A to be an object placed before the *convex lens* BC, at a greater distance than the *focus*, the rays after they have passed thro' the glass will converge towards some point as D; if then an eye be placed at a little distance behind the glass, suppose at E, it will perceive the object confused, because as the rays fall upon it converging, they will be made to meet before they arrive at the fund of the eye, and consequently, will be scattered on the *retina*, and thereby render the appearance confused; if the eye be moved gradually backward to F, G, and D, or which is the same thing, if by carrying the object forward, the rays be made to fall upon the eye at less and less distances from the *focus*, they will be scattered more and more upon the *retina*, because the convergency wherewith they fall upon the eye

Pl. 13.
Fig. 6.

eye is by so much the greater, by how much the nearer the eye is placed to the *focus* or the point D; consequently, the object as it is more and more removed from the glass, will appear more and more confused; for which reason, the mind which has been used to connect nearer distances with greater degrees of confusion, will in this case judge the object to approach, tho' in reality it recedes; and what fully confirms this is, that if by placing a concave glass at a proper distance between the eye and the convex, the convergency of the rays be taken off, and the appearance thereby rendered distinct, the object will then appear at its due distance.

If an eye be removed from a *convex lens* beyond the place where the image is projected, that is, if the eye be farther from the *lens* than is the point D, the object will appear in an inverted position, and seem to be situated between the eye and the glass; for in this case, the eye sees only the image or representation of the object, which, as I shewed in a former lecture, is projected at D in an inverted position; upon looking at the image with both eyes, it appears double, and upon shutting either eye, the image on the contrary side disappears; the reason of which is this, the eye at O perceives the image by means of the rays ODC, and therefore sees it on the same side with C, whereas the eye at P perceives it by means of the rays PDB, and on that account sees it on the same side with B; as the head is moved farther back, the distance between the two images must decrease, and at length vanish; for since the interval between the eyes continues unvaried, the rays which exhibit the image to each eye, will diverge less and less as the head is more and more removed from D, as is evident from the bare inspection of the scheme; consequently, the distance between the two images must continually decrease, and at last become so small as to be insensible.

Pl. 10.
Fig. 6.

Pl. 10.
Fig. 7.

As to concave glasses, since it is their property to make the rays which flow from any point to diverge, in such a manner as if they had issued from a point less distant, it is evident, that an object seen thro' a *concave lens* must appear nearer than it really is, and it must likewise appear diminished; for the extremam points of the object AB, are seen by the naked eye by means of the rays AO and BO, which when the *concave lens* CD is interposed, are made to diverge, so as not to meet at O, consequently, upon the interposition of the glass, the eye will not perceive the extremities of the object by those rays, but by some others as AK and BL, which falling within the former, are by the refractive power of the glass made to proceed in the lines KO and LO, so as to meet at O; wherefore continuing OK and OL backward to the object, the extremities of the object will be seen at E and F, that is, the object will appear to be less than it really is; and by the spreading of the rays in their passage thro' the glass, some of them are made to escape the eye, which if the glass were removed, would fall upon the pupil; for which reason, the object must appear less luminous; so that the property of concave glasses is to make objects appear smaller, nearer, and more faint and obscure, than they do to the naked eye.

LECTURE XXIII.

OF CATOPTRICKS.

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IN this lecture, wherewith I shall close this course, I shall explain to you the *Doctrine of CATOPTRICKS*, or that part of *Opticks* which treats of the reflexion

reflection of light; in doing of which, I shall first say something concerning the cause of that reflexion; Secondly, I shall lay down two principles, which are the chief foundation of *Catoptricks*; and lastly, I shall lay before you the most remarkable properties of plain and spherical mirrors.

As to the first, before Sir ISAAC NEWTON published those wonderful and surprising discoveries which he made, concerning the nature and properties of light, it was an opinion generally received by the writers of *opticks*, that the rays of light were reflected in the manner of other bodies, by striking on the solid and impervious parts of bodies; but that great Philosopher has fully proved this opinion to be erroneous; and has shewn, that the particles of light are turned back before they touch the reflecting body, by some power of the body which is equally diffused all over its surface; what he has delivered concerning this matter, is to be met with in the eighth *Proposition* of the *second Book of his Opticks*, wherein, after he has offered several reasons to prove, that light is not reflected by striking against bodies, he at last expresses himself in the following manner; “ Were the rays of light reflected
 “ by impinging on the solid parts of bodies, their
 “ reflexions from polished bodies could not be so
 “ regular as they are; for in polishing glass with sand,
 “ putty, or tripoly, it is not to be imagined, that
 “ those substances can, by grating and fretting the
 “ glass, bring all its least particles to an accurate polish, so that all their surfaces shall be truly plain,
 “ or truly spherical, and look all the same way, so
 “ as together to compose one even surface. The
 “ smaller the particles of those substances are, the
 “ smaller will be the scratches by which they continually fret and wear away the glass until it be
 “ polished; but be they never so small, they can wear
 “ away the glass no otherwise than by grating and
 “ scratching

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“ scratching it, and breaking the protuberances,
 “ and therefore polish it no otherwise than by bringing its roughness to a very fine grain; so that the scratches and frettings of the surface become too small to be visible. And therefore, if light were reflected by impinging on the solid parts of the glass, it would be scattered as much by the most polished glass, as by the roughest. So then it remains a *Problem*, how glass polished by fretting substances can reflect light so regularly as it does; and this *Problem* is scarce otherwise to be solved, than by saying, that the reflection of a ray is effected, not by a single point of the reflecting body, but by some power of the body, which is evenly diffused all over its surface, and by which it acts upon the ray without immediate contact.”

Now taking it for granted, that this repelling power is the true cause of reflection, if it be supposed to act upon the rays of light in lines perpendicular to the surface of the reflecting body; it will thence follow, that the angle of incidence, or the angle contained between the incident ray, and a line drawn perpendicular to the reflecting surface at the point of incidence, is equal to the angle of reflexion, or the angle contained between the same perpendicular and the reflected ray. For if we suppose a ray of light to move in the direction AC, towards the reflecting surface BCD; and if we suppose that motion to be resolved into two, one in the direction AE, parallel to BD, and the other in the direction AB, perpendicular to BD, it is manifest, that of those two motions, the latter only is opposed to the repelling force; and of consequence, the ray after reflexion, will go on in the parallel direction, with the same velocity it did before; and forasmuch as the repelling force which opposes the perpendicular motion, acts incessantly, it no sooner destroys the motion of the ray towards the body, but

Pl. 10.
Fig. 8.

It gives it an equal degree of motion the contrary way; that is, it throws it back with the same perpendicular velocity wherewith it approached. If therefore EG be taken equal to AE, and from G be let fall GD equal and parallel to AB, EG will express the parallel motion of the ray after reflexion, and DG its perpendicular motion; and the diagonal line CG, will be actually described by the ray, by virtue of its compound motion; and from the nature of similar triangles, the angle of incidence ACE, must be equal to ECG, the angle of reflexion; and this is the first of those principles whereon the doctrine of *Catoptricks* is founded. The second is, that every radiant point when seen by reflexion, appears in that place where the reflected ray meets the perpendicular, drawn from the radiant point to the reflecting surface; for instance, if from a radiant point as R, placed before the plain *speculum* AB, be let fall the line REM, perpendicular to the plane of the *speculum*; and if RC and CD be so drawn, as that the former may denote the incident ray, and the latter the reflected; and if DC be continued on, till it meets the perpendicular REM; an eye at D will perceive the radiant point, as placed at M, the point of intersection of the reflected ray, and the perpendicular; and thus it is in all cases of reflexion, except two, wherein this principle seems to fail; one whereof relates to plain glass *speculums*, and the other to concave spherical mirrors; the latter has been observed by TAQUET, Doctor BARROW, and others; but the former has not been mentioned by any one of the *optick* writers that I know of; I shall take notice of each in its proper place, and proceed now to consider the chief properties of mirrors; and first, of such as are plain.

Pl. 10.
Fig. 8.

When an object is seen by reflection from a plain *speculum*, its image appears as far behind the *speculum*, as the object is before; for the proof of which
let

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Fig. 9.

let R be an object placed before the plain *speculum* A B, and let it be seen by reflexion from the point C, by an eye situated somewhere in the line C D, then producing CD, till it meets the perpendicular R E M, the image will, by the second principle, appear at M; now the angles of incidence and reflexion being equal, their complements are so too, that is to say, the angle RCE is equal to DCB or MCE; so that in the two right-angled triangles, the angles at C being equal, and the side E.C common to both, the triangles must be equal, and the side M E, that is, the distance of the image behind the *speculum* must be equal to R E, the distance of the object before the *speculum*; and the same thing is in like manner demonstrable, tho' the point of reflexion be taken different from C; for the reflected ray will constantly meet the perpendicular in the point M; whence it follows, that however the situation of the eye with respect to the mirror may be changed, yet if the object and mirror remain unmoved, the image will always appear in the same place; it likewise follows, that there cannot appear more than one image of one and the same object; but then this is to be understood with respect to such mirrors, as being opaque, have but one reflecting surface; for in looking-glasses, which by reason of their transparency, have a double reflexion in some certain positions of the eye and object, several images may be seen. Thus if A B be a looking-glass, R the flame of a candle, placed at a small distance before A H, the plane of the glass produced, an eye being placed at Q, shall see several images standing at small distances one beyond another, in the same position with the letters, C, D, E, F, whereof the first and second appear bright and luminous, and the rest but faint and obscure; for the several images taken in their order from the second, grow more and more dark and obscure, till at length they

Pl. 10.
Fig. 10.
Exp. 1.

they become too weak and feeble to affect the sight, and of consequence vanish.

In order to account for this multiplicity of images, let $ABCD$ be a looking-glass, whose nearest surface, or that which lies next the eye is AB , and its farther or silvered surface is DC , R the place of the candle, and Q the place of the eye, RS a line drawn from the candle perpendicular to AO and DY the two surfaces of the glass produced; the angle REA being made equal to QEB , and the line QE being produced till it cuts the perpendicular RS in T , the eye shall see the first image at T , by means of the reflexion from the outward surface AB , the ray RE being reflected to the eye from the point E . Let a second ray as RG , pass into the glass at G , and being refracted to the point H of the farther surface, let it thence be reflected to K , and there passing out of the glass, let it by refraction be carried to the eye; let then QK be produced, and the eye shall see a second image situated in that line, and that at a little distance beyond the perpendicular RS ; for if the rays suffered no refraction in passing in and out of the glass, the second image would not be seen by means of the ray RG , but by means of the ray RH , which passing directly from R to H , is thence reflected directly to Q , and being produced till it cuts the perpendicular in X , would exhibit the second image at X ; but forasmuch as the place of the image is changed by the refraction, and brought nearer to the glass, if we suppose the line QX to be moved upward about the point Q , till it coincides with the line QV , in which the second image really appears, the point X must necessarily fall beyond the perpendicular, and so of consequence, must the place of the image. Let now a third ray as RF , pass into the glass at F , and be refracted to L , and from thence let it be reflected to E , and from E to M , and from M to

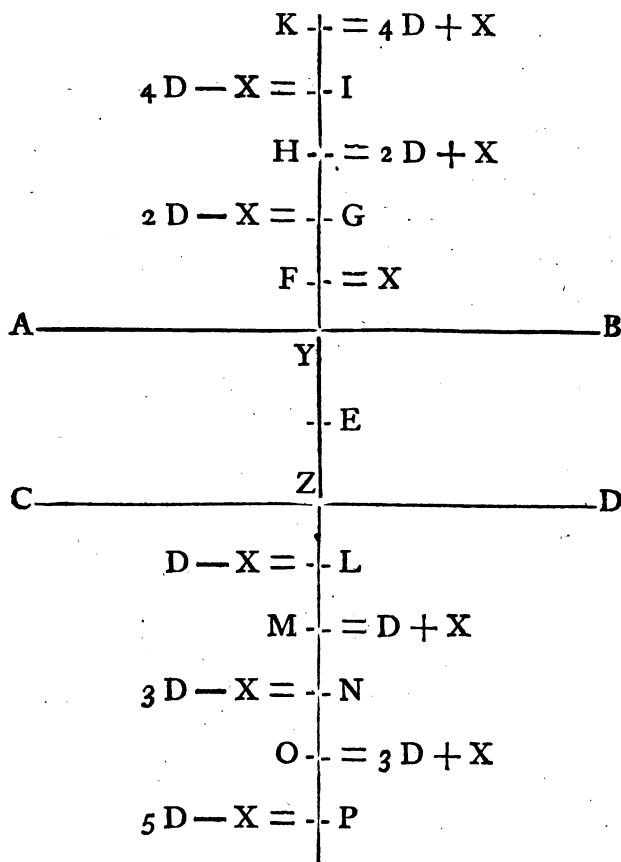
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Pl. 10.
Fig. 11.

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M to N, where let it go out, and be refracted to the eye at Q; then producing QN to W, a third image will appear in that line somewhere beyond the perpendicular; for were there no refraction, the ray which after three reflexions exhibits the third image, would when produced cut the perpendicular in the point S; and therefore, since the line QS is raised up by the refraction, and made to coincide with the line QW, the point S, that is, the place of the third image, must fall beyond the perpendicular.

As a third image is seen by means of three reflexions, so is a fourth by five reflexions, a fifth by seven, a sixth by nine, and so on, according to the progress of the odd numbers, every succeeding image being seen by two reflexions more than the preceding; and this is the true reason why, setting aside the first and second, which being seen each by one single reflexion, appear almost equally bright, every succeeding image appears more dim and faint than the foregoing, the rays of light being rendered more weak and feeble by reflexion.



If two plane *speculums* as AB and CD , be set parallel to one another, and an object be placed any where between them as at E , the rays of light which issue from the object and fall upon each *speculum*, will be reflected backward and forward from one to the other a great number of times; by which means, there will appear in each *speculum* a great number of images situated one behind another, in a right line perpendicular to the *speculums*, and passing thro' the object at E ; in order to determine the

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the distances of the several images from the *speculum*, let EY , the distance of the object from the *speculum* AB , be denoted by X , and let ZY , the interval of the glasses, be denoted by D ; and let us first consider the reflexion which begins from the *speculum* AB ; if YF be taken equal to X , then F will be the place of the first image; and forasmuch as the image at F may be looked upon as an object placed before the *speculum* CD , if ZM be taken equal to FZ , that is, to $D + X$, there will appear an image at M ; which being considered as an object with respect to the other *speculum* AB , and YH being taken equal to MY , or $2D + X$, another image will be seen at H ; and for the same reason, if ZO be taken equal to HZ , or $3D + X$, there will another image appear at O , and so on; again, if we consider the reflexion which begins from the *speculum* CD , by taking ZL equal to EZ , or $D - X$, we shall have L for the place of an image in the *speculum* CD ; and by making YG equal to LY , or $2D - X$, we shall have the place of another image in the *speculum* AB ; and again, by taking ZN equal to GZ , or $3D - X$, we shall have the place of another image in the *speculum* CD , and so on. From this manner of determining the places of several images, it is evident, that if D , which stands for the distance of the glasses, be multiplied into each of the even numbers taken in their order, and if X , which denotes the distance of the object from AB , be subducted from each product, and likewise added to each, the differences, and the sums taken in their order, will express the distances of the several images from the *speculum* AB , the distance of the first being X ; that is to say, the distance of the second image will be $2D - X$, of the third $2D + X$, of the fourth $4D - X$, of the fifth $4D + X$, and so on, according to the first Table.

TABLE

TABLE I.

*Distances of the several images from the speculum
AB.*

$$\begin{aligned}
 1 &= X \\
 2 &= 2D - X \\
 3 &= 2D + X \\
 4 &= 4D - X \\
 5 &= 4D + X \\
 6 &= 6D - X \\
 7 &= 6D + X \\
 8 &= 8D - X \\
 &\quad \&c.
 \end{aligned}$$

If D be multiplied into each of the odd numbers taken in their order, and if X be deducted from each product, and likewise added to each as before, the differences and the sums taken in their order, will express the distances of the several images from the *speculum* CD, as in the second Table.

TABLE II.

*The distances of the several images from the speculum
CD.*

$$\begin{aligned}
 1 &= D - X \\
 2 &= D + X \\
 3 &= 3D - X \\
 4 &= 3D + X \\
 5 &= 5D - X \\
 6 &= 5D + X \\
 7 &= 7D - X \\
 8 &= 7D + X \\
 &\quad \&c.
 \end{aligned}$$

If by moving the object nearer to AB, X becomes less, then all those images whose distances are expressed by those symbols wherein X is affirmative, will come nearer to the *speculums*, whilst those whose distances are expressed by symbols
Z
wherein

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Exp. 2.

Pl. 10.
Fig. 12.
Exp. 3.

wh rein X is negative, move farther off; thus in the *speculum* AB, the first, third, fifth, seventh and so on, will approach, and the second, fourth, sixth, eighth, and so on, will recede; so that the several images, beginning from the first, will approach and recede alternately; and on the other hand, those in the *speculum* CD will recede and approach alternately, beginning from the first; hence, if a man puts his hand between the two *speculums*, and moves his palm towards one of them, a person looking into the other, shall see several pairs of hands, palm to palm, approaching each other.

If two plane *speculums* as AC and BC, be inclined to one another, so as to meet in an acute angle at C; and if an object be placed any where between them, suppose at F, an eye looking into either, shall see several images standing in the circumference of a circle, whose center is at C the concourse of the *speculums*, and its *radius* equal to CF the distance of the object from the concourse; for if from F be drawn FD perpendicular to the *speculum* CA, and KD be made equal to FK, D will be the place of an image in the *speculum* CA; and if from D be drawn DE perpendicular to the *speculum* CB, and produced till HE is equal to DH, E will be the place of an image in the *speculum* CB, and thus by drawing perpendiculars continually from the place last found to the opposite *speculum* may the places of all the images be found which are seen by means of those reflexions, the first whereof is made from the *speculum* CA; and in the same manner, by drawing the perpendiculars FG, GL, and so on, may the places of all those images be found which are seen by means of the reflexions whereof the first is made from the *speculum* CB. Now that the points D and E are in the circumference of the circle whose *radius* is CF, I thus prove; in the triangles CFK and CDK, the sides FK and DK are equal by the construction, and CK is common to both,

thus both, and the angles at K are right ones, where-
 fore the two triangles are equal, and of consequence
 CD is equal to CF; again, the triangle CDH is
 equal to the triangle CEH, the sides DH and EH
 being by construction equal, as are also the angles
 at H, wherefore CE is equal to CD, which is equal
 to CF, consequently, a circle described on the cen-
 ter C, with the radius CF, will pass thro' the points
 D and E; and by the same way of reasoning, it
 will be found to pass thro' G and L, and thro' the
 extremities of all the other perpendiculars; and
 therefore the several images must of necessity appear
 in the circumference of a circle whose center is at
 the concourse of the *speculums*, and whose radius is
 equal to the distance of the object from that con-
 course. From what has been said, it follows, that
 if the distance of the object from the concourse of
 the *speculums* be given the images will still appear
 in the circumference of the same circle, notwith-
 standing any alteration that may be made in the
 angle whereat the *speculums* meet; if that be en-
 larged, the images will be fewer in number, and at
 greater distances from one another; and on the
 other hand, if it be made less, the images will be
 more in number, and stand closer together, but the
 circle in whose circumference they appear, will be
 the same in both cases; for that is not to be lessened
 or enlarged otherwise, than by lessening or en-
 larging the distance of the object from the con-
 course of the *speculums*.

Having laid before you the chief properties of
 plain *speculums*, I come now to consider such *spe-*
culums as are spherical; and they are of two sorts,
concave and *convex*; concerning which it must be
 observed, that as all the rays which fall upon them
 from a radiating point, are reflected in such man-
 ner as to meet the perpendicular very nearly in one
 and the same point; in order to find out the *focus*,

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XXIII.Pl. 10.
Fig. 13.

or the place where the reflected rays cross one another, nothing more is necessary, but to determine the point wherein any one reflected ray meets the perpendicular; which may be done in the following manner; let A be a radiating point, exposed directly before the concave glass FG, whose center is C, AB a perpendicular from the radiating point to the *speculum*, which likewise denotes the distance of the radiating point from the *speculum*, AD a ray falling on the *speculum* at D, whose distance from B is indefinitely small, DE the reflected ray meeting the perpendicular in E, CD a *radius* drawn to the point of incidence, and of consequence bisecting the angle ADE in the triangle ADE; since the angle at D is bisected by the line DC, which cuts the opposite side, AD is to DE, as AC to CE; but forasmuch as the points D and B are supposed to be indefinitely near, AD is equal to AB, and ED is equal to EB, wherefore AB is to EB, as AC is to CE; that is the distance of the radiating point from the *speculum*, is to the distance of the point E, where the reflected ray cuts the perpendicular, commonly called the *point of intersection*, as the distance of the radiating point, lessened by the *radius*, is to the *radius*, lessened by the distance of the point of intersection; that is, putting D for the distance of the radiating point, F for the distance of the point of intersection, and R for the *radius*, $D : F :: D - R : R - F$; consequently, reducing this analogy into an equation, and clearing F, F will be found equal to $\frac{DR}{2D - R}$; that is, the distance of the point of intersection from the *speculum*, and consequently, the distance of an image formed by reflexion from a concave *speculum*, is equal to a rectangle under the distance of the object from the *speculum*, and the *radius*, divided by twice the distance of the object lessened by the *radius*.
Hence

Hence it follows, that if an object be placed before a concave *speculum*, at an infinite distance, that is, if the distance be so great as that the *radius* of the *speculum* bears no sensible proportion to it, the image will appear on the same side of the *speculum* with the object, at the distance of one half the *radius* from the *speculum*; for in this case, D being infinite, $2D - R$ becomes equal to $2D$, and of consequence, F is equal to R divided by 2; so that one half the *radius* is the least distance at which an image can be projected from a concave *speculum* on the same side with the object; and forasmuch as the sun's image, which, by reason of the immense distance of his body, is formed at the distance of half the *radius* from the *speculum*, is there apt to burn, that place is usually called the *focus* or *burning point*.

As the object approaches the *speculum*, the image recedes; for as in one and the same *speculum*, the *radius* is a standing quantity, it is manifest, that as D lessens, the proportion of DR to $2D - R$ must increase, consequently, F , or the distance of the image from the *speculum* must do so too; and when the object has approached so near the *speculum* as to be at the center, the image will have receded so far as to be there likewise; for in this case, D being equal to R , $2D - R$ is equal to R , and of consequence, F is equal to D ; so that the object and its image meet at the center of the *speculum*; upon the object's passing from the center towards the glass, the image is projected beyond the center, and when the object has approached so near the *speculum*, as to be distant from it but half the *radius*, the image is at an infinite distance; for in this case, D being equal to half the *radius*, $2D - R$ is nothing, consequently, F , that is the distance of the image, is infinite; or to speak more properly, the rays after reflexion proceed parallel; for which reason, if the flame of a candle be placed directly before a con-

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cave *speculum*, at the distance of half the *radius*, the *speculum* will seem to be in flames, and the reflected light will be so intense, as that by the help of it one may be able to read at a very considerable distance from the *speculum*. TAQUET asserts, that he has read at the distance of no less than 400 feet; and to say the truth, the distance would be without limits, were it not for the atmosphere, whose particles continually intercept the rays, and by so doing at length totally extinguish the light. It sometimes happens, that when the flame of a candle is placed in the *focus* of a concave *speculum*, its image is projected on a distant wall, which seems to invalidate the truth of what I just now proved concerning the parallelism of the rays after reflexion, but this is occasioned by the flames being too large to be contained totally within the *focus*; for were it so small as to lie wholly within the *focus*, it would not project an image, but the rays after reflexion, would form a cylindrical body of light, which when projected on a distant wall, would have a circular figure, of an equal circumference with the *speculum*.

When the distance of the object from the *speculum* is less than half the *radius*, the image appears behind the *speculum*; for in this case, $2D - R$ is a negative quantity, and of consequence, so is F , which shews, that the distance of the image which is denoted by F , must be taken on the other side of the *speculum*, with respect to the object; as the object moves nearer to the *speculum* before, so likewise does the image behind; and when the object is so near as to touch the *speculum*, the image does the same; for in this case, D being nothing, F , that is, the distance of the image from the *speculum*, is likewise nothing.

As to the position of the images which are seen by reflexion from a concave *speculum*, those which appear

appear on the same side of the *speculum* with the object must be inverted; and those which appear behind the *speculum* must be erect. For the proof of which, let AB be an object placed before the concave *speculum* F'G, at any distance beyond the center C, in which case the image will be seen between the center and the *speculum*, suppose at DE; from A and B, the extrem points of the object, let the lines AH and BI be drawn perpendicular to the *speculum*, and of consequence crossing one another at the center; this being done, since the image is supposed to be at DE, and since every point of an image is seen in the perpendicular drawn from the corresponding point in the object, it is manifest, that D, the lowest point of the image, will correspond to A, the highest point of the object, and E, the highest point of the image, will correspond to B, the lowest point of the object, that is, the image will appear inverted. And by the same way of reasoning, if DE be the object, situated at such a distance between the *speculum* and the center, as to have its image projected beyond the center at AB, the image must appear inverted. On the other hand, where an object as DE, is placed between the *speculum* and the center, and consequently, projects an image behind the *speculum*; for it must be observed, that the same object DE, which when situated between the center and the *speculum*, at a less distance from the center than half the *radius*, projects an image as AB beyond the center, does likewise project another image as HI, behind the *speculum*; and as the former image is visible to an eye placed beyond it, so the latter image is visible to an eye placed between the object and the *speculum*, and it must appear erect, inasmuch as the perpendicular CH, which passes thro' D the highest point of the object, does likewise pass thro' H, the highest point of the image.

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Fig. 13.Pl. 10.
Fig. 14.

As to the magnitudes of an object and its image they are to one another in the same proportion with the squares of their distances from the *speculum*; for if the line LCM be drawn thro' the center C, perpendicular to BA, DE, and HI, and consequently, bisecting the angle at G, if BA be the length or breadth of an object, and DE the length or breadth of its image projected on this side the *speculum*, then LA and OD will be half the length or breadth of the object and its image, and the triangle CLA being similar to COD, LA is to OD, and consequently BA to ED, as LC to OC, that is, the length or breadth of the object, is to the length or breadth of its image, as the distance of the object from the center of the *speculum*, to the distance of its image from the same center; but it has been proved, that as AC, the distance of the object from the center, is to EC, the distance of the image from the center, so is AB, the distance of the object from the *speculum*, to EB, the distance of the image from the *speculum*; consequently, the length or breadth of an object, is to the length or breadth of its image, as the distance of the object from the *speculum*, to the distance of the image from the *speculum*; and forasmuch as similar surfaces are to one another, as the squares of their homologous sides, the magnitude of the object, is to the magnitude of the image, as the square of the object's distance from the *speculum*, to the square of the image's distance. And by the same method of arguing, if DE be an object whose image behind the *speculum* is HI, the magnitude of the former will be found to be to the magnitude of the latter, as the square of KO, to the square of KM. Hence it follows, that the object during its continuance beyond the center, must appear larger than its image, as being more distant from the *speculum*, and when it is in the center, where it meets the image, it must

must appear equal to it, but being on the same side of the center with the *speculum*, it must be less than its image, which in that case lies beyond the center, and consequently, is at a greater distance from the *speculum*.

It likewise follows, that the image which appears behind the *speculum* is ever larger than the object; for since MK, the distance of the image behind the *speculum*, is to OK, the distance of the object before the *speculum*, as MC, the distance of the image from the center, to OC, the distance of the object from the center; and since in this case, the object is always less distant from the center than its image, during the appearance of the image behind the *speculum*, it is evident, that the image must appear larger than the object; but then this is to be understood with respect to such images only, as are projected by objects less distant than the center; for if an object be beyond the center, an eye being close to the *speculum*, shall see the image at the same distance, and of an equal magnitude with the object; and in this case, the several parts of the image do not appear in those points where the perpendiculars from the corresponding points of the object meet with the reflected rays; the reason of all which seems to be this, the portion of the *speculum* which the eye makes use of in this case is so exceedingly small, that notwithstanding the spherical figure of the *speculum*, it may be looked upon as plane, and consequently, the appearances must be the same as in other plain *speculums*; that is, the image must appear as far behind the *speculum* as the object is before it, and of the same magnitude with the object.

If an image formed on this side a concave *speculum* be looked at with both eyes, it will appear double, provided the distance of the eyes from the image be but small, and upon shutting either eye, the

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the contrary image will disappear; for since the reflected rays which form the several points of an image meet and cross one another at the image, those which enter the right eye must be reflected from the left side of the *speculum*, and those which fall upon the left eye, must be reflected from the right side of the *speculum*, and of consequence, one and the same point of the image must appear to the right eye, as situated before the left side of the *speculum*, and to the left eye, as situated before the right side of the *speculum*; that is, it must appear double, and the right or left image must vanish upon closing the contrary eye. Thus, if the point C of the image A B, be looked at with both eyes, one whereof is at O, and the other at Q, the eye at O shall see it by means of the rays O N, which are reflected from N, and of consequence, shall see it as placed before N, but the eye at Q seeing it by means of the rays Q M, which proceed from M, shall see it as situated before M, for which reason, the point C will appear double; and what has been thus shewn with respect to the point C, may in the same manner be shewn, with regard to all the other points in the image, and therefore the whole image must appear double; as the eyes are more and more removed from the images, they approach nearer together, and at length coincide; the reason of which is plain, from the bare inspection of the figure; for since the interval of the eyes continues the same, it is evident, that when they are farther removed from the image, the rays whereby they see the point C must be reflected from parts of the *speculum* less distant from one another than M and N, and the distance of the parts of the *speculum* which reflect the rays to each eye, must continually lessen as the eyes are more and more removed from the image, and at certain distances of the eyes, must become so small as not to be sensible.

And

And thus much concerning such spherical *speculums* as are concave. As to *convex speculums*, in order to determine the places of images formed by reflexion from them, Let A be a radiating point, exposed directly before the *convex speculum* H K, whose center is C, A B a perpendicular from the radiating point to the *speculum*, which likewise denotes the distance of the radiating point from the *speculum*, A D a ray falling on the *speculum* at D, whose distance from B is indefinitely small, D E the reflected ray meeting the perpendicular in E, C D a *radius* drawn to the point of incidence, and of consequence bisecting the angle F D E; let the angle F C D be made equal to E C D, and let C F be continued till it meets A D produced; this being done, it is evident, that the angle at C in the triangle A C F, is bisected by the line C D, which cuts the opposite side, consequently, A C is to F C, as A D to D F; but forasmuch as D and B are supposed to be indefinitely near, A D is equal to A B, and D E to B E; and because the triangles C F D and C E D are equal, D F is equal to D E, and F C is equal to C E; wherefore, A B is to B E, as A C to C E; that is, the distance of the radiating point from the *speculum*, is to the distance of the point E where the reflected ray cuts the perpendicular, which is called the *point of intersection*, as the sum of the distance of the radiating point and the *radius*, to the *radius* lessened by the distance of the point of intersection; that is, putting D for the distance of the radiating point, F for the distance of the point of intersection, and R for the *radius* as before, $D : F :: D + R : R - F$; consequently, reducing this analogy into an equation, and clearing F, F will be found equal to $\frac{DR}{2D+R}$; that is, the distance of the point of intersection behind the *speculum*, and consequently the distance of an image behind

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behind the *speculum*, is equal to a rectangle under the distance of the object from the *speculum* and the *radius*, divided by the sum of twice the distance of the object added to the *radius*. Hence it follows that if an object be placed so near a *convex speculum* as to touch it, its image will do so too; for in this case, D being nothing, F is likewise nothing; as the object recedes from the *speculum*, the image goes off behind; and when the object is removed to an infinite distance, the image appears behind in the midway between the *speculum* and its center; for in this case, D being infinite, $2D + R$ becomes $2D$, and of consequence, F is equal to $\frac{R}{2}$, so that ob-

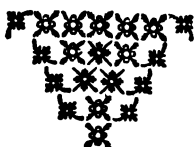
Pl. 10.
Fig. 14.

jects seen by reflexion from convex spherical *speculums*, appear constantly behind the *speculum*, within the limits of half the *radius*; and forasmuch as the images constantly appear on the same side of the center with the objects, they must be less than the objects; for if we suppose HI to be an object placed before the *convex speculum* FG, and projecting its image at DE, it is manifest, that the image subtends the same angle at a smaller distance, than the object does at a larger distance, and consequently, must be less; and the disproportion between the object and its image, must increase as the object recedes, and decrease, as it approaches; because, as the object recedes from the center, the image approaches, and as that approaches; the image recedes; but as the image can never be more distant from the center than the object, it can in no case appear larger. The proportions which the magnitudes of the object and its image bear one to another, is the same with the squares of their distances from the *speculum*, as in the case of concave *speculums*; the proof of which being exactly the same with that made use of in the case of concaves, I shall not here repeat it.

As

As to the position of such images as are seen by reflexion from convex spherical *speculums*, they must always appear erect; for as they ever appear on the same side of the center with the objects, the perpendiculars which are drawn from the uppermost parts of the objects, must pass thro' the uppermost parts of the images; and those from the lower parts of the objects, must likewise pass thro' the lower parts of the images; thus, the perpendicular HC, which comes from H, the highest point in the object, passes thro' D, the highest point of the image, and IC, which comes from I, the lowest point of the object, passes thro' E, the lowest point of the image; and so it is with regard to the perpendiculars which come from the intermediate points; so that the several parts of the image have the same situation with the corresponding parts of the object, and of consequence the image appears erect.

Pl. 10.
Fig. 14.



APPEN-

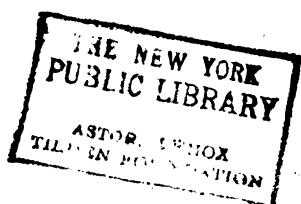
A P P E N D I X.

OF THE COLLISION OF NON-ELASTICK AND ELASTICK BODIES.

PROBLEM I.

IF two bodies be either entirely void of elasticity or perfectly elastick, and one strike the other directly; if A and B denote the quantities of matter or weights of the two bodies, a and b their velocities before the stroke; and if A be the swifter body when the bodies move the same way, the body which has the greater motion when they move contrary ways, and the moving body when one of them is at rest before the stroke; to determine the ratio of the bodies when their velocities before the stroke are given, or the ratio of their velocities before the stroke when the bodies are given; that is, to determine $\frac{A}{B}$ when a and b are given, or $\frac{a}{b}$ when A and B are given; so as that the motion of A before the stroke, shall be to its motion after the stroke, in the given ratio of m to 1.

To give a solution of this *Problem*, it is necessary to know the motions of A before and after the stroke, both when the bodies are entirely void of elasticity, and when they are perfectly elastick; and likewise to know the motion of A after the stroke, when the bodies move the same way, when they move contrary ways, and when B is at rest before the stroke. The motion of A before the stroke, is Aa in all cases. And from what has been delivered by



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our Author, when the bodies are entirely void of elasticity, the motion of A after the stroke, is $\frac{AAa + ABb}{A + B}$ when before the stroke the bodies move

the same way, $\frac{AAa - ABb}{A + B}$ when they move diffe-

rent ways before the stroke, and $\frac{AAa}{A + B}$ when be-

fore the stroke B is quiescent. And when the bodies are perfectly elastick, the motions of A after the stroke, when before the stroke the bodies move the same way, contrary ways, and B is quiescent, are $\frac{2ABb + AAa - ABa}{A + B}$, $\frac{AAa - ABa - 2ABb}{A + B}$, and

$\frac{AAa - ABa}{A + B}$. Hence, this *Problem* contains six

Cases, three when the bodies are entirely void of elasticity, and three when they are perfectly elastick; which *Cases* are thus solved.

When the bodies are entirely void of elasticity.

CASE I. If the bodies move the same way, Aa will be to $\frac{AAa + ABb}{A + B}$, as m to 1; whence we have $\frac{A}{B} = \frac{a - mb}{ma - a}$, and $\frac{a}{b} = \frac{mB}{A + B - mA}$.

CASE II. If the bodies move contrary ways, Aa will be to $\frac{AAa - ABb}{A + B}$, as m to 1; whence we have $\frac{A}{B} = \frac{mb + a}{ma - a}$, and $\frac{a}{b} = \frac{mB}{mA - A - B}$.

CASE III. If B be at rest before the stroke, then will Aa be to $\frac{AAa}{A + B}$, as m to 1; whence we have

have $\frac{A}{B} = \frac{1}{m-1}$. In this case b is nothing, and consequently $\frac{a}{b}$ is infinite.

When the bodies are perfectly elastic.

CASE IV. If the bodies move the same way, Aa will be to $\frac{2ABb + AAa - ABa}{A+B}$, as m to 1 ; whence we have $\frac{A}{B} = \frac{ma + a - 2mb}{ma - a}$, and $\frac{a}{b} = \frac{2mB}{mB + A + B - mA}$.

CASE V. If the bodies move contrary ways, Aa will be to $\frac{AAa - ABa - 2ABb}{A+B}$, as m to 1 ; whence we have $\frac{A}{B} = \frac{2mb + ma + a}{A+B}$, and $\frac{a}{b} = \frac{2mB}{mA - A - B - mB}$.

CASE VI. If B be at rest before the stroke, Aa will be to $\frac{AAa - ABa}{A+B}$, as m to 1 ; whence we have $\frac{A}{B} = \frac{m+1}{m-1}$. In this case b is nothing, and consequently $\frac{a}{b}$ is infinite.

EXAMP. I. If the bodies be entirely void of elasticity, and move the same way, A with a velocity of 7 , and B with a velocity of 3 ; and A lose half its motion by the stroke, or, which amounts to the same, if the motion of A before the stroke be to its motion after, as 2 to 1 . In this case, a
 $b, m,$

b, m, are 7, 3, 2; and $\frac{A}{B}$, which is equal to $\frac{a-mb}{ma-a}$ by *Case 1*, will be equal to $\frac{1}{7}$; so that A and B will be as 1 and 7. Here Aa, the motion of A before the stroke, is 7, and $\frac{AAa+ABb}{A+B}$, its motion after the stroke is $3\frac{1}{2}$; but 7 is to $3\frac{1}{2}$, as 2 to 1.

EXAMP. II. If the bodies be entirely void of elasticity, and move the same way, if A and B be as 1 and 4, and the motion of A before the stroke be to its motion after, as 3 to 1, in which case m will be 3; then $\frac{a}{b}$, which is equal to $\frac{mB}{A+B-mA}$ by *Case 1*, will be equal to $\frac{6}{1}$; so that a and b will be as 6 and 1. Here Aa, the motion of A before the stroke, is 6, and $\frac{AAa+ABb}{A+B}$, its motion after the stroke by *Case 1*, is 2; but 6 is to 2, as 3 to 1.

EXAMP. III. If B be at rest before the stroke, and the motion of A before the stroke, be to its motion after, as 10 to 1, in which case m will be 10; then will $\frac{A}{B}$ be $\frac{1}{9}$, or A and B will be as 1 and 9. If the velocity of A before the stroke be expressed by 1, that is, if a be 1, then will Aa be 1, and $\frac{AAa}{A+B}$ be $\frac{1}{10}$; but 1 is to $\frac{1}{10}$, as 10 to 1.

It is to be observed, that A can never communicate all its motion to B, except when it is infinitely greater than B, in which case B will become nothing. For if A communicate all its motion to B, m will be 1; and $\frac{A}{B}$, which is as $\frac{1}{m-1}$, will be as $\frac{1}{0}$; but $\frac{1}{0}$ is infinite, and therefore A must be infinitely

finitely greater than B, to lose all its motion by the stroke.

EXAMP. IV. If the bodies be perfectly elastick, and move the same way with velocities which are as 3 and 2; and if the motion of A before the stroke be to its motion after, as 2 to 1; then will a, b, m, be 3, 2, 2; and $\frac{A}{B}$, which is as $\frac{ma+a-2mb}{ma-a}$ by *Case* 4, will be $\frac{1}{3}$; so that A and B will be as 1 and 3. Here Aa, the motion of A before the stroke, is 3; and $\frac{2ABb+AAa-ABa}{A+B}$, its motion after the stroke, is $\frac{3}{2}$; but 3 is to $\frac{3}{2}$, as 2 to 1.

EXAMP. V. If the bodies be perfectly elastick, and move the same way, if A and B be as 4 and 5, and the motion of A before the stroke be to its motion after, as 3 to 1; then will A, B, m, be 4, 5, 3; and $\frac{a}{b}$, which is as $\frac{2mB}{mB+A+B-mA}$ by *Case* 4, will be $\frac{5}{2}$, so that a and b will be as 5 and 2. Here, Aa, the motion of A before the stroke, is 20; and $\frac{2ABb+AAa-ABa}{A+B}$, the motion of A after, is $\frac{60}{3}$; but 20 is to $\frac{60}{3}$, as 3 to 1.

EXAMP. VI. If A and B be perfectly elastick, and B be at rest before the stroke, if A move with a velocity of 4, and its motion before the stroke be to its motion after, as 3 to 1; then will a, b, m, be 4, 0, 3; and $\frac{A}{B}$, which is as $\frac{m+1}{m-1}$ by *Case* 6, will $\frac{4}{2} = \frac{2}{1}$; so that A and B will be 2 and 1. Here, Aa, the motion of A before the stroke, is 8; and $\frac{AAa-ABa}{A+B}$, its motion after the stroke, is $\frac{8}{3}$; but 8 is to $\frac{8}{3}$, as 3 to 1.

S C H O L I U M.

If it be required to know the motion of B after the stroke in the six Cases before mentioned, that motion may be had, from what our Author has delivered, when the weights of the bodies, and their velocities before the stroke, are given.

If the bodies be intirely void of elasticity; the motion of B after the stroke, when before the stroke, the bodies move the same way, when they move contrary ways, or when B is quiescent, is $\frac{BAa + BBb}{A + B}$, $\frac{BAa - BBb}{A + B}$, or $\frac{BAa}{A + B}$.

And if the bodies be perfectly elastick; the motions of B after the stroke, when before the stroke the bodies move the same way, when they move contrary ways, or when B is quiescent, is $\frac{2BAa + BBb - BAb}{A + B}$, $\frac{2BAa - BBb + BAb}{A + B}$, or $\frac{2BAa}{A + B}$.

PROB. II. If two bodies A and B be given, and be perfectly elastick, if A be the lesser body, and B be at rest before the stroke; it is required to find an intermediate body of such a weight or quantity of matter, which I shall denote by x, as that A striking x at rest, and x with the motion acquired by the stroke striking B at rest, the motion produced in B shall be greater than can be produced by an intermediate body of any other weight, or, in other words, that the motion in B shall be a maximum.

The motion of x after it is struck by A, is $\frac{2Aax}{A + x}$, and the motion of B after it is struck by x, is $\frac{4ABax}{AB + Ax + Bx + xx}$, by Schol. Prob. 1.

A a 2

But

But by supposition the motion of B is a *maximum*, and consequently its fluxion is nothing. The fluxion therefore of $\frac{4ABax}{AB+Ax+Bx+xx}$ is nothing;

that is, $\frac{4A^2B^2ax - 4ABxax^2}{AB+Ax+Bx+xx^2} = 0$. Consequently,

$4A^2B^2ax - 4ABxax^2 = 0$; and, by dividing by $4ABax$, $AP - x^2 = 0$; and $AB = x^2$; whence x is a mean proportional between A and B.

Our Author has given a clear solution of this *Problem*, but in a different manner.

COR. I. If a number of bodies be in a continual geometrical progression, if the least of the bodies be A, the *ratio* of the increase be e, and the number of bodies n; and if A strike the second body at rest, and the second with the motion acquired strike the third body at rest, and so on to the last; the bodies, their velocities and motions, will be thus expressed.

Bodies - - A, eA, e²A, e³A &c. $\left[e \right]^{n-1} A$.
 Velocities - - a, $\frac{2a}{1+e}$, $\frac{4a}{1+e^2}$, $\frac{8a}{1+e^3}$, &c. $\left[\frac{2}{1+e} \right]^{n-1} a$.
 Motions - - Aa, $\frac{2Aae}{1+e}$, $\frac{4Aae^2}{1+e^2}$, $\frac{8Aae^3}{1+e^3}$, &c. $\left[\frac{2e}{1+e} \right]^{n-1} Aa$.

EXAMP. I. If the number of bodies increasing in geometrick proportion be 20, and the common *ratio* of the terms be 2, n will be 20, and e be 2. The last body will be 524288 times greater than the first; the velocity of the last will be to the velocity of the first, as 1 to $2216\frac{4}{5}$; and the motion of the last will be about $236\frac{4}{5}$ times greater than the motion of the first.

EXAMP.

COR. II. If the motion of the first body be to the motion of the last, as 1 to D, that is, If Aa

For example, if the number of bodies be 20, and the motion of the last be 100000 times greater than the motion of the first, n will be 20, D will be 100000, and e will be 10.9746 nearly; so that each preceding body in the 20 bodies must be 10.9746 times greater than the body lying next behind it.

$\left. \frac{2e}{1+e} \right|^{n-1}$ Aa, as 1 to D, D will be equal to
 $\left. \frac{2e}{1+e} \right|^{n-1}$, and putting R for $\frac{2e}{1+e}$, and L for lo-

For example, if e be 4, and n be 25, $\frac{2e}{1+e}$ will be

be $\frac{1}{3}$, the logarithm of which number is 0.2041299 \equiv L, R; and $n - 1 \times$ L, R \equiv 4.8988795 \equiv L, D. The natural number of this logarithm is 79228 nearly; so that in this case the motion of the last body will be nearly 79228 times greater than the motion of the first.

COR. IV, If D and R be given, n may be found by being equal to $\frac{L, D + L, R}{L, R}$; for by the last Corollary L, D \equiv $n - 1 \times$ L, R, and consequently $n = \frac{L, D + L, R}{L, R}$.

For example, if D be 100000, and e be 2, in which case R will be $\frac{1}{3}$; then will L, D, be 5.000000 and L, R 0.1249387; and $\frac{L, D + L, R}{L, R}$, will be 41.02 \equiv n; so that more than 41 bodies will be necessary to make the motion of the last 100000 times greater than the motion of the first.

Of the Motion of a Globe in a Fluid Medium.

PROB. III. *If the diameter and density of a Globe moving in a fluid medium, if the density of the medium, the velocity with which the globe sets out, and the time of the motion, be all given; to determine the part of the velocity which is destroy'd by the resistance of the medium, the remaining part of the velocity, and the space described by the globe in the given time.*

Let D denote the diameter of the globe, d its density, δ the density of the fluid medium, V the velocity with which the globe sets out, t the time of the motion expressed in seconds, m the part of a diameter or number of diameters of the globe which it would describe with the velocity V in the time t, and

and T the time in which the globe with the velocity V would *in vacuo* describe a space which is to $\frac{8D}{3}$ as d to δ ; and then the part of the velocity destroyed by the resistance of the medium, will be

$\frac{m\delta V}{\frac{8}{3}d + m\delta}$; the remaining part of the velocity will be $\frac{\frac{8}{3}dV}{\frac{8}{3}d + m\delta}$; and the space described in the *medi-*

um in the time t , will be $\frac{8Dd}{3\delta} \times \text{Log. } 1 + \frac{m\delta}{\frac{8}{3}d} \times 2.302585093$.

For Sir ISAAC NEWTON has proved, that the part of the velocity which is destroy'd by the resistance of the medium in the time t , is $\frac{Vt}{T+t}$; that

the remaining part, is $\frac{VT}{T+t}$; and that the space

described in the time t , is $TV \times \text{Log. } \frac{T+t}{T} \times 2.302585093$. But by construction, T is as

$\frac{8Dd}{3\delta V}$, and V is as $\frac{mD}{t}$. And therefore, by sub-

stituting $\frac{8Dd}{3\delta V}$ and $\frac{mD}{t}$ instead of T and V in the foregoing expressions, the part of the velocity destroyed by the resistance of the medium in the time t will

be $\frac{m\delta V}{\frac{8}{3}d + m\delta}$, the remaining part of the velocity will be $\frac{\frac{8}{3}dV}{\frac{8}{3}d + m\delta}$, and the space described in the time

t will be $\frac{8dD}{3\delta} \times \text{Log. } 1 + \frac{m\delta}{\frac{8}{3}d} \times 2.302585093$.

COR. I. If the density of the globe be equal to the density of the *medium*, that is, if d be equal

A a 4

to

to δ , the velocity destroy'd by the resistance of the *medium* in the time t , will be $\frac{mV}{\frac{8}{3} + m}$.

This *Corollary* will obtain, if the globe and the *medium* be perfectly dense or void of pores; for by being intirely void of pores, they will have equal densities. And such a globe moving in such a *medium* the length of 3 times its diameter, will lose above half its velocity; for if m be 3, $\frac{mV}{\frac{8}{3} + m}$ will be $\frac{9V}{17}$. And this will always be the velocity lost in moving three times the length of the diameter, when the globe and the *medium* have equal densities.

COR. II. If a globe in moving through m times its diameter in a fluid *medium* lose the n part of its velocity; then will $n = \frac{m\delta}{\frac{8}{3}d + m\delta}$, $d = \frac{\delta \times m - nm}{\frac{8}{3}n}$, $\delta = \frac{\frac{8}{3}dn}{m - nm}$, and $m = \frac{8nd}{3\delta - 3n\delta}$. For $\frac{m\delta V}{\frac{8}{3}d + m\delta} = nV$; whence $n = \frac{m\delta}{\frac{8}{3}d + m\delta}$, $d = \frac{\delta + m - nm}{\frac{8}{3}n}$, $\delta = \frac{\frac{8}{3}dn}{m - nm}$, and $m = \frac{8nd}{3\delta - 3n\delta}$.

EXAMP. I. If a globe lose $\frac{3}{4}$ of its velocity in moving the length of 10 times its diameter in water, in which case n will be $\frac{3}{4}$, m will be 10, and δ will be 1; then d will be $\frac{3}{4}$, that is, the globe will be denser than water in the proportion of 5 to 4.

EXAMP. II. If a globe 10 times as dense as water, lose $\frac{3}{4}$ ths of its velocity in moving 10 times its diameter in a fluid; the density of that fluid will

will be 8 times as great as the density of water.

In this case d is 10, m is 10, and n is $\frac{3}{4}$; and δ , which is equal to $\frac{3dn}{m - nm}$, is 8.

EXAMP. III. If a globe twice as dense as water, lose $\frac{3}{4}$ ths of its motion by moving in a fluid 14 times as dense as water; it will suffer this loss of velocity in moving the length of $1\frac{1}{7}$ D. For in this case d , δ , n , are 2, 14, $\frac{3}{4}$; and m , which is equal to $\frac{8nd}{3\delta - 3nd}$, will be $\frac{8}{7} = 1\frac{1}{7}$.

EXAMP. IV. If a perfectly solid globe move 24 times the length of its diameter in a perfectly solid medium, it will lose 9 parts in 10 of the velocity it had at the beginning of the motion. For in this case d is equal to δ , and m is 24; and n , which is equal to $\frac{m}{\frac{8}{3} + m}$, will be equal to $\frac{72}{80} = \frac{9}{10}$.

EXAMP. V. If a globe of equal density with water, move half the length of its diameter in air, it will lose the $\frac{1}{4587\frac{2}{3}}$ part of its velocity, on supposition that the density of water is to the density of air, as 860 to 1. For in this case, d , δ , m , are 860, 1, $\frac{1}{2}$; and n , which is equal to $\frac{m\delta}{\frac{8}{3}d + m\delta}$, will be $\frac{1}{4587\frac{2}{3}}$.

EXAMP. VI. If the earth moved round the sun in a fluid medium of equal density with the air at the surface of the earth, it would by the resistance of the medium lose almost all its motion in 10000 years, on supposition that the densities of the earth, of water, and of the medium, are 5, 1, $\frac{1}{860}$, or in decimals

decimals 0.0011628. For the earth moves in its orbit with a velocity that carries it at the rate of 4893938782791 miles, or 617142343 times the length of its own diameter in 10000 years, on supposition that the sun's horizontal parallax is $10\frac{1}{2}$ seconds. In this case therefore d , δ , m , are 5, 0.0011628, and 617142343; and consequently n , which is equal to $\frac{m\delta}{\frac{1}{2}d + m\delta}$, will be a $\frac{717713}{717717}$ th part, which is nearly the whole, of its present velocity.

By the *French* measures, a degree of a great circle of the earth contains 342366 *Paris* feet, or 365403.3158 *English* feet, on supposition that a *Paris* foot is to an *English* foot, as 1142 to 1070. And consequently the diameter of the earth, supposing the earth to be spherical, will be 41870881 *English* feet, or 7930 miles. The mean distance of the sun from the earth, reckoning the parallax at $10\frac{1}{2}$ seconds, is about 19644.2675 semidiameters of the earth, or 77889520.6375 miles; consequently the circumference of the earth's orbit is 489393878.2791 miles, which the earth describes in one year, or 29558161.6 seconds of time.

EXAMP. VII. If the earth move in an *Æther* 700000 rarer than the air at the surface of the earth, it will lose about $\frac{1}{14}$ th part of its present velocity in 10000 years; for in this case d , δ and m , are 5, 0.00000000166, and 617142343; and consequently n , which is equal to $\frac{m\delta}{\frac{1}{2}d + m\delta}$ will be equal to a $\frac{1.02445628938}{14.35778962271}$ th part, that is $\frac{1}{14}$ th part of the present velocity very nearly.

And if the earth moves 100000 years in this *Æther*, it will lose almost half of its present motion in that time.

EXAMP.

EXAMP. VIII. If we suppose the earth to lose the $\frac{1}{1000}$ th part of its present velocity by moving in an *ætherial medium* for 400000 years, in which time it will have described 24685693680 times its diameter, the density of the *medium* will be above 200 millions of times less than the density of the air at the surface of the earth. For in this case d , n , m , are 5, 0.01, 24685693680, and consequently δ , which is equal to $\frac{\frac{8}{3}dn}{m - nm}$, will be

$$\frac{0.1333333}{24438836743.2000000} = \frac{1}{183366000000}$$

But the density of water being 1, the density of air is $\frac{1}{800}$; and consequently, the density of the air at the surface of the earth will be to the density of this *medium*, as above 213200000 to 1.

Of the Motion of Wheels over Obstacles.

PROB. IV. If a wheel moving on an horizontal plane, meet with an immoveable obstacle in its way, over which it is to be drawn by a force fixed to its center; if the weight and diameter of the wheel, the height of the obstacle, and the direction of the force drawing the wheel, be all known; thence to determine the force that is sufficient to draw the wheel over the obstacle.

Let G P M E be the wheel, N D the horizontal plane on which it moves from N towards D, E F the obstacle over which it is to be drawn; let the wheel arrive at the obstacle, and touch its top E; and there let it be supposed to stand pressing the horizontal plane at G with its whole weight. Draw OEK a tangent to the wheel in the point E, draw the diameter ACG perpendicular to the horizontal plane, and produce it till it meet the tangent in O; from

Pl. 11.
Fig. 1.

from E draw the *radius* EC; draw EH perpendicular to AG; and mr, MC, perpendicular to E and consequently parallel to the tangent OK; a lastly, draw the *radius* Cm; if the whole weight the wheel be expressed by CO, in the direction which line that weight acts when the wheel is whole supported by the horizontal plane at G, that weight may be resolved into two others CE and OE, acting according to the directions of those lines, the weight CE pressing against the top of the immovable obstacle, and being wholly sustained by it, and the weight OE drawing the wheel down in a direction parallel to the tangent OEK. Let W denote the whole weight of the wheel, r its *radius*, h the height of the obstacle, and x the part of the whole weight which draws the wheel down in a direction parallel to OEK; and then we shall have this analogy; as x is to W, so is OE to CO, or HE to CE, from the similarity of the triangles CEO, and CEH; whence $x = \frac{W \times HE}{r}$; but HE from the nature of the circle, is equal to $\sqrt{AH \times HG}$, or to $\sqrt{AH \times EF}$, that is, in symbols, to $\sqrt{2rh - hh}$; and therefore $x = \frac{W \times \sqrt{2rh - hh}}{r}$. A force just equal to this weight, and acting in direct opposition to it, that is, drawing the wheel upward in the direction CM parallel to OK, will just be able to make the wheel rest on E the top of the obstacle, without suffering any part of its weight to rest on the horizontal plane at G. This force must be increased to produce the same effect, if it act in any other direction than that of CM. For let it draw the wheel in the direction Cm, m lying between E and M, and then the force acting in this direction may be resolved into two forces, which will be as Cr and rm, whereof Cr draws the wheel directly against E the

top of the obstacle, and so is lost, and mr draws it up in a direction parallel to OK. But mr is less than Cm or CM, and to become equal to it, and consequently sufficient to support the wheel against the top of the obstacle without suffering any part of its weight to rest on the horizontal plane, it must be increased in the *ratio* of Cm or CM to rm, that is, putting s for the sine of the angle which the direction of the force makes with CE, in the *ratio* of r to s; but the force rm cannot be increased, but the whole force CM must be increased in the same proportion. And therefore the force $\frac{W \times \sqrt{2rh - hh}}{r}$ must be increased in the proportion of r to s, and then, putting F for the force, acting in the direction Cm, which is just sufficient to support the wheel on the obstacle without suffering it to press on the plane ND, $F = \frac{W \times \sqrt{2rh - hh}}{s}$; and the smallest addition to this force will make it draw the wheel over the obstacle.

Since the resistance given by the obstacle, is equal to the force that is just sufficient to make the wheel rest on the obstacle without suffering any part of its weight to press on the plane of the horizon, that is, putting R for the resistance given by the obstacle, since R is equal to F; R will be equal to $\frac{W \times \sqrt{2rh - hh}}{s}$.

It is to be observed that the direction of the force must lie between CE and CA; for if the force draw the wheel in the direction CE it will be wholly spent upon the obstacle, and not in the least contribute to draw the wheel over it; and if it draw the wheel directly upwards from C to A, it will not make it to press against the obstacle, and

confe-

consequently, however great we may suppose it to be, can never draw it over it.

COR. I. If the direction of the moving force change continually, passing from CE to CM, and thence to CP, the line of the angle which the line of direction makes with CE, will increase in the passage of that line from CE to CM, and decrease in its passage from CM to CP; but as s increases or lessens, $\frac{W \times \sqrt{2rh - hh}}{s}$ will lessen or increase;

and consequently the force F will lessen in the passage of the line of its direction from CE to CM, and thence increase in the passage of that line to CA. So that the force will be least when it acts in the direction CM, in which case the whole force will be employed in drawing the wheel over the obstacle; whereas in all other directions, part of the force will be lost by drawing directly against the top of the obstacle. Hence the most advantageous direction of the force, will be that which makes a right angle with CE, in which case s will be equal to r , and $F = \frac{W \times \sqrt{2rh - hh}}{r}$.

COR. II. If the height of the obstacle be given, in which case h will be as 1, and the force draw the wheel in the direction CM parallel to OK; then F will be as $\frac{W \times \sqrt{2r - 1}}{r}$.

If the radii of four wheels be 1, 2, 3, 4, then will $\frac{\sqrt{2r - 1}}{r}$, be 1, $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{5}}{3}$, $\frac{\sqrt{7}}{4}$, that is, as the numbers 1000, 866, 745, 661; and the forces requisite to support these wheels on the point E, so as not to suffer any part of their weight to rest on the horizontal plane, will be as their weights multiplied



plied into these numbers respectively. The force requisite to support the first wheel, will be as its weight multiplied into 1000, the force requisite to support the second wheel as its weight multiplied into 866; and so of the rest. And if the weights of all the wheels be equal, the forces necessary to support them, and consequently the resistances given by the obstacle to which these forces are equal, will be as the numbers 1000, 866, 745, 661. So that in wheels of a given weight, the lesser the wheel is, the greater will be the resistance which is given to it by an obstacle of a given height.

COR. III. If the height of the obstacle be indefinitely small and given, in which case the tangent OK will coincide with the horizontal plane ND, and the point E coincide with the point G; and if the force draw the wheel in a direction parallel to OK or ND; then will F be as $W \times \frac{\sqrt{2r}}{r}$;

or, because 2 is a given quantity, as $\frac{W}{\sqrt{r}}$; and if the weight of the wheel be given, F will be as $\frac{1}{\sqrt{r}}$.

If the radii of four wheels of equal weights be 1, 2, 3, 4, and the wheels be drawn on a smooth plane parallel to the horizon; the forces necessary to put them in motion, when they draw in directions parallel to that plane, will be as 1, $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{4}}$; that is, as the numbers 1000, 707, 577, 500. And therefore, of wheels drawn on the plane of the horizon by forces acting in directions parallel to that plane, lesser wheels will require a greater force to put them in motion than greater.

COR.

COR. IV. If the height of the obstacle be proportional to the *radius* of the wheel, and if the force draw the wheel in a direction parallel to OK; that is, if h be as r , and F be as $\frac{W \times \sqrt{2rh - hh}}{r}$; then will the force, and consequently the resistance given by the obstacle, be as the weight of the wheel; for $\frac{\sqrt{2rh - hh}}{r}$ will be as $\frac{\sqrt{2rr - rr}}{r}$, that is, as 1; and therefore F will be as W .

COR. V. If the direction of the force drawing the wheel be parallel to the horizontal plane, that is, if mC be parallel to ND ; then will the force that is requisite to sustain the wheel on the point E , be $\frac{W \times \sqrt{2rh - hh}}{r - h}$. For in this case the angle mCE is equal to the angle CEH , and consequently, their sines are equal, that is, s is equal to CH , which in symbols is $r - h$. And therefore F , which universally is as $\frac{W \times \sqrt{2rh - hh}}{s}$, is in this case as $\frac{W \times \sqrt{2rh - hh}}{r - h}$.

If the height of the obstacle be given, in which case h will be as 1, then will F be as $\frac{W \times \sqrt{2r - 1}}{r - 1}$.

If the *radii* of four wheels of equal weight, be 1, 2, 3, 4; then will F with respect to these four wheels, be as $\frac{1}{0}, \frac{\sqrt{3}}{1}, \frac{\sqrt{5}}{2}, \frac{\sqrt{7}}{3}$, that is, as infinite, 1732, 1128, 882. The height of the obstacle is equal to the *radius* of the first wheel, inasmuch as I have supposed them both to be as 1; and consequently the force must be infinite to make the wheel rest against E , and hinder any part of its

its weight from pressing on the horizontal plane at G.

COR. VI. The force, is to the weight of the wheel, as the sine of the angle ECH, is to the sine of the angle which the line of direction of the force makes with EC; that is, $\frac{F}{W} = \frac{\sqrt{2rh - hh}}{r}$.

If the force be one half of the weight of the wheel, that is, if F be one half of W, $\sqrt{2rh - hh}$ will be one half of s; if P be equal to W, $\sqrt{2rh - hh}$ will be equal to s; and if F be as W, $\sqrt{2rh - hh}$ will be as s.

Of the Motion of Water through Orifices and Pipes.

PROB. V. *To determine the motion of water running out of a hole made in the bottom of a vessel.*

Sir ISAAC NEWTON has given a general solution of this *Problem* in the following paragraph, which is contained in *prop. 36. prob. 8. lib. 2.*

“ Sit ACDB vas cylindricum, AB ejus orificium Pl. 11.
 “ superius, CD fundum horizonti parallelum, EF Fig. 2.
 “ foramen circulare in medio fundi, G centrum fo-
 “ ramini, et GH axis cylindri horizonti perpendi-
 “ cularis. Et finge cylindrum glaciei APQB ejus-
 “ dem esse latitudinis cum cavitate vasis, et axem
 “ eundem habere, et uniformi cum motu perpetuo
 “ descendere, et partes ejus quam primum attingunt
 “ superficiem AB liquefcere, et in aquam conversas
 “ gravitate suâ defluere in vas, et cataractam vel
 “ columnum aquæ ABNFEM cadendo formare,
 “ et per foramen EF transire, idemque adæquate
 “ implere. Ea vero sit uniformis velocitas glaciei
 “ descendentis ut et aquæ contiguæ in circulo AB,
 B b “ quam

“quam aqua cadendo et casu suo describendo alti-
 “tudinem IH acquirere potest; et jaceant IH et
 “ HG in directum, et per punctum I ducatur recta
 “ KL horizonti parallela et lateribus glaciei occur-
 “rens in K et L . Et velocitas aquæ effluentis
 “per foramen EF ea erit quam aqua cadendo ab I
 “et casu suo describendo altitudinem IG acquirere
 “potest. Ideoque per theorematum GALILÆI erit
 “ IG ad IH in duplicata ratione velocitatis aquæ
 “per foramen effluentis ad velocitatem aquæ in
 “circulo AB , hoc est, in duplicata ratione circuli
 “ AB ad circulum EF ; nam hi circuli sunt reci-
 “proce ut velocitates aquarum quæ per ipsos eo-
 “dem tempore et æquali quantitate, adæquatè
 “transeunt. De velocitate aquæ horizontem versus
 “hic agitur. Et motus horizonti parallelus quo
 “partes aquæ cadentis ad invicem accedunt, cum
 “non oriatur a gravitate, nec motum horizonti
 “perpendicularem a gravitate oriundum mutet,
 “hic non consideratur. Supponimus quidem quod
 “partes aquæ aliquantulum cohærent, et per co-
 “hæsiōnem suam inter cadendum accedant ad in-
 “vicem per motus horizonti parallelos, ut unicam
 “tantum efforment cataractum et non in plures
 “cataractas dividantur: sed motum horizonti pa-
 “rallelum, a cohæsiōne illâ oriundum, hic non
 “consideramus.”

This *Theory* Sir ISAAC corrected by experiments,
 proved it in six different cases, and drew several
 corollaries from it. The reason why a correction
 was necessary will be shewn in the *Scholium*. And
 the truth of his and other corollaries flowing from
 this theory, will more easily appear by expressing
 the foregoing proportions of the velocities in sym-
 bols; to do which let A denote the *area* of the
 circle AB , a the *area* of the hole EF , H the line
 HG , which is the perpendicular height of the water
 in the vessel above the hole, x the height IH , from
 which water or any other body must fall by the
 force

force of gravity from a state of rest, to acquire the velocity of the water in AB, V the velocity of water in its passage through the hole EF, and v its velocity in the surface AB; and then the proportions will be thus expressed, $H + x . x :: V^2 . v^2 :: A^2 . a^2$; whence, $\sqrt{H + x} . \sqrt{x} :: V . v :: A . a$.

COR. I. The height from which a body must fall to acquire a velocity equal to the velocity of the water in the surface AB, is equal to $\frac{v^2 H}{V^2 - v^2}$, or

$\frac{a^2 H}{A^2 - a^2}$. For by inversion and division of proportion, $x . H :: v^2 . V^2 - v^2 :: a^2 . A^2 - a^2$;

whence $x = \frac{v^2 H}{V^2 - v^2} = \frac{a^2 H}{A^2 - a^2}$. But x denotes IH.

And therefore $IH = \frac{V^2 H}{V^2 - v^2} = \frac{a^2 H}{A^2 - a^2}$.

COR. II. The perpendicular height of the water in the vessel, denoted by H , is equal to $\frac{IH \times V^2 - v^2}{v^2}$ or $\frac{IH \times A^2 - a^2}{a^2}$, by Cor. I.

COR. III. The height from which a body must fall, to acquire a velocity equal to that with which the water flows through the hole, is equal to

$\frac{V^2 H}{V^2 - v^2}$, or $\frac{A^2 H}{A^2 - a^2}$. For by division of proportion, $H + x = IG . H :: V^2 . v^2 :: A^2 . A^2 - a^2$, whence $IG = \frac{V^2 H}{V^2 - v^2} = \frac{A^2 H}{A^2 - a^2}$.

COR. IV. The perpendicular height of the water in the vessel, denoted by H , is equal to $\frac{IG \times V^2 - v^2}{v^2}$, or to $\frac{IG \times A^2 - a^2}{A^2}$, by Cor. 3.

COR. V. If the *area* of the surface be equal to the *area* of the hole, H will be nothing in comparison of IH and IG which will be equal. For if A be equal to a, H will be nothing, by Cor. 2. and IH and IG will be equal and infinite, by Cor. 1, and Cor. 3.

The truth of this *Corollary* may likewise appear from the nature of gravity. For if A be equal to a, V must be equal to v. But V can never be equal to v while there is any acceleration of the motion of the water in its descent thro' the vessel, as there will always be till H becomes nothing in comparison of the equal lines IH and IG, which in this case must be considered as infinite.

COR. VI. If a be greater than A, in which case $A^2 - a^2$ will be negative, H will be negative, by Cor. 4; and IG, and consequently V, will be affirmative, by Cor. 3. But a negative perpendicular height of the water in the vessel, and an affirmative velocity of the water flowing through the hole, require an inversion of the vessel or a turning of its bottom upwards; by which inversion the hole will become the upper orifice, and the upper orifice the hole; a will become A, and A become a; and the velocity will be affirmative, that is, the water will move downwards, as it ought to do from the nature of gravity. Farther, when a is greater than A, the vessel will be conical with its wider end downwards; but from the nature of gravity, water poured in at the top or narrower end of such a vessel, will descend in a cylindrical column, which will not fill the base, as the foregoing account of this motion requires; and therefore, to give this case the conditions required, there must be an inversion of the vessel.

COR. VII. If the hole be small, and the surface of the water infinitely large, both a and v may be

be considered as 0 with respect to A and V; consequently IH will be 0, by *Cor. 1.* and IG will be equal to H, by *Cor. 3.*

In this case, and this only, the superficial parts of the water have no velocity at the very beginning of the motion, but begin to descend from a state of rest, as quiescent bodies do when the support is taken away. In all other cases, in which a and v have some magnitudes when compared with A and V, the superficial parts of the water set out with some velocity, and do not begin to descend, on the water's beginning to flow through the hole, as heavy bodies near the surface of the earth begin to descend from a state of rest.

COR. VIII. If the *ratio* of the surface to the hole be given, as it will be when each of them continues the same, or when both of them change in the same proportion; the velocity in the surface will be proportional to the velocity through the hole, and both will be proportional to the velocity which would be acquired by a body in falling through a height equal to the perpendicular height of the water in the vessel. If $\frac{A}{a}$ be given, $\frac{V}{v}$ will be given; and consequently v will be as V. And since $\frac{A}{a}$ is given, $\frac{a^2}{A - a^2}$, and $\frac{A^2}{A^2 - a^2}$, will both be given; and consequently both IH and IG will be as H, by *Cor. 1.* and *Cor. 3.* But v and V are as \sqrt{IH} and \sqrt{IG} . And therefore, both v and V will be as \sqrt{H} .

By this *Corollary*, when A and a continue invARIABLE, and the heights of the water in the vessel are 1, 4, and 16 feet; the velocities in AB and EF will be as 1, 2, and 4. But bodies placed at small distances from the surface of the earth, do all begin to descend with the same velocity very nearly,

B b 3

as

Of the Motion of Water

as has been proved by experiments. And therefore the superficial parts of the water, in this case, begin to descend in a very different manner, or with very different velocities from that with which a heavy body placed at those heights, begins to descend from a state of rest. The velocity in AB is regulated by the velocity in EF, and the velocity in EF is always measured by \sqrt{H} , when $\frac{A}{a}$ is given.

COR. IX. The velocity of the water in the surface AB is always the $\frac{a}{A}$ part of the velocity thro' the hole, that is, v is the $\frac{a}{A}$ part of V , or in other words, $v = \frac{aV}{A}$. When a is nothing in proportion to A , as we may suppose it to be, when a is very small, and A is exceedingly great, then will v be no sensible part of V , that is, it will be nothing; and consequently, the superficial parts of the water will in this case begin their motion, as heavy bodies do, from a state of rest.

COR. X. The whole motion of the descending column AMEFNB, is equal to the motion of a cylinder of water, whose base is a , whose altitude is H , and whose velocity is V , that is, to the motion $aH \times V$. For Va is equal to vA , that is, the motion of the water in EF is equal to its motion in AB; and from the nature of the descending column, each of them is equal to the motion in any section of the column parallel to EF or AB; and consequently, the motion in all the sections, supposing them to be indefinitely many, that is, the whole motion of the descending column, will be equal to the motion in the hole multiplied into the number of

of sections, that is, to $Va \times H$, or $aH \times V$. This property has been proved by Dr. JURIN.

COR. XI. The force which can generate the whole motion of the water running out of the hole, is equal to the weight of a cylinder of water whose base is a , and altitude is $2IG$, by *Cor. 3*; that is, equal to the weight of a cylinder of water, whose magnitude is $2aH \times \frac{A^2}{A^2 - a^2}$. For in the same time, in which the water running out is equal to this cylinder, this cylinder, by falling from the height IG by the force of its gravity, will acquire a velocity equal to that with which the water runs out. But when the quantities of matter and velocities of two bodies are equal, their motions, and consequently the forces which can generate those motions in equal times, will likewise be equal. And therefore the force which can generate the whole motion of the water running out of the hole, is equal to the weight of a cylinder of water whose magnitude is $2aH \times \frac{A^2}{A^2 - a^2}$.

COR. XII. The weight of the descending column $AMEFNB$ is equal to the weight of a cylinder of water, whose base is a , and whose height is $\frac{2HA}{A+a}$, that is, whose magnitude is $2aH \times \frac{A}{A+a}$. For let IO be a mean proportional between IH and IG , and then $\sqrt{IH} \cdot \sqrt{IG} :: IH \cdot IO :: IO \cdot IG :: a \cdot A$; and by division of proportion, $HO \cdot IH :: OG \cdot IO$; and by alternation and composition, $HO + OG \cdot 2HO :: IH + IO \cdot 2IH :: a + A \cdot 2a$. But, by *Cor. 11*. in the time a drop of water falls by its own gravity from I to G , the quantity of water discharged by the hole will be equal to $a \times 2IG$, or $A \times 2IO$; and in the time the drop

B b 4

descends

descends from I to H, the quantity of water passing through the surface AB, and discharged by the hole, will be equal to $A \times 2IH$, and the difference of these quantities, namely $A \times 2HO$, will be the quantity discharged in the time the falling drop descends from H to G, which quantity is the column AMEFNB; for in the time the drop descends from H to G, the superficial parts of the water, setting out with the velocity of the drop at H, and descending freely and without resistance, will reach the hole. And therefore, all the water in the vessel will be to the water in the column AMEFNB, as $A \times H$ is to $A \times 2HO$, or as $H = HO + OG$ to $2HO$; or as $a + A$ to $2a$; whence, putting Q for the quantity of water in the descending column, $A \times H . Q :: A + a . 2a$; and consequently,

$$Q = 2AH \times \frac{A}{A+a}.$$

This *Corollary* may be proved in another manner, thus. The cataract is the difference of the two hyperboloids KAMEFBL and KABL, supposing the affymptote KL to be infinitely extended both ways, and the *area* AB to be infinite; but by fluxions, as Dr. JURIN has shewn, the hyperboloid KAMEFNBL is equal to $2a \times H + x$, or to $\frac{2A^2x}{a}$, because H is equal to $\frac{A^2x - a^2x}{a^2}$ by *Cor. 2*; and the hyperboloid KABL, is equal to $2Ax$, and the difference of the two is $\frac{2A^2x}{a} - 2Ax = \frac{2A^2x - 2Aax}{a}$. All the water in the vessel is AH or, by substituting $\frac{A^2x - a^2x}{a^2}$ in the room of H, $\frac{A^3x - Aa^2x}{a^2}$; and consequently, the water in the vessel is to the water in the cataract, as A^3x

$\frac{A^3x - Aa^2x}{a^2}$ is to $\frac{2A^2x - 2Aax}{a}$, that is, after due reduction, as $A + a$ is to $2a$. Therefore $AH \cdot Q : : A + a \cdot 2a$: whence, $Q = 2aH \times \frac{A}{A + a}$.

COR. XIII. The weight of all the water in the vessel, is to the weight of that part of it which is sustained by the bottom, as the sum of the circles AB and EF is to their difference. For, since $A \times H \cdot Q : : A + a \cdot 2a$, by *Cor. 12.* $A \times H \cdot A \times H - Q : : A + a \cdot A + a - 2a = A - a$ by division of proportion.

COR. XIV. The weight of the water which the bottom sustains is to the weight of the cataract, as the difference of the circles AB and EF , to twice the lesser circle EF . For $A \times H \cdot Q : : A + a \cdot 2a$, by *Cor. 12.* And by division of proportion, $A \times -Q \cdot Q : : A + a - 2a = A - a \cdot 2a$.

COR. XV. The weight of water which the bottom sustains, is to the weight of water perpendicularly incumbent thereon, as the circle AB , is to the sum of the circles AB and EF . For the weight of water which the bottom sustains is $A \times H - Q = AH - \frac{2aHA}{A + a}$, by *Cor. 12.* $= \frac{A^2H - aAH}{A + a}$; and the weight perpendicularly incumbent on the bottom is $A - a \times H = AH - aH$. But $\frac{A^2H - aAH}{A - a} \cdot AH - aH : : A^2 - aA \cdot A^2 - a^2 : : A \cdot A + a$ by dividing by $A - a$.

COR. XVI. The quantity of water in the descending column is to the quantity perpendicularly incumbent on the hole, as twice the circle AB , is to the sum of the circles AB and EF . For the quantity

quantity of water in the descending column is $2aH \times \frac{A}{A+a}$. But $2aH \times \frac{A}{A+a} \cdot aH :: \frac{2A}{A+a} \cdot 1 :: 2A \cdot A + a$.

Hence, when a is nothing, as we may suppose it to be when A is infinitely great, the descending column will be equal in magnitude to $2aH$, as Dr. JURIN has shewn it to be by determining its magnitude by fluxions.

COR. XVII. The weight of the descending column, is to the weight of water which can generate the whole motion of the water running out of the hole, as the difference of the circles AB and EF , is to the greater circle AB . For, putting F for the force or weight which can generate the whole motion of the water running out of the hole, and supposing Q to denote the weight of the descending column, we shall have F equal to the weight of a quantity of water whose magnitude is $2aH \times \frac{A^2}{A^2 - a^2}$, by Cor. 11. and Q equal to the weight of a quantity, whose magnitude is $2aH \times \frac{A}{A+a}$, by Cor. 12. and therefore, $Q \cdot F :: 2aH \times \frac{A}{A+a} \cdot 2aH \times \frac{A^2}{A^2 - a^2} :: 1 \cdot \frac{A}{A-a} :: A - a \cdot A$. Hence $Q = \frac{F + A - a}{a}$, and $F = \frac{QA}{A - a}$; and consequently, the force which can generate the whole motion of the water running out of the hole, will always exceed the weight of the descending column, except when a becomes 0, as we may suppose it to do, when it is very small, and A exceedingly great.

COR. XVIII. The force which can generate the whole motion of the water running out of the hole,

hole, is to the weight of water perpendicularly incumbent on the hole, as twice the square of the greater circle AB, to the difference of the squares of the circles AB and EF. For the force which can generate the whole motion of the water running out of the hole, is the weight of $2aH A \frac{A^2}{A^2 - a^2}$ quantity of water, by Cor. 11. and the weight of water perpendicularly incumbent on the hole, is the weight of the cylinder aH . But $2aH \times \frac{A^2}{A^2 - a^2} : aH :: 2A^2 : A^2 - a^2$. In the same ratio is the whole motion of the effluent water to the motion of the water in the cataract.

COR. XIX. If in the middle of the hole be placed a little circle PQ parallel to the horizon, whose center is G, and if the area of this circle be called o ; the weight of water which it sustains during the efflux of the water through the ring surrounding it, is to the weight of half the cylinder oH , as a to $a - \frac{1}{2}o$; if R denote the weight sustained, R is to $\frac{oH}{2}$, as a to $a - \frac{1}{2}o$, and R is equal to $\frac{aoH}{2a - o}$. For if we suppose A to be contracted till it becomes equal to a , in which case IH will be infinite, by Cor. 1. the water, notwithstanding this, will descend about the column PQH which the little circle sustains with velocities, which are every where in the subduplicate ratio of the distance from KL, and likewise in the reciprocal ratio of the several sections through which it passes; consequently, the cataract AEPHQFB, is equal to the difference of the two hyperboloids PEAKLBFQH and AKLB. But the hyperboloid PEAKLBFQH $= 2a - 2o \times H + x = 2aH - 2oH + 2ax - 2ox$; and the hyperboloid AKLB is $2ax$; and the difference

Pl. 11.
Fig. 3.

rence of the two is $2aH - 2oH - 2ox$, which is the cataract $AEPHQFB$. The *ratio* of all the water in the vessel to this annular cataract, is

$\frac{aH}{2aH - 2oH - 2ox}$. But from the nature of the mo-

tion of the descending water, a is to $a - o$, as

$\sqrt{H + x}$. \sqrt{x} , whence $H = \frac{2aox - oox}{a^2 - 2ao + oo}$. The

foregoing *ratio*, when this value of H is substituted

in its room, will, after due reduction, become $\frac{2a - o}{2a - 2o}$.

Therefore aH , the whole quantity of water in the vessel, is to the annular cataract, as $2a - o$ to $2a - 2o$;

whence the annular cataract is $\frac{2a^2H - 2aoH}{2a - o}$, which

being subducted from aH , leaves $\frac{aoH}{2a - o}$ for the

quantity sustained by the little circle o . Conse-

quently, $R = \frac{aoH}{2a - o}$; and $R. \frac{oH}{2} : : a. a - \frac{1}{2} o$.

S C H O L I U M.

Upon examining this motion by experiments, Sir ISAAC NEWTON found the velocity of the water in its passage through the hole to be less than it ought to be, if the water in the vessel descended from the surface to the hole freely and without resistance, in the proportion of 1 to $\sqrt{2}$. For he observed the vein of the effluent water, and found it to contract and grow narrower, to the distance of about a diameter of the hole below it, at which place he measured the diameter of the vein, and found it to be less than the diameter of the hole in the proportion of 21 to 25, and consequently, the *area* of a section of the vein at that place to be less than the *area* of the hole, in the proportion of 441 to

to 625, that is, of 1 to $\sqrt{2}$. But as the vein contracts the velocity increases. And therefore, at the distance of a diameter of the hole below it, the velocity will be greater than in the hole in the proportion of $\sqrt{2}$ to 1. If IG be four feet or 48 inches, and the diameter of the hole be 1 inch, 1 added to 48 will make the height from the place where the velocity is greatest to be 49 inches; and if the velocities of the descending column in the hole and that place, were truly measured by the subduplicate ratios of those heights, as they would be if the water descended freely and without resistance, they would be nearly equal, being as the numbers 69 and 70. And therefore, the velocity of the water in the hole is less than it would be if it was proportional to \sqrt{IG} , in the ratio of 1 to $\sqrt{2}$. This diminution of velocity can be owing to nothing but the lateral motion of the descending water, retarding its perpendicular motion downwards, and making it less than it otherwise would be, in the said ratio of 1 to $\sqrt{2}$. Hence, the velocity with which the water flows through the hole is very nearly equal to the velocity which a body, by falling freely and without resistance from a state of rest at I, would acquire in descending through $\frac{1}{2}IG$. For the velocity acquired in falling through $\frac{1}{2}IG$, is to the velocity acquired in falling through IG, as 1 to $\sqrt{2}$.

Pl. 11.

Fig. 2.

According to Sir ISAAC NEWTON, a body falling *in vacuo* from a small height above the surface of the earth, will describe $193\frac{1}{3}$ inches, or $16\frac{2}{3}$ feet in one second minute of time, and will have acquired a velocity at the end of the fall, which being continued uniform, would carry it through twice that space, that is, $386\frac{2}{3}$ inches or $32\frac{2}{3}$ feet, in an equal time. But uniform velocities are as the spaces described by them in the same time, and the velocities acquired by a body falling *in vacuo* through the

the spaces $16\frac{1}{2}$, and GI or $\frac{A^2H}{A^2-a^2}$, are in the subduplicate *ratios* of those spaces; and therefore $32\frac{2}{3} \cdot V :: \sqrt{16\frac{1}{2}} \cdot \sqrt{\frac{A^2H}{A^2-a^2}}$. Whence, $V = 8.02773$

$\sqrt{\frac{A^2H}{A^2-a^2}}$ feet, $= 96.33276 \sqrt{\frac{A^2H}{A^2-a^2}}$ inches. And

lessening these measures of the velocity of the water flowing through the hole, the *ratio* of 1 to $\sqrt{2}$, that is, dividing each by 1.414, we shall have $V = 5.6773196$

$\sqrt{\frac{A^2H}{A^2-a^2}}$ feet, $= 68.1278352 \sqrt{\frac{A^2H}{A^2-a^2}}$ inches.

These are the true measures of the velocity of the water in its passage through the whole; which velocity is therefore such as carries it at the rate

of $5.6773196 \sqrt{\frac{A^2H}{A^2-a^2}}$ feet, or 68.1278352

$\sqrt{\frac{A^2H}{A^2-a^2}}$ inches, in a second minute of time.

These expressions may be shortened, if A be considerably greater than a , for in all such cases $\frac{A^2H}{A^2-a^2}$

will be so nearly equal to H , that $\frac{A^2}{A^2-a^2}$ may be

safely rejected; and then the foregoing measures of the velocity will become $5.6773196 \sqrt{H}$ feet, or $68.1279352 \sqrt{H}$ inches. To shew the truth of this by an example, let A be 100 square inches, and

a 1 square inch, and then $\frac{A^2H}{A^2-a^2}$ will be $\frac{10000H}{9999}$;

if H be four feet or 48 inches, $\frac{10000H}{9999}$ will be

48.0048 inches, which is only greater than 48 by 48 parts of an inch divided into 10000. The excess is so small, that it may be safely rejected.

Another true measure of the velocity of the water flowing through the hole, will be had by dividing the quantity of water discharged, by the *area* of the
hole

hole and time of the discharge, taken together; the quantity of water discharged being expressed in cubick inches, the *area* of the hole in square inches or parts of a square inch, and the time of the discharge in seconds. Let Q denote the quantity discharged, d the diameter of the hole, and t the time of the discharge, and then V will be measured by $\frac{Q}{at} = \frac{Q}{0.78539816d^2t}$ inches, which will be the space described in one second of the time.

This measure is equal to the former, that is, $\frac{Q}{0.78539816d^2t} = 68.1278352 \sqrt{H}$; and consequently, $Q = 53.5074764d^2t\sqrt{H}$ cubick inches; or $13555.227d^2t\sqrt{H}$ grains; because a cubick inch of water weighs $253\frac{1}{3}$ grains. If W denote the weight of water discharged, then will $W = 13555.32d^2t\sqrt{H}$ grains.

In order to know, whether the velocities of water flowing through circular holes of different diameters, when placed at the same perpendicular distance from the surface of the water, be all equal; what relation the velocity of water flowing through a hole, bears to the velocity of water flowing through an horizontal pipe of an equal diameter, inserted into the side of a vessel at an equal perpendicular distance from the surface of the water; and under what circumstances the measure of the velocity laid down in my *Animal Economy* obtains; I say, in order to know these things, I caused a proper *apparatus* to be made, and from the experiments made with it, I composed the following Tables.

TABLE

TABLE I.

t	H	d	W	w	$\frac{w}{W}$
10	4	$\frac{1}{10}$	2711	2944	1086
		$\frac{4}{10}$	43377	47040	1084
		$\frac{5}{10}$	67776	72960	1076
		$\frac{8}{10}$	173507	178560	1029
		$\frac{1}{10}$	1917	2087	1088
	2	$\frac{4}{10}$	30672	33600	1095
		$\frac{5}{10}$	47925	51840	1082
		$\frac{8}{10}$	122688	128400	1046

TABLE II.

d	l	w	d	l	w	d	l	w
$\frac{2}{10}$	0	12736	$\frac{4}{10}$	0	47040	$\frac{8}{10}$	0	178560
	d	14385		d	54720		d	204720
	2d	14400		2d	56160		2d	224640
	3d	13792		3d	52800		3d	217440
	4d	13728		4d	52220		4d	212160
	5d	13663		5d	51600		5d	203520
	10d	12683		10d	47040		16d	188160
							23d	178560

The first Table contains, in the first column, under *t*, the time of the discharge in seconds; in the second column, under *H*, the perpendicular heights of the water above the hole in *London* feet; in the third, the diameters of the hole in parts of an inch; in the fourth, under *W*, the weights of water in grains, which ought to have been discharged by the theory or foregoing rule; in the fifth, under *w*, the weights of water in grains which were discharged by experiment, each weight being a mean taken from five or six experiments; and in the sixth column, under $\frac{w}{W}$, the ratio of the weight discharged by experiment,

periment to the weight which ought to have been discharged by the theory.

The second Table consists of three parts, and each part of three columns. The first column of each part contains the diameter of the pipe in parts of an inch; the second contains the lengths of the pipe in the terms of the diameter, beginning with the hole, which may be considered as a pipe of an infinitely small length expressed by 0; and the third column contains the weights in grains discharged in ten seconds, each weight being a mean taken from particular experiments. The holes and pipes were all at the perpendicular distance of four feet from the surface of the water, so that here t was 10 seconds, and H four feet.

TABLE III.

d	H	l	W	w	$\frac{w}{W}$	H	W	w	$\frac{w}{W}$
$\frac{1}{8}$	2	1	2180	2180	1000	$\frac{1}{2}$	1090	982	901
		2	1541	2080	1349		770	922	1196
		3	1258	2057	1634		629	877	1393
		4	1090	1874	1719		545	762	1398
		5	980	1759	1804		490	720	1469
		6	890	1690	1899		445	665	1494
		7	824	1564	1898		412	620	1505
		8	770	1520	1972		385	585	1519
		9	727	1440	1982		363	553	1522
		10	689	1410	2045		344	525	1523
		12	629	1320	2098		314	470	1493
		14	582	1225	2102		291	430	1476
		16	545	1163	2134		272	383	1405
		18	514	1086	2113		257	350	1362
		20	487	1030	2113		243	320	1313
		24	445	866	1946		222	260	1168
		25	436	860	1972		218	253	1160
		28	412	844	2048		206	230	1116
		32	385	758	1967		192	202	1048
		36	363	659	1814		181	185	1018
		48	314	509	1618				
		60	281	421	1496				
		72	257	345	1342				

C c

TABLE

TABLE IV.

d	H	l	W	w	$\frac{w}{W}$	H	W	w	$\frac{w}{W}$
$\frac{2}{10}$	2	1	12332	10040	814	$\frac{1}{2}$	6166	5018	814
		2	8720	9270	1063		4360	4630	1062
		3	7120	8820	1238		3560	4400	1235
		4	6166	8570	1389		3083	4270	1385
		5	5515	8240	1494		2758	4040	1465
		6	5034	7840	1557		2517	3880	1541
		7	4661	7580	1626		2330	3766	1616
		8	4360	7360	1688		2180	3668	1682
		9	4111	7150	1739		2055	3570	1737
		10	3900	6950	1782		1950	3414	1751
		16	3083	5776	1873		1541	2955	1918
		25	2466	4785	1940		1233	2460	1995
		36	2055	4048	1970		1027	2120	2064
		49	1762	3480	1975		881	1730	1963
		64	1542	3250	2108		771	1326	1720
		81	1370	3062	2235		685	1120	1635
		97	1252	2700	2156		626	940	1502

The third and fourth Tables consist each of two parts corresponding to different perpendicular heights of the water in the vessel, and different diameters of the pipes. In both Tables, H denotes the perpendicular height of the water in the vessel above the pipe in feet; l the length of the pipe in inches; W the weight in grains which ought to be discharged by the first *Proposition* of my *Animal Economy*; w the weight in grains which was discharged by experiment; and $\frac{w}{W}$ the ratio of the weight discharged by experiment to the weight which ought to have been discharged by that *Proposition*. The diameter of all the pipes in the third Table was $\frac{1}{10}$ of an inch, and of all the pipes in the fourth Table $\frac{2}{10}$ of

of an inch. And the time of the discharge was 10 seconds in all the experiments of both Tables.

The quantity or weight of water which ought to be discharged by the first *Proposition* of the *Animal Economy*, may be thus found. I there proved, that the velocity of water flowing thro' a pipe, is as $\sqrt{\frac{F}{dl}}$. But if the force which can generate the

motion of water flowing through a pipe lying parallel to the horizon, be equal to the force which can generate the motion of water flowing through a hole of an equal diameter with the pipe, when placed at an equal perpendicular distance from the surface of the water; F , by *Cor. 11.* of this *Problem*, will be as $2d^2H$, on supposition that the *area* of the hole is extremely small in comparison of the *area* of the surface of the water. And therefore the velocity of water flowing through a pipe lying parallel

to the horizon, is as $\sqrt{\frac{2dH}{1}}$. The weight of water discharged, is as the orifice of the pipe, the time of the discharge, and velocity, taken together; that is, as $d^2t\sqrt{\frac{2dH}{1}}$. And therefore, W is as

$$d^2t\sqrt{\frac{2dH}{1}}.$$

A pipe of $\frac{1}{8}$ of an inch in diameter, and 1 inch in length, discharged 2180 grains of water in 10 seconds, when it was inserted into the side of the vessel at the perpendicular distance of two feet from the surface. In this case therefore, d , t , H , l , were

0.1, 10, 2, 1; and $d^2t\sqrt{\frac{2dH}{1}}$ was equal to 0.06326.

Hence we may find W in other cases by this analogy; $2180 : 0.06325 :: W : d^2t\sqrt{\frac{2dH}{1}}$; whence

$$W = 48746.3d^2t\sqrt{\frac{dH}{1}} \text{ grains.}$$

In the first part of the third Table, W is $\frac{2180}{\sqrt{1}}$,
 and in the first part of the fourth Table, $\frac{12332}{\sqrt{1}}$,
 and W in the second part of each Table is one half
 of W in the first part.

OBSERVATIONS on the TABLES.

OBS. I. By the first Table the discharges by experiment are nearly proportional to the discharges by the theory, that is, w is nearly proportional to W , or $\frac{w}{W}$ is nearly the same, whatever be the diameter of the hole, provided the time of the discharge, and the perpendicular height of the water in the vessel above the hole, be given. The discharges by experiment were all sometimes larger than the discharges by the theory, which might be partly owing to the pouring in of the water at the top of the vessel, in order to keep the vessel constantly full during the time of the discharge; for the pouring, tho' it was done gently, might a little increase the velocity wherewith the water ran out of the hole.

OBS. II. By the second Table, the weight of water discharged, and consequently the velocity, increases from the hole till the length of the pipe becomes equal to about twice its diameter, that is, till l becomes equal to about $2d$, and is greater there than at any other length of the pipe. The greatest velocities in these pipes in proportion to the velocities through their respective holes, are as the numbers 1130, 1258 to 1000.

OBS. III. From the length of twice the diameter, that is from the length $2d$, the velocity lessens continually on increasing the length of the pipe, and becomes equal to the velocity through the hole when the length of the pipe becomes equal to about $22.3657d\sqrt{d}$ inches. For, by the second Table,
 the

the velocities of the water flowing through the pipes, were nearly equal to the velocities through their respective holes, when the lengths of the pipes were 10d, 16d and 23d, that is 2 inches, 6.4 inches, and 18.4 inches. But 2, 6.4, and 18.4, are nearly as 1, 2.8, and 8, the sesquiplicate *ratios* of 1, 2 and 4, and 1, 2 and 4, are as the diameters $\frac{2}{10}$, $\frac{4}{10}$ and $\frac{8}{10}$. And therefore the velocities of the water flowing through the pipes, were nearly equal to the velocities through their respective holes, when the lengths of the pipes were in the sesquiplicate *ratios* of their diameters. The diameter of the smallest pipe being $\frac{2}{10}$ of an inch, $d\sqrt{d}$ is 0.0894; and if d be of any other magnitude, and l be the length of a pipe of that diameter through which the water flows with a velocity equal to that with which it flows through its corresponding hole, we shall have this proportion; as 2 is to 0.0894, so is l to $d\sqrt{d}$, whence $l = 22.3657d\sqrt{d}$.

Obs. IV. By the third and fourth Tables, the quantity of water discharged by experiment in proportion to the quantity which ought to have been discharged by the theory, that is $\frac{w}{W}$, increases gradually till the pipe comes to be of a certain length, and after that it decreases gradually on increasing the length of the pipe. In the two parts of the third Table this *ratio* was greatest, when the lengths of the pipes in inches were about 20 and 10, and it was greatest in the two parts of the fourth Table, when the lengths of the pipes were 81 and 36. But from the course of the numbers expressing $\frac{w}{W}$ in the second part of the fourth Table, I think this *ratio* would have been greater in a pipe of 40 inches in length, than in the one I used of 36, and therefore shall suppose that it would have been greatest at the lengths of 81 and 40. Consequently, putting x for the length of the pipe in inches, at which

C c 3

this

this *ratio* is greatest, x will be as \sqrt{H} when d is given, and as d^2 when H is given, and when neither d nor H is given, as $d^2\sqrt{H}$. Hence we may form a rule for finding the length of the pipe, at which this *ratio* shall be a *maximum*; for it was a *maximum* in a pipe of $\frac{1}{10}$ th of an inch in diameter, when its length was 20 inches, and the perpendicular height of the water in the vessel two feet. In this case therefore x , d and H , are 20, 0.1, and 2, and $d^2\sqrt{H}$ is 0.01414; and in other cases, x may be found by this analogy; as 20 is to 0.01414, so is x to $d^2\sqrt{H}$; whence x is equal to $1414d^2\sqrt{H}$. To see whether this rule be universal, and obtain in pipes of greater diameters, and at greater distances from the surface of the water, I shall suppose d and H to be 0.5 and 3, as in our Author's Table p. 227, and then $1414d^2\sqrt{H}$ will be about 600 inches or 50 feet, which length is twice as great as it was in reality; for the *ratio* was a *maximum* by that Table, when the length of the pipe was 25 feet; so that the value of x here determined seems to obtain only in pipes of small diameters.

Obs. V. By the third and fourth Tables, the quantity discharged by experiment in proportion to the quantity which ought to have been discharged by the theory, that is $\frac{w}{W}$, does not differ much in pipes whose lengths are within certain limits. $\frac{w}{W}$, in the pipes, whose lengths were 6 and 32 in the first part of the third Table, is less than in the pipe where this *ratio* is a *maximum*, in the proportions of 100 to 112 and 110, and the difference of $\frac{w}{W}$ and the *maximum* is still less in pipes of all other lengths between 6 and 32; so that in this part of the Table, 6 and 32 are the limits, at and within which there is a near agreement between

tween theory and experiment. $\frac{w}{W}$ in the pipes whose lengths are 4 and 16 in the second part of this Table, is less than in the pipe where this *ratio* is a *maximum*, in the proportion of about 100 to 109 and 108, and it is still less in pipes of all other lengths within these limits. And $\frac{w}{W}$ in the pipes whose lengths are 9 and 64 in the second part of the fourth Table, for the pipes were not carried to such lengths as were necessary to settle the limits in the first part, is less than the *maximum* in the proportion of 100 to 118 and 120; and it is still less in pipes of all other lengths within these limits.

Obs. VI. By the third and fourth Tables, the quantity of water discharged by experiment always exceeds the quantity which ought to be discharged by the theory; it was near double within the limits of the first part of the third Table and second part of the fourth, and greater in the second part of the third Table in the proportion of 5713 to 3858. If we suppose it to be double within the limits, in pipes of all lengths, then will w be equal to $2W$, or to $97492.6d^2t\sqrt{\frac{dH}{l}}$ grains, W being equal to $48746.3d^2t\sqrt{\frac{dH}{l}}$ grains, as was shewn above.

Of the Foci of Optick Glasses.

PROB. VI. *If the distance of an object from a double convex lens whose surfaces are spherical; if the radii of both the spherical surfaces, the thickness of the lens, and the sines of incidence and refraction, be all given; thence to determine the distance behind the lens of the principal focus or concurrence of the rays issuing from the object and falling perpendicularly, or very nearly so, on that surface of the lens which is turned towards the object.*

Let MN be a lens, E and e the centers of its spherical surfaces MCN and MDN , Q an object placed

C c 4

Pl. 11.
Fig. 5.

placed directly before the *lens*, Qq a line drawn from the object perpendicular to the surfaces of the *lens*, and consequently passing through the centers e and E; let the point A be indefinitely near to C, in which case QA and QC may be looked upon as equal; let q be the *focus* or concourse of the rays QA and QC after the first refraction by the surface MCN, and z their *focus* or concourse after the second refraction by the surface MDN. Put D for QC the distance of the object from the *lens*, r for the *radius* CE, ρ for the *radius* eD, x for Dq, the distance of the *focus* behind the *lens*, after the first refraction, and z for Dz its distance behind the *lens* after the second refraction; and lastly, let I and R denote the sines of incidence and refraction of the rays passing out of air or any other *medium* into the first surface MCN, and consequently R and I the sines of incidence and refraction in their passage out of the second surface MDN into air or that other *medium*.

Pl. 11.
Fig. 4.

To determine z, we must first determine the measure of x in known terms, to do which draw AF perpendicular to Qq, EI perpendicular to QAI the incident ray produced, and ER perpendicular to the refracted ray Aq; and then, from the similarity of the two triangles QAF and QIE, and also of the triangles qAF and qER, and from QA being equal to QC, and qA equal to qC, we shall have AF equal to $\frac{QC \times EI}{QE}$, or, in symbols, to $\frac{DI}{D+r}$, by the two first triangles, and by the two last triangles, equal to $\frac{Cq \times ER}{Eq}$, or, in symbols, to $\frac{Rx}{x-r}$.

Consequently, $\frac{DI}{D+r}$, is equal to $\frac{Rx}{x-r}$ and $x =$

$$\frac{DIr}{DI - DR - Rr}.$$

Pl. 11.
Fig. 5.

Having found the measure of x or Cq in known terms, z or Dz may be thus determined. For that measure,

measure, that is for $\frac{DIr}{DI-DR-Rr}$, put A, to za and qa produced draw the perpendiculars eI and eR, and draw am perpendicular to Qq. And then, from the similarity of the triangles qam and qRe, and also of the triangles zam and zIe, and from qa being equal to qD, and za equal to zD am will be equal to $\frac{eR \times qD}{qe}$ from the first triangles, and equal to

$\frac{eI \times zD}{ze}$ from the second. eR is the sine of incidence of the ray Aa falling on the second surface MDN, and eI the sine of its refraction; and therefore eR will be I, and eI will be R. qD is equal to qC - CD = A - t, putting t for CD the thickness of the lens; and qe is equal to qC + eC = qC + eD - CD = A + p - t; consequently

$$\frac{R \times A - t}{A + p - t} = \frac{Iz}{z + p}$$

and from this equation, $A = \frac{Iz + Rt_z + Rt_p - It_z}{Rz + R_p - Iz}$. But A denotes $\frac{DIr}{DI-DR-Rr}$.

And therefore $\frac{Iz + Rt_z + Rt_p - It_z}{Rz + R_p - Iz} = \frac{DIr}{DI-DR-Rr}$.

By clearing z in this equation, we shall have $z = \frac{DIR_p + RR_{rt} + DRR_{pt} - DIR_{pt}}{DIr_p + zDIR_t - DR_{ht} - DIR_p - DR_{rt} - IR_{rt} - RR_{rt} + IR_{rt} - DIr_t + DIr_r}$.

To give this equation a more simple form, divide both numerator and denominator by $1-R$;

and then the numerator will become $\frac{R}{1-R} \times IDr_p$

$+ \frac{R}{1-R} \times Rr_{pt} - DR_{pt}$, or by putting B instead of

$\frac{R}{1-R}$, $BIDr_p + BRr_{pt} - DR_{pt}$, and the denomi-

nator will become $IDr + ID_p - ID_t + RD_t + Rrt - BI_r$; and the equation will be reduced to another form, and stand thus;

$$z = \frac{BIDr_p + BRr_{pt} - DR_{pt}}{IDr + ID_p - ID_t + RD_t + Rrt - BI_r}$$

This

Of the Foci of Optick Glasses.

This is Dr. HALLEY's universal *Theorem* for finding the principal *focus* of rays falling diverging on a double *convex lens*, published in the *Philosophical Transactions*.

If the rays instead of falling diverging, fall parallel on a double *convex lens*, as they will nearly do, when the object is at an immense distance from the *lens*, D in this case may be considered as infinite; and consequently, all the terms in which D is not found, may be thrown out of the equation,

$$\text{and then } z = \frac{\text{BIDr}_e - \text{DR}_{et}}{\text{IDR} + \text{ID}_e - \text{IDt} + \text{RDt}} = \frac{\text{BIR}_e - \text{R}_{et}}{\text{Ir} + \text{I}_e - \text{It} + \text{Rt}}.$$

And lastly, if the rays fall converging on a double *convex lens*, the signs of all the terms in which D is found must be changed; for when the rays fall converging, the point behind the *lens* to which they tend at their incidence, must be considered as the place of the object, which, from its being differently situated with respect to the *lens* from what it is when the rays fall diverging, requires the signs of all the terms in which D is found to be changed, which being done, we shall have

$$z = \frac{\text{DR}_{et} + \text{BR}_{et} - \text{BIDr}_e}{\text{IDt} - \text{IDr} - \text{ID}_e - \text{RDt} + \text{Rrt} - \text{BIR}_e}.$$

These are the three general *Theorems* for finding the principal *focus* of rays falling, diverging, parallel, or converging on a double *convex lens*.

If the *lens* be made of glass, as *lenses* usually are, and the object be placed in air, then, since the sine of incidence of a ray passing out of air into glass, is to the sine of refraction, as 3 to 2, I, R and B will be 3, 2, and 2; and the foregoing general *Theorems* for finding the *foci* of rays falling diverging, parallel, and converging on a double *convex*

$$\text{glass, will be } z = \frac{6\text{Dr}_e + 4\text{r}_{et} - 2\text{DR}_{et}}{3\text{Dr} + 3\text{D}_e - 3\text{Dt} + 2\text{Dt} + 2\text{rt} - 6\text{r}_e},$$

$$z = \frac{6\text{r}_e - 2\text{r}_{et}}{3\text{r} + 3\text{e} - 3\text{t} + 2\text{t}}, \text{ and}$$

$$z =$$

$$z = \frac{2Dpt + 4rpt - 6Drp}{3Dt - 3Dr - 3Dp - 2Dt + 2rt - 6rp}.$$

And if the *radii* be equal, and the thickness of the glass be neglected, or considered as 0, then will these *Theorems* stand thus, $z = \frac{Dr}{D-r}$, $z = r$, and

$$z = \frac{-Dr}{-D-r}.$$

If the *lens* be a double concave glass, the *radii* of whose two spherical surfaces are equal, and if the thickness of the *lens* be considered as 0, the *radii* will lye on different sides of the *lens* with respect to the object from what they did before, and consequently, the signs of the *radii* must be changed; and then the last *Theorems*, in which the *radii* were supposed to be equal, and the thickness of the glass was neglected or considered as 0, will stand thus,

$$z = \frac{-Dr}{D+R}, z = -r, \text{ and } z = \frac{Dr}{r-D}.$$

By these *Theorems*, z is always negative when the rays fall upon the double concave, diverging, or parallel, and when they fall converging it is negative when D is greater than r . When z is negative, the *focus* falls on the same side of the glass with the object, contrary to what it does in all cases of a double convex *lens*, excepting that of diverging rays, when the distance of the object is less than the *radius*, or D is less than r . For in that case, z , which is equal to

$$\frac{Dr}{D-r}, \text{ will be negative.}$$

By this *Problem* we may determine how far a radiating point must be distant from the eye, to have the principal *focus* of the rays issuing from it placed in the *retina*, on supposition that the coats and humours of the eye are unchangeable as to their figures, magnitudes, and densities.

Let $ABGz$ represent a human eye, in which ABG is the *cornea*, $AMCNGB$ the cavity containing the aqueous humour, $MCND$ the crystalline humour, and

Pl. 11.
Fig. 6.

and AMDNGz the cavity containing the vitreous humour. According to Doctor JURIN, the *radii* of the spherical surfaces of the *cornea* and of the crystalline humour, that is, of the spherical surfaces ABG, MCN, and MDN, are in 10th parts of an inch, 3.3294, 3.3081, and 3.5056; and the distance of the *cornea* from the anterior part of the crystalline, the thickness of the crystalline, the distance of the posterior part of the crystalline from the *retina*, and the distance of the *cornea* from the *retina*, are in the same parts of an inch, 1.0358, 1.8525, 6.2617 and 9.15. Let Q be the radiating point, q the principal *focus* of the rays by the first refraction of the aqueous humour, by virtue of which refraction they fall converging on the crystalline, and let z be their *focus* after their refractions by the crystalline and vitreous humours. By taking the specific gravities of the humours of the eye, I have found that the specific gravities of the aqueous and vitreous humours are very nearly equal, and each much the same with that of water; and that the specific gravity of the crystalline is greater than the specific gravity of water, in the proportion of about 11 to 10. For the mean specific gravities of five crystalline humours of oxen's eyes, and of three crystalline humours of sheeps eyes, were 11134 and 11033, the specific gravity of water being 10000, and the mean of these two means, is 11083, which I shall suppose to be the specific gravity of the crystalline humour of a human eye. But the refractive power of the crystalline is very nearly proportional to it's density, and the sine of incidence of rays passing out of the aqueous humour into the crystalline, is to the sine of refraction, very nearly as 21 to 20, as I shall shew in the *Scholium*. And consequently, I will be 21, and R will be 20. From these measures I now proceed to determine the distance of a radiating point from the *cornea*, that is, the distance of Q from B, so as that the

focus

focus of the rays, issuing from it and falling diverging on the *cornea*, may by the refractive powers of the aqueous, crystalline, and vitreous humours, be placed in the *retina* at *z*. By the refraction of the aqueous humour, the rays fall on the crystalline with such a degree of convergence as would make them unite at *q*. In the universal *Theorems* therefore for finding the principal *focus* of rays falling converging on a double convex *lens*, *Cq* is *D*, *Dz* equal to 6.2617 is *z*, the *radius* of *MCN* is *r*, the *radius* of *MDN* is *p*, *CD* the thickness of the crystalline is *t*, and *I* and *R* are 21 and 20. And by clearing *D* in that *Theorem*, we shall have $D = \frac{BIrpz + BRpt - Rrtz}{BIrp + Irz - Irz - Ipz - Rpt - Rtz} = 10.3102 = Cq$. And $Cq + BC = 11.346 = Bq$.

In the *Theorem* for finding *x*, *Bq* is *x*, *QB* is *D*, *I* is 4, *R* 3, the sine of incidence of rays passing out of air into water or into the aqueous humour, being to the sine of refraction, as 4 to 3, and the *radius* of the *cornea* is 3.3294 10th parts of an inch; consequently, *D* is 57.48, that is, about 5 inches and 3 quarters. So that supposing the eye to be unchangeable, a radiating point placed at the distance of $5\frac{3}{4}$ inches from it, will have its image placed in the *retina*.

SCHOLIUM.

“ Let *AB* represent the refracting plane surface
 “ of any body, and *IC* a ray incident obliquely on
 “ the body at *C*, so that the angle *ACI* may be
 “ infinitely little, and let *CR* be the refracted ray.
 “ From a given point *B* perpendicular to the re-
 “ fracting surface erect *BR* meeting the refracted
 “ ray *CR* in *R*, and if *CR* represent the motion of the
 “ refracted ray, and this motion be distinguished in-
 “ to two motions *CB* and *BR*, whereof *CB* is parallel
 “ to the refracting plane, and *BR* perpendicular to
 “ it: *CB* shall represent the motion of the incident
 “ ray,

Pl. 11.
Fig. 7.

“ ray, and BR the motion generated by the refraction.” NEWT. *Opt. Prop.* 10. p. 245, 246. CBR is equal to the angle of incidence, and CRB is equal to the angle of refraction; consequently, if R be made the center, and a circle be supposed to be drawn with the *radius* CR, CR will be the sine of the angle of incidence, and CB the sine of the angle of refraction; and, putting I and R for those sines, we shall have this analogy, $I : R :: CR : CB$. Hence $\frac{I^2 - R^2}{R^2} = \frac{CR^2 - CB^2}{CB^2}$, or $\frac{I^2}{R^2} - 1 = \frac{BR^2}{CB^2}$. But the motion of the ray at its incidence represented by CB, is given; and therefore, $\frac{I^2}{R^2} - 1$ is as BR^2 .

But by the aforesaid proposition BR^2 expresses the refractive force, and is nearly as the density of the body; as Sir I. NEWTON found, by computing BR^2 from the sines I and R in several bodies, and then comparing it with their respective densities. And consequently, putting D for the density of the body, $\frac{I^2}{R^2} - 1$ is as D, and $\frac{I}{R}$ as $\sqrt{D + 1}$. In passing

out of air into water $\frac{I}{R}$ is $\frac{4}{3}$, and, the density of water being 10000, $\sqrt{D + 1}$ is 10004: And in passing out of air into the crystalline, whose density is to that of water as 11083 to 10000, $\sqrt{D + 1}$ is 10528. Therefore in passing out of air into the crystalline $\frac{I}{R}$ will be $\frac{7}{5}$; for $10004 \cdot 10528 :: \frac{4}{3}$;

$\frac{4 \cdot 2 \cdot 1 \cdot 1 \cdot 2}{3 \cdot 5 \cdot 0 \cdot 1 \cdot 2} = \frac{7}{5}$ very nearly. $\frac{I}{R}$ in passing out of the aqueous humour into the crystalline, will be compounded of the *ratio* $\frac{3}{4}$ and $\frac{7}{5}$ by the second *Theorem* of the *Opticks*, p. 113; and therefore $\frac{I}{R}$ will be equal to $\frac{21}{20}$; or I will be to R, as 21 to 20.

F I N I S.



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